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ALTERNATIVE (1*): A CRITERION OF IDENTITY FOR INTENSIONAL ENTITIES

Abstract. The problem of formulating an adequate criterion of identity for propositions and other intensionalia was at one time considered the principle obstacle to the construction of an acceptable general intensional logic. The objection was urged time and again, principally by W. V. Quine, and still has its influence.[1] Alonzo Church responded directly to the challenge and attempted to incorporate various criteria of identity into fully formalized intensional logics, different implementations of his logic of sense and denotation. The logistic system which was to be based on the most important of Church's ideas, synonymous isomorphism, remains unfinished.

In this paper I attempt to clarify the point and nature of demands for criteria of identity and to evaluate various objections against Church's criterion. My conclusion is that a modification suggested by certain alleged counterexamples appears to fit the data better and so merits further study. Important parts of the overall project require the development of a detailed logistic treatment meeting current standards of rigor. This will not be attempted here, but is reserved for a technical sequel.

1. CRITERIA OF IDENTITY

To fix ideas,[2] define a (*potential*) *one-level criterion of identity for being an* F^1 to be a relation R_o satisfying:

$$(\mathrm{CI}_1)\ \ F^1(z_1)\,\&\,F^1(z_2) \supset \text{.}\ z_1 = z_2 \equiv R_o(z_1, z_2).$$

As an example of this, we may take the familiar one, the *Axiom of Extensionality* of set theory. Here F^1 is being a set and '$R_o(z_1, z_2)$' means that all the members of z_1 are members of z_2, and vice-versa. In this case and analogous ones to be discussed below we shall speak of criteria of identity of this sort as *ontological* criteria of identity.

Now suppose that G^1, F^2, and R_σ satisfy:

$$(\mathrm{CI}_2)\ \ G^1(x)\,\&\,G^1(y) \supset \text{.}\ F^2(x, z_1)\,\&\,F^2(y, z_2) \supset \text{.}\ z_1 = z_2 \equiv R_\sigma(x, y),$$

where F^2 is a function (on G^1's). That is, F^2 obeys the two conditions:

$$(\mathrm{E})\ \ G^1(x) \supset \exists y F^2(x, y),$$

$$(\mathrm{U})\ \ G^1(x) \supset \text{.}\ F^2(x, u) \supset \text{.}\ F^2(x, v) \supset u = v.$$

[1] See, for example, *Grim 1951*, page 20.

[2] And following *Williamson 1990*, pages 144–148.

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Then the relation R_σ will be said to be a *(potential) two-level criterion of identity*. If certain other conditions are satisfied, we might be willing to say that R_σ is a (two-level) criterion of identity for the-range-of-F^2-with-its-domain-confined-to-G^1.

Many of the examples in the literature, proposed as illustrative of the general idea of a criterion of identity can be made to fit one or the other of these patterns. One much discussed two-level criterion is the case where the G^1's are predicates or open sentences (of some given language) and F^2 is the relation between these and their *extensions*. Here R_σ is the relation which holds between two open sentences when they are *true-of*, or *satisfied by*, the same objects. Since we will be concerned with this example and its analogues for intensions, we call such cases of two-level criteria of identity by the more suggestive name, "semantical criteria of identity".[3] The most pleasant case occurs when we have criteria of both sorts and if F^1 is related to F^2 and G^1 in this way:

$$\text{(T)} \quad F^1(z) \supset \exists x(G^1(x) \,.\, F^2(x,z)) \qquad \text{("Totality Condition").}$$

There are various interesting logical relationships between these things.[4] If R_σ is well understood, we may even be able to usefully define an appropriate R_o. In some cases, if other things are favorable, we can "construct" the F^1's as equivalence classes of G^1's. Here we are interested in obtaining further conditions which can guide us in our search for criteria of identity for intensions.

Consider first our paradigm of an ontological criterion, the Axiom of Extensionality just mentioned. There are various ways of choosing an appropriate companion two-level criterion, but we take our cue from Quine:

> Observe, in contrast, how well the corresponding requirement is met in the individuation of classes. I began by saying that classes are identical when their members are identical; but what we now want is a satisfactory formulation of a relation between two open sentences 'Fx' and 'Gx' which holds if and only if 'Fx' and 'Gx' determine the same class. The desired formulation is of course immediate: it is simply '$(x)(Fx \equiv Gx)$'. It does not talk of classes; it does not use class abstraction or epsilon, and it does not presuppose classes as values of

[3] The terminology of *semantical/ontological* criteria of identity is not appropriate for many of the examples discussed in the literature. John Myhill almost recognizes the distinction between semantical and ontological criteria of identity for intensions in his discussion of Church's Alternative (0):

> This [Church's criterion] does not determine the identity or diversity of those senses which are not senses of expressions, but it does suggest a series of axioms concerning the identity and diversity of senses from which various interesting consequences can be deduced. (*Myhill 1958*, page 82)

[4] Timothy Williamson (*1986*) has begun a study of logical questions about criteria of identity which should be emulated.

