

FREE LOGIC AND QUANTIFICATION IN SYNTACTIC MODAL CONTEXTS

1. INTRODUCTION

The field of modal logic spans a very wide range of philosophical notions and motivations, with the result that to construct any particular formal system will require that a number of choices be made between alternative intuitions and goals. This is particularly true of quantified modal logic, since here disparate modal notions combine with alternative interpretations of quantification to produce a labyrinth of technical possibilities. Unfortunately, even though the number of possibilities is large, the theoretical ingredients seem to interact in ways which force internal compromises. For example, choices in favor of standard first-order logic conflict with modal intuitions about assertions involving possible objects, while the “primary” reading of necessity as logical validity runs up against the problem of not being recursively axiomatizable.

In the present paper I will explore an approach to quantified modal logic (QML) where the pattern of technical choices is guided by two basic, interrelated themes. The first is that modality will be interpreted metalinguistically, that is, as a predicate of object language expressions. The assertion that ϕ is necessary will be construed as the claim that ϕ is valid in some appropriately specified class of structures. In turn, the structures in question should possess the traits of quantification and predication that we wish the resulting modal context to exhibit. This leads to the choice of free logic rather than standard first-order logic (FOL) as the appropriate source of base level semantics.

This method of constructing the logical machinery differs somewhat from the method often adopted, wherein a system of propositional modal logic is taken as the starting point, and a theory of quantification is then “pasted on” to extend the system of possible worlds semantics. This type of hybrid approach is not feasible on the metalinguistic account, because rather than starting with propositional modal semantics and adding quantification, one must start with the quantificational structures and define modal contexts in terms of the original models. This design feature forces one to be explicit at the outset about the type of quantificational structures

needed, which motivates the choice of a semantics for free logic as opposed to ordinary first-order models. In section 2 of the paper I will discuss the metalinguistic approach in greater detail, and briefly review its employment in the simpler case of propositional modal logic. In section 3 I will examine the applicability of free logic to the quantificational case, and in the remainder of the paper I will develop the metalinguistic approach to QML using outer domain semantics.

2. METALINGUISTIC MODALITY

One of the first choices to be made in a formal treatment of modality concerns the basic *logical form* of necessity statements. There is a long history surrounding the view that necessity should be construed as having the form of a predicate which applies to formulas taken as syntactical objects (for example, Carnap (1937), Quine (1953)). Under this development syntactical expressions are designated by singular terms, and the necessity device attaches to these terms to yield new atomic formulas. If ϕ is a formula in the object language, $[\phi]$ a singular term denoting ϕ , and N the necessity device, then the modal assertion that ϕ is necessary will have the form $N[\phi]$. In this manner modal statements are viewed as predications in which a metalinguistic property is attributed to a linguistic expression treated as an object of discourse. In the present formulation the metalinguistic property in question will be truth in a certain set of models.

However, this interpretation of necessity requires some caution in its development. The expressive power of a metalinguistic predicate greatly exceeds that of an object level operator. Thus on the standard account, where necessity has the logical form of a monadic *connective*, structurally comparable to negation, its expressive power is rather limited. For example, the type of “self-reference” achieved through Gödel’s diagonal lemma will not apply to an operator, since monadic connectives must apply directly to expressions rather than to their names. Because of this, application of \Box to a formula ϕ must always result in a *new* formula $\Box\phi$, which means that iterated operator modality is inherently stratified. This hierarchical feature of the iterated object level device will be utilized in the present construction in order to define a consistent formulation of predicate modality that exactly mirrors the deductive strength of the operator. There is a fine balance to be maintained between expressive and deductive power, and the modal inconsistencies derived in Montague (1963) can be seen as violations of this balance.

Following the same basic strategy as the Gödel and Tarski theorems, Montague establishes that if necessity is formalized as a predicate of sentences in some theory T extending Robinson arithmetic, then the theory will be inconsistent, given that the predicate N obeys even very weak modal principles. In fact, a formal contradiction can be derived if the logic of the modal predicate is governed merely by principles (i) and (ii) below, corresponding to the Law of Necessity and the inference rule of Necessitation:

- (i) $\vdash_T N[\phi] \rightarrow \phi$
- (ii) $\vdash_T N[\phi]$, if $\vdash_T \phi$

However, this result depends on several auxiliary assumptions. First, it must be assumed that the privileged terms used to designate object language formulas, which are required in order to state the above two schemas, are structurally rich enough to support the diagonal lemma. This will be true of Gödel numerals, but it may well not be true of more “primitive” methods, such as formal quotation names. This will depend on the particular axioms (if any) which govern these terms. And given this first auxiliary assumption, it must then be required that the above metavariable ϕ ranges over all sentences involving the modal predicate N , even “ungrounded” sentences which cannot in principle be articulated with the resources of the operator. In particular, the “self-referential” fixed point θ , where $\vdash_T \theta \leftrightarrow \neg N[\theta]$, must count as an instance, even though no corresponding sentence can be formulated with the \Box connective. This second move introduces radically new modal *axioms* into the predicate system, which then leads to an (unlimited) increase in deductive power.

This analysis of the problem suggests defining a grounded hierarchy of languages, where the base language is modality free, and successive levels are attained by adding primitive terms which denote the *closed* formulas of the previous level, which is all that is needed for *propositional* modal logic. The extension of the necessity predicate is then successively specified as the set of all valid sentences of the lower level. This leads to a cumulative language L_ω which properly contains its own necessity device, in the sense that for all closed formulas $\phi \in L_\omega$, it is the case that $N[\phi] \in L_\omega$. In this manner the claims which it is possible to articulate and prove using the predicate formulation will exactly correspond to the standard systems of propositional modal logic expressed *via* the operator. See Schweizer (1992) for a detailed discussion of the construction for the propositional case. An analogous construction for quantified modal logic will be pre-

sented in sections 4 and 5 below, with the additional subtlety that outer domain semantics will supply the base level structures for interpreting quantification, and special provisions must be made for defining free variables in syntactic modal contexts.

3. FREE LOGIC AND MODAL SEMANTICS

Free logics are primarily motivated in response to certain infelicities of first-order logic which can be expressed without appeal to specialized object language methods for constructing contexts in which first-order formulas are embedded. However, it is well known that the principles of free logic are especially suited to some of the philosophical intuitions underlying quantified modal logic, since these same infelicities of standard logic are put into sharp focus by the analysis of statements involving possible but non-actual objects. Indeed, one of the basic motivations behind the development of QML is to provide a framework within which to evaluate as *true* certain statements about things which do not exist in this world, but which do exist in alternative “states of affairs”. Thus in Kripke’s (1963) semantics several choices are made which embody principles of free rather than classical logic. The most distinctive of these choices is that the interpretation function in a semantical structure can assign an object to the extension of a predicate at some world, even though that object is not an element of the domain at that world. And conversely, a predication can turn out false in a world when evaluated with respect to an object that does not exist at that index, but which does exist at a world which has access to the first one. In addition, Kripