PREFACE

Alonzo Church was undeniably one of the intellectual giants of the Twentieth Century. These articles are dedicated to his memory and illustrate the tremendous importance his ideas have had in logic, mathematics, computer science and philosophy. Discussions of some of these various contributions have appeared in *The Bulletin of Symbolic Logic*, and the interested reader is invited to seek details there. Here we just try to give some general sense of the scope, depth, and value of his work.

Church is perhaps best known for the theorem, appropriately called "Church's Theorem", that there is no decision procedure for the logical validity of formulas of first-order logic. A decision procedure for that part of logic would have come near to fulfilling Leibniz's dream of a calculus that could be mechanically used to settle logical disputes. It was not to be. It could not be. What Church proved precisely is that there is no lambda-definable function that can in every case provide the right answer, 'yes' or 'no', to the question of whether or not any arbitrarily given formula is valid. To draw the more sweeping conclusion, that there is no "mechanical" procedure, no "algorithm", no "effectively calculable" way of deciding the question requires an identification the class of formally defined methods, with those methods characterized in these more informal ways. The proposal, the hypothesis or conjecture, that this identification is correct is "Church's Thesis".

"Lambda-definability" is a notion defined using another of Church's important contributions—the Lambda Calculus. Church began the construction of this calculus in the hope of showing that Gödel's Incompleteness Theorems are somehow not as conclusive as they seem. The available analyses of the notion of a correct proof all made use of some calculus or other. Yet Gödel had apparently shown that every such formal system, adequate for a certain portion of mathematics, will fail to capture some correct methods of proof and cannot be shown to be consistent except by using assumptions stronger than those it endorses. For Church this must have seemed intolerable. How then can we make the notion of proof precise? Must not we be able to at least give a convincing consistency proof for a formal system adequate for mathematics?

The Lambda Calculus was hoped to be a system that can be proved to be consistent (which it can and has been—by the "Church-Rosser Theorem") and yet somehow escape, or at least mitigate, the limitations on formal systems Gödel's Incompleteness Theorems seemed to impose. (I heard Church

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say, in later years, that once you understand Gödel's Theorem, it's obvious (!) really that it is correct and there is no escape. But he also said his endeavor was not entirely in vain for it resulted in the Lambda Calculus. This calculus was and continues to be of enormous importance in computer science. So here even Church's failure was a kind of success.) Using the Lambda Calculus and a definition of "lambda-definability" in terms of it, Church proposed to identify this notion or (better) its extension with that of "effectively computable function" and thereby provide an analysis, or definition, of this latter, informal notion. Church's Thesis is certainly not obviously correct. It seems quite amazing that on the basis of a certain amount of experimentation in defining intuitively computable functions, Church's intuition told him that *all* had been captured. Later work by Turing and Post would provide more intuitively accessible characterizations of effective computability, but it is now virtually universally agreed that Church was correct and that the Thesis is true (whatever exactly that may mean!). This much, his proposed analysis of effective computability and his proof that first-order logic does not yield in these terms a decision procedure, already ensure Church a permanent place as a major figure in the development of symbolic logic. But there was more, much more.

Church contributed early-on to the foundations of theoretical circuit synthesis. He also formulated a theory of weak implication which became part of the basis for work by Alan Ross Anderson, Nuel Belnap, and others, on non-classical conceptions of implication, "entailment" and "relevant implication." He made an important proposal for analyzing the concept of a random sequence. The simple theory of types, a modification of the ramified theory of types proposed by Chwistek and Ramsey, received from Church its first precise syntactical formulation in an elegant version using some of the ideas gleaned from the Lambda Calculus. In later years he would give the first clear formulation of the ramified theory of types itself and show an important relationship between Russell's solution to the semantical paradoxes and Tarski's solution by means of the distinction of object-language and metalanguage. And he formulated and gave a relative consistency proof for a set theory that allows for the existence of a universal set.

One would be remiss if *Introduction to Mathematical Logic* were not mentioned for containing important contributions to logic. It contains, for example, what was to become the standard axiomatic formulation of second-order logic (not complete, alas, as follows from Gödel's Incompleteness Theorem.) And Church gives there a correct formulation of the rule of substitution for functional variables, a matter which had eluded Hilbert and Ackermann and others.

In philosophy, besides setting an admirable standard of rigor in philosophical argumentation, Church contributed most of all to defending and developing intensional logic and related matters of general semantics. His own favored approach, the Logic of Sense and Denotation, develops ideas of

Gottlob Frege. But he also gave considerable effort to precisely formulating what he considered to be a viable alternative—Russell's intensional logic as embodied in *Principles of Mathematics* and in *Principia Mathematica.*

He was an able defender of realism in mathematics and logic and a telling critic of various nominalist projects. The sharpening of Frege's argument that sentences denote truth-values, the use of Langford's Translation Test to defeat analyses and arguments about meaning and propositions, an improved formulation of Quine's criterion of ontological commitment, and a refutation of Ayer's formulation of the positivist criterion of empirical significance, are just a few of his distinctively philosophical contributions.

Church wrote several excellent papers on various historical matters concerning logic, for example, on Schröder's partial anticipation of the theory of types and on the history of the notion of a proposition. His explications of the philosophical ideas of Russell and Frege constitute historical scholarship at its very best. Chapter 0 of *Introduction to Mathematical Logic* contains a crystal clear exposition of Frege's ideas about semantics combined with a keen sense of what is worth saving and what ought to be emended.

His monumental *Bibliography of Symbolic Logic* contains every known item on the subject of symbolic logic from the time of Leibniz to 1935. In effect, he continued to work on the *Bibliography* as editor of the reviews section of the *Journal of Symbolic Logic.* The purpose of those reviews was in part to defend symbolic logic against falling into disrepute as a result of misuse and abuse. And he was one of the founders of the Association of Symbolic Logic, playing a large role in the ever increasing respectabilty of the subject.

We rest our case. Church's intellectual legacy plainly establishes for him an honored and permanent place in logic, mathematics, computer science, philosophy, and scholarship about the history of logic.

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