> variables. It is as pure as the driven snow. Classes, whatever their foibles, are the very model of individuation on this approach.
> (*Quine 1975*, page 7)

Unfortunately, there is a use-mention error here which obscures an important point. Quine merely cites a formula of first-order logic as specifying the relation between the open sentences 'Fx' and 'Gx', but the formula itself does not speak of these syntactical entities. Rather the (semantical) criterion in question must be the relation which holds between these open sentences when '$(x)(Fx \equiv Gx)$' is true. Equivalently, we may speak of the relation which holds between these open sentences when they are *true-of* or *satisfied by* the same objects. It is evidently required that truth-of (or satisfaction) either be taken as primitive in the metalanguage, or be defined in set-theoretical terms therein for the given language.[5] And there is a hidden parameter in the criterion as stated by Quine, namely the language whose open sentences are in question.

So let us take as our instance of (CI_2), the case where G^1 is *being an open formula (of such-and-such a language L)*, F^2 is *has as extension* (in L) (or *determines-* or, perhaps, *denotes-in-L*), and R_σ is the relation which holds between x and y when they are open formulas *true-of-in-L* (or *satisfied-by-in-L*) for exactly the same objects. Thus, in semi-English:

$(\mathrm{CI}_2)^{\mathrm{S}}$ If x and y are open sentences of language L, then if z_1 is the extension of x (in L) and z_2 is the extension of y (in L), then $z_1 = z_2$ if and only if x and y are satisfied (in L) by the same objects.

The superscript 'S' is supposed to suggest that we are ultimately interested in a theory of sets. Here, alas, if we take F^1 as *being a set* in (CI_1), then the Totality Condition with respect to G^1 and F^2 does not hold. For if L is a language of the familiar kind, there will be sets which are not the extensions of any open formulas in L. Still, this seems to be the semantical criterion of interest.[6]

The semantical principle is supposed to have a kind of epistemic priority over the corresponding ontological principle.[7] We imagine that open sentences and their properties are familiar to us. Perhaps in the general case we

[5]This observation might lead one to question the "purity" of $(\mathrm{CI}_2)^{\mathrm{S}}$ below. I am aware that one might try to finesse the semantical concepts by means of some sort of "deflationary" tactic. But even apart from doubts about the feasibility of such projects, we are seeking a *general* criterion of identity for sets as extensions of predicates, and this seems to require the use of semantical concepts.

[6]There are interesting "intermediate" principles that might serve as criteria of identity for sets. For example, we might see something like Frege's Basic Law V as telling us that concepts have the same sets as extensions if they are "materially equivalent". This does not quite qualify as semantical or as ontological in our sense. It does rightly emphasize the epistemological priority of attributes or concepts over sets. It suffers from the fact that we currently have no detailed theory of concepts.

[7]Compare *Anderson 1980*, page 220.

might just require that the entities in question be treated by a systematic and well-understood theory. And it is supposed that we either understand directly, or by way of a theory, the relation of satisfaction or being-true-of for a given language. Extension-of-in-L is just a restriction of the idea of a set to those supposed to correspond to open sentences of the given language and is to be antecedently understood, or at least clarified by the criterion itself, perhaps with the aid of an accompanying theory.

In the present case, the epistemic priority comes out clearly when we consider the possibility of *proving* that two sets are identical. Typically this is done by showing, in a particular language, a certain biconditional—the constituent formulas of which are taken as determining as extensions the sets in question.

Now what is the desired relationship between $(\mathrm{CI}_2)^{\mathrm{S}}$ and $(\mathrm{CI}_1)^{\mathrm{S}}$? Well, it is evidently that we are to find R_o, formulated in set theoretical terms, using $\in$ if desired, so that $(\mathrm{CI}_1)^{\mathrm{S}}$ has the semantical criterion $(\mathrm{CI}_2)^{\mathrm{S}}$ as "observational consequence". Of course this will be of little assistance if we do not have a well-developed theory about sets—but we in fact do, namely, set theory with the Axiom of Extensionality functioning as ontological criterion.

