INTRODUCTION

This is the second volume of *Collected Papers of Stig Kanger with Essays on his Life and Work.* The first volume contains Kanger's own published papers, most of which have become virtually inaccessible even in his own country, together with one previously unpublished manuscript: 'Choice based on preference'. In this second volume we have collected critical essays on the various aspects of Kanger's work as well as some biographical sketches.

Stig Kanger made groundbreaking contributions to a broad range of areas within both mathematical and philosophical logic:

(i) *General proof theory*: In 1955–57, several logicians – Beth, Hintikka, Kanger and Schütte, independently of each other – brought about a kind of synthesis between the proof-theoretic methods of Gentzen and the model-theoretic ones of Tarski. Exploiting the close correspondence between the rules of inference of Gentzen's calculus of sequents and the semantic clauses of Tarski's definition of truth, they obtained very natural and simple proofs of Gödel's completeness theorem for classical first-order predicate logic. The fundamental idea was to view a proof of a logically valid formula as an unsuccessful attempt to find a counter-model to it. Kanger's completeness proof in his 1957 dissertation *Provability in Logic* – perhaps the most elegant – established in a direct way the connection between Gentzen's sequent calculus and Tarski's model theory. As an immediate corollary, Kanger's completeness proof yielded a simple (but non-constructive) proof of Gentzen's *Hauptsatz*.

Kanger's work in general proof theory is described in Göran Sundholm's contribution to this volume: 'The proof theory of Stig Kanger: a personal recollection'. Sundholm also describes how the Beth–Hintikka–Kanger–Schütte proof method has been extended beyond elementary logic after Kanger. In addition, Sundholm's article contains information about Kanger's early work in mathematical logic.

Additional light is thrown on Kanger's proof theory and semantics by Kaj Børge Hansen in his 'Kanger's ideas on non-well-founded sets'. Hansen describes how, at one point in his dissertation, Kanger outlines a theory of non-wellfounded sets, and makes use of it in a proof of a version of his completeness theorem for predicate logic. Hansen gives a careful and thorough analysis of Kanger's proof and points out that the particular version of the

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completeness theorem does not hold without the assumption of non-well-founded sets. Nowadays, non-wellfounded sets have of course become a topical research subject due especially to the work of Peter Aczel. This was far from the case when Kanger wrote his book.

(ii) *Efficient proof procedures and automated deduction*: As a by-product, Kanger's completeness proof yields a *proof procedure* that is *effective* in the sense of providing an algorithm for finding a proof of any given logically valid sequent. To construct a proof of a valid sequent $\Gamma \Rightarrow \Delta$, we start from below with the given sequent and construct a tree of sequents above it by means of repeated backwards applications of the rules of the cut-free sequent calculus. We continue until the process terminates and we have reached an axiom at the top of each branch in the tree. The resulting tree is then a proof of the valid sequent that we started with. Kanger's completeness proof guarantees that the process terminates after finitely many applications of the rules, provided, of course, that the sequent we started with was indeed valid. In the paper 'A simple proof procedure for elementary logic', Kanger describes how the proof procedure can be extended to predicate logic with identity and how it can be made more efficient for actual implementation on a computer.

Kanger's work on efficient proof procedures was carried further by Dag Prawitz. In 'A note on Kanger's work on efficient proof procedures' Prawitz gives a personal account of Kanger's and his own work to make proof procedures more efficient. He describes how in the late 1950s Kanger's first proof procedure was implemented on a computer and the difficulties that then arose. Prawitz gives a fascinating account of the genesis of the so-called "dummy method" at one of Kanger's seminars. The subsequent fate of the Kanger–Prawitz method of "dummies" for making proofs more efficient is also described in Prawitz's article.

In 'Kanger's choices in automated reasoning' Anatoli Degtyarev and Andrei Voronkov discuss how Kanger's classical 1963 paper 'A simple proof procedure for elementary logic' fares compared to modern work on automated deduction. They write: "Now, when we are equipped with the impressive amount of techniques developed in this are, we are amazed by the incredible intuition of Kanger that allowed him to choose elegant, interesting (and correct) solutions among many possible choices. This article explains these choices and their place in modern automated deduction".

(iii) *Algebraic logic.* In the 1960s, Kanger came into contact with the group of logicians around Tarski at Berkeley and the work that they pursued on the algebraic study of first-order predicate logic by means of so-called cylindric algebras. Intuitively, cylindric algebras play a role in the study of predicate logic that is analogous to that played by Boolean algebras in the study of

sentential logic. Kanger was very impressed by this work and it inspired him to develop an *algebraic logic calculus*, where the ordinary sentences of predicate logic are treated as terms, the statements are equations between terms, and the only rule of inference is substitution of equals by equals.

In his contribution to this volume, 'The proper treatment of quantifiers in ordinary logic', Jaakko Hintikka reviews Kanger's algebraic approach to standard first-order logic and discusses whether it can be applied more generally, in particular to Hintikka's own generalization of standard first-order logic, so-called *independence-friendly first-order logic* (*IF-logic*).

There are several reasons why Kanger's equational approach does not seem well suited for the study of IF-logic. First of all, the set of valid formulas of IF-logic is not recursively enumerable. Hence, IF-logic does not admit of a complete *proof procedure*. On the other hand, there exists a complete *disproof procedure*: If A is an unsatisfiable formula of IF-logic, then there is a taubleaux-type (i.e., Gentzen-type) demonstration of this fact. In ordinary two-valued logic, the existence of a complete disproof procedure is tantamount to the existence of a complete proof procedure. Ordinarily, if a formula is irrefutable, i.e., lacks a counter-model, then it is valid. Due to the failure of the law of excluded middle, this implication does not hold in IF-logic.

But couldn't Kanger's equational approach still be applied to IF-logic – simply by formulating the rules of logical disproof as an equational calculus? This is not a simple matter either, due to the apparent failure in IF-logic of the principle of compositionality. Intuitively speaking, the semantic interpretation of a formula of IF-logic depends, not only on the semantic interpretations of its subformulas, but also on the context in which the formula occurs. Consequently, substitution of equals by equals (applied to formulas) does not, in general, preserve satisfiability.

Hintikka points to a way around this problem. Kanger's algebraic methods can still be used, once predicate symbols and quantifiers have been eliminated in favor of so-called Skolem functions. By means of this technique, the problem of testing a finite set of formulas of IF-logic for satisfiability can be reduced to the problem of testing whether a certain Boolean combination of equations is derivable in an equational calculus à la Kanger. Hintikka remarks: "Kanger's calculus of functional equations can handle more than he himself pointed out".

(iv) *Semantics for modal logic*. In Kanger's dissertation from 1957, appears, for the first time in print, a detailed exposition of a Tarski-style model-theoretic semantics for quantified modal logic. A crucial innovation was the use of accessibility relations in the semantic evaluation clauses for modal operators. Kanger points out that by imposing various formal requirements on

the accessibility relation one can make the operator satisfy corresponding well-known axioms of modal logic. In this way, the introduction of accessibility relations made it possible to apply semantic and model-theoretic methods to the study of a variety of modal notions.

Kanger's early semantics for modal logic differs in interesting ways from the semantic frameworks developed, at about the same time, by Hintikka, Kripke and Montague. Kanger's work on modal logic is discussed in Sten Lindström's paper 'An exposition and development of Kanger's early semantics for modal logic'.

(v) *Deontic logic.* In 'New Foundations for Ethical Theory' from 1957, Kanger developed a model-theoretic semantics also for normative concepts, the so-called *deontic modalities* "It ought to be that ...", "It is right that ...", and to *imperatives*: "Let it be the case that ...!". Kanger's formal language contains quantifiers as well and he discusses the interplay between these and deontic operators. It is noteworthy that Kanger already in this early paper discusses the notion of agency. In terms of the deontic operators and the notion of agency, Kanger, already in 1957, takes the first steps in developing a theory of rights.

Kanger's contributions to deontic logic are discussed in Hilpinen's paper 'Stig Kanger on deontic logic'.

(vi) *Theory of rights and actions.* Kanger's work in deontic logic led him to develop a *typology of rights*, inspired by the work of the American jurist W. N. Hohfeld, within the framework of a formal language containing among its primitive concepts, in addition to deontic operators, the *action operator* "X sees to it that ...". Kanger's theory of rights is arguably his most substantial and influential contribution outside of the field of pure logic.

In this volume, Kanger's theory of rights is dealt with in Lars Lindahl's 'Stig Kanger's theory of rights' and in Lennart Åqvist's 'Stig Kanger's theory of rights: bearers and counterparties, sources-of-law, and the Hansson Petaluma example'. Kanger's contributions to the theory of action are described in Ghita Holmström-Hintikka's 'Stig Kanger's actions and influence'. Holmström-Hintikka also discusses Kanger's attempts at developing a typology of different kinds of influence that is analogous to his typology of rights.

(vii) *Theory of preference and choice.* The theory of preference and rational choice occupied Kanger intermittently during the last 20 years of his life. A comprehensive overview of Kanger's contributions to this area is given by Sven Ove Hansson in 'Kanger's theory of preference and choice'. Hansson discusses Kanger's attempts to develop a preference logic in the tradition of Halldén, his so-called *paradox of exclusive disjunction* (more extensively

treated by Rabinowicz, see below), as well as Kanger's contribution to the theory of rational choice (more extensively treated by Sen, see below).

In his contribution 'Preference logic and radical interpretation: Kanger meets Davidson', Wlodek Rabinowicz discusses a paradox in preference logic (referred to by Hansson as 'the paradox of exclusive disjunction') that was formulated by Kanger and that led Donald Davidson to modify his theory of radical interpretation. Rabinowicz argues that although Kanger's paradox can be dissolved, Davidson's theory of radical interpretation still confronts serious difficulties.

Finally, Amartya Sen in 'Non-binary choice and preference: a tribute to Stig Kanger' discusses Kanger's contribution to the theory of rational choice in 'Choice based on preference'. In this paper Kanger generalizes the standard theory of preference and choice to choice functions that select a set of alternatives from a "menu" of available alternatives against a "background set" of alternatives. As the background set varies, the selected set may vary as well, even if the menu of available alternatives is kept fixed. Sen compares Kanger's approach to the standard theory of rational choice and discusses the reasons that Kanger might have had for adopting his alternative approach.

The process of publishing these two volumes dedicated to the work of Stig Kanger has been a genuinely joint venture in which many people have contributed in essential ways. We are grateful to them all, including the contributors of essays on Kanger's life and work. We owe special thanks to Jaakko Hintikka and Krister Segerberg for their enthusiastic and steadfast support of the project as well as for their inspiration and good advice. We also wish to thank Jaakko Hintikka for including the two volumes in the Synthese Library Series and Krister Segerberg for editing the section of biographical sketches. We are grateful to Ms. Annie Kuipers and Mr. Rudolf Rijgersberg at Kluwer for their patience and cooperation, Ms. Kaipainen at the Department of Philosophy of the University of Helsinki for doing an excellent work in transforming Kanger's typographically difficult texts into camera-ready copy, Sharon Rider and Kaj Børge Hansen for translating some of Kanger's Swedish texts into English, and Sven Ove Hansson and Lars Lindahl for valuable editorial assistance. Kaj Børge Hansen helped us prepare the indexes for the two volumes and Anders Berglund assisted us with proof reading.

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