CHAPTER 1

HERMENEUTICS AND MATHEMATICS EDUCATION

The language used can only be the language we have. So one must not look for meanings - only for the things that are meant (Brookes, 1971, p 159).

How far can mathematics be drawn into a linguistic domain and how far does such a move enable us to clarify the way in which students share their mathematical thinking with their peers and with their teacher? Such concerns will be addressed throughout the book. Before proceeding, however, I feel an example might help in further clarifying some of the issues of concern. I offer an account of someone working in a university seminar specifically concerned with exploring how words are introduced in holding on to evolving ideas (first reported by Brown, 1996 a, pp. 62-64). In the seminar there was an invitation to imagine a particular geometrical configuration. In an attempt to give a more graphic example of the issues of concern in this book I offer the three sentences which initiated the exercise and an account of someone's developing understanding during the exercise, written immediately afterwards.

"Imagine a circle with two tangents meeting at a fixed point. Imagine the circle getting bigger and smaller. Attend to the locus produced by two points where the tangents meet the circle."

I remained unsure of whether the centre of the circle was fixed or movable. My initial sense was that if it was fixed then the circle could only enlarge or shrink within a very limited range. It stopped growing as it hit the point (which I had located above the circle) and finished shrinking as it became no more than the point at its centre, at which point the two tangential points met. The radius could not exceed the distance between the centre of the circle and the top point where the tangents met. I was having difficulty in picturing the locus, mainly because there seemed to be more degrees of freedom than I could cope with. At first the circle growing seemed to be associated with its centre moving down. I could not easily hold the centre as fixed as the circle moved. I could not be sure about the shape of the locus as too many things were moving at once. I thought of a U-shaped curve, a parabola maybe. Or, possibly an ellipse since I had some sense of the locus joining at the top as well.

I reported on this to my colleagues. Some seemed surprised that *my centre could move. A discussion between my colleagues ensued which left me somewhat bewildered. There was talk of fixing radii or of fixing the centre. I heard one colleague speak of a tulip. It was this latter image which I used in firming up my image of a parabola - I could picture a tulip as being rather like the base of the truncated parabola I had in mind* (see Figure 1).

I commenced the exercise feeling comfortable with the words circle and tangent as used in the original direction. These terms were very familiar to me and I regularly use them in describing things I see. Ellipses and parabolas became things I attempted to fit on to my picture of the locus. This, however, was unsuccessful since the image was too dynamic for me to capture it clearly in words. After the brief discussion I became aware of not being able to follow my colleagues and began working on clarifying my original picture. I began attempting to fit a "tulip" onto the images I was having. At this point I felt I was placing my images on the words of others. I caught fragments of the discussion as it continued; like - "you take one of the lines and you get half of the tulip" but I had too few straws to grasp hold of and continued working on my own. It later turned out, in fact, that my colleague had not said tulip but rather had said "tulip-vase".

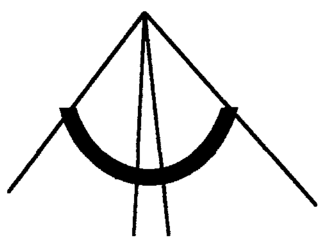

Figure 1.

Initially he works with the words offered in the original description. His thoughts are framed in words with which he is familiar as he seeks to build up his own picture of the locus. "Circle" and "tangent", are among the conventional mathematical terms used in the original description which guide his thoughts. "Radius", "ellipse" and "parabola" are conventional words he seeks to introduce in making sense of the picture. "Tulip" is introduced into his thinking both as an aid in developing his own picture but also in an attempt to share the images offered by his colleagues.

The person offering the original "imagining" may have had an image in his head but the significance to participants depended on a number of issues;

- he will have a particular understanding of, and ways of working with, words like "circle" and "tangent",

- he will have the ability to introduce other conventional mathematical terms and personal images into his analysis to support his thinking,

- he will introduce particular interpretations into the original wording, for example, allowing the centre of the circle to move or not,

- he has to find ways of capturing the shapes and movement he senses in a way that can be stored and communicated,

- he also had a need to reconcile his own mental imagery with descriptions offered by others about how they were seeing things (for example, "Tulip").