Here there may arise the threat of a regress. Quine (*1975*) observes that the identity of sets is relative to the identity of their members. But in a set theory allowing individuals, or non-sets of some kind, the question may arise as to the ontological criterion of identity for these things. Somewhere the regress must halt and explanations must come to an end.

To illustrate the point, let us consider the simpler case of the extensional entities corresponding to sentences. The desired criteria of identity in that case would seem to be:

$(\mathrm{CI}_2)^{\mathrm{T}}$ If x and y are sentences of language L and if z_1 and z_2 are the truth-values determined (or denoted) by x and y, respectively, then $z_1 = z_2$ if and only if x and y are materially equivalent in L—that is, x and y are both true in L or both false in L.

and

$(\mathrm{CI}_1)^{\mathrm{T}}$ If z_1 and z_2 are truth-values, then $z_1 = z_2$ if and only if $z_1 = \mathbf{t}$ and $z_2 = \mathbf{t}$, or $z_1 = \mathbf{f}$ and $z_2 = \mathbf{f}$.

Here we have the desired relationship between the criteria—we could (and often do) even define the relation of material equivalence by reference to the truth-values.

Suppose we continue to press our Socratic question: "But what is the criterion of identity for being either **t** or **f**?" Here the only possible answer is that these two things are distinct and we can tell this in some cases. If we take **t** and **f** as being the values of something known to be true and something known to be false—say as of the propositions $\forall x(x = x)$ and $\exists x(x \neq x)$, respectively—then in some sense we know what the truth-values are and that they are distinct.

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Presumably, the desired analogues for attributes are of these forms:

$(\mathrm{CI}_2)^{\mathrm{A}}$ If x and y are open formulas (of a language L), then if z_1 is the attribute expressed by x and z_2 is the attribute expressed by y, then $z_1 = z_2$ if and only if $R_\sigma^{\mathrm{A}}(x, y)$,

and

$(\mathrm{CI}_1)^{\mathrm{A}}$ If z_1 and z_2 are attributes, then $z_1 = z_2$ if and only if $R_o^{\mathrm{A}}(z_1, z_2)$,

where Totality may not be assumed and the relation R_σ^{A} is well-understood. Using our analogy, it is apparently permissible that our theory of intensional entities employ the relation R_o^{A} which is not necessarily definable by appeal to R_σ^{A}. But the theory of intensions using R_o^{A}, like the theory of extensions using $\in$, should meet various formal and informal criteria of adequacy. $(\mathrm{CI}_2)^{\mathrm{A}}$ is to be "pure as the driven snow"—it may not employ the initially unfamiliar theoretical primitives involved in R_o^A and the quantification involved in R_σ^{A}, if any, shall not be explicitly[8] over intensions.

The analogy with the extensional case of sets is not quite exact. The ontological criterion for sets appeals to the identity of their elements and these elements may all be (but need not be) of a new kind. The expected relation R_o^A between complex intensions may well appeal to a comparison of "constituents", some of which are themselves necessarily intensional entities. We should certainly demand that the theory give a detailed accounting of the alleged constituent relation, but the ontological criterion is not defeated simply by the fact that it involves reference to intensions.

No regress is forthcoming if we suppose that in some fundamental cases we can just tell whether two simple expressions expressing intensions are synonymous or, what is the same, that the meanings they express are identical or distinct. But of course we can, and when we do, we may suppose that the synonymy is stated by, or is a consequence of, the semantical rules of the language—the rules which codify the knowledge attributable to a competent, perhaps ideally competent, user of the language.

To my knowledge no one has offered a compelling, or even plausible, argument that if a putative kind is not accompanied by criteria of identity, then the alleged kind is unacceptable on grounds of obscurity or other defect. Until and unless a clear, careful, and complete set of conditions for what makes something an acceptable (actual, instead of potential) criterion of identity is given, there is scant hope for such an argument. But the debate will at least be advanced if we can supply what we take to be criteria of identity for intensions, closely parallel to the two principles about sets and extensions,

[8]In the promised logistic treatment, we propose to quantify over everything whatever. However, the condition on quantification is designed to avoid circularity and it does not appear that any conceptual or epistemological circularity is reintroduced by our procedure.

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