In tackling this exercise the participant is attempting to reconcile his mental dynamics with descriptions in words and in so doing move from his personal account to one that could be shared with his colleagues. Not only is he trying to bring images to the words of the seminar leader and his peers but also find ways of capturing his own imagery in words. This locates the struggle between understanding the world as it is and operating on it to make it different - the very attempt to describe the world as it is, changes the world. The participant who introduced "tulip-vase" as a way of describing what he saw affected the perceptions of others and the descriptions which they offered. Any attempt to move from the personal to the social or vice versa requires such a *moving forwards* as interpretations combine. The geometrical construction lends itself to formal statements yet such statements are arranged by the individual user. The statements have an interpretive feel and are offered with varying degrees of certainty and one is only able to focus on specific parts of the construction at any one time.

In more general terms the individual is seeking to share some images with his colleagues but, on this occasion is obliged to tackle this task through the medium of spoken words. He attempts to connect his own mental imagery with the words of his "teacher" and, in turn translate this imagery into his own words. Both "teacher" and "student" are using the language of their culture to capture their personal way of seeing things. But what does "personal" mean when it has to be so carefully channelled through the social filter of language?

In this chapter I discuss how interpretation might assume a more central role in mathematical understanding. By shifting the emphasis from what a statement might mean to how it has been used by someone in a specific situation, the human agent becomes implicated, and a certain perspective gets revealed. This is reflected in recent discussion in mathematics education research which seems generally opposed to notions of absolute meaning and has shifted towards understanding meaning as an individually or socially constructed phenomena. Nevertheless, whilst such a move seems consistent with theoretical shifts in other academic fields there is still an underlying difficulty resulting from a history of seeing mathematical meaning as in some ways independent of time and context, a notion associated with concepts rather than with conceiving, a fixed point to which the learner converges. Whilst radical constructivism, for example, tries to move away from such stable notions of meaning, in doing this they explicitly avoid entering into theories about how things are. I will argue here that by using notions of objectivity derived from phenomenology, a theory of how things are can be introduced without undermining the constructivist theory of how we know.

Hermeneutics, the theory and practice of interpretation, is governed by a belief that whilst the world may exist independently of humans, it cannot present itself directly to the human gaze. It attends to the process through which we develop an understanding of the world. The hermeneutic task can be seen as an uncovering of meaning, but a historically situated meaning dependent on the media and experiences through which it is observed. Further, we can never uncover a meaning free of the conditions that gave rise to us and of the particular perspective we assume. Hermeneutics readily lends itself to the disciplines within the human sciences, which in general, "deal with the world of meaningful objects and actions (as opposed to physical objects and events in themselves)" (Culler, 1976), where the human subject is assumed to have a particular position and perspective rather than some God-like overview.

The principal task of this chapter is to outline the phenomenological roots of contemporary hermeneutics and show how such a perspective can assist us in describing mathematical activity. By seeing mathematical expressions as being used by humans in particular situations, rather than as things with inherent meaning, emphasis is placed on seeing mathematical activity as a subset of social activity and, as such, is subject to the methodologies of the social sciences. This will be organised as follows: Firstly, I shall outline some issues arising through seeing mathematical expressions as being necessarily contained in action, resulting in meanings that transcend mathematical symbolism. This is followed by a discussion of phenomenology and hermeneutics. I then show how different conceptions of hermeneutics lead to alternative educational practices. Finally, I consider how we might understand mathematical learning as a hermeneutic process and show how the creation of mathematical phenomena in human understanding is a consequence of a linguistic process of classifying according to an individual phenomenology. A reader wishing to minimise a fuller theoretical development may prefer to skip a bit. Such a reader might take a preview of discussions of classroom mathematics in Chapters 3, 4, 5 and 7 and then orientate the reading of Chapters 1 and 2 around the examples taken from mathematics education.

ACTION AND MEANING

In his book "*Keywords*" Raymond Williams (1976) took a hundred or so words from the contemporary scene and discussed their usage over the last century or so and in each case demonstrated how this usage had evolved over the years. In commenting on this book Brookes (1978) pointed out how the word "meaning" simply was not used by Williams. It seems that this was a deliberate attempt to show how the use of a word in speech is more important than any supposed intrinsic meaning. Brookes argued that to assert an intrinsic meaning is to underplay the context of what one says and to presume the universality of certain ways of seeing things. This emphasis on usage echoes Wittgenstein (1958, p. 20) who suggested that the meaning of a word might be seen as its usage in language and is thus dependent on both situation and time. This view offers an alternative to seeing words as having inherent meaning and a key to analysing expressive activity as action; to say a sentence then is to perform an action, an action that takes place through time. The meaning of a

http://www.springer.com/978-1-4020-0013-3

Mathematics Education and Language
Interpreting Hermeneutics and Post-Structuralism
Brown, T.
2001, X, 306 p. 1 illus., Hardcover
ISBN: 978-1-4020-0013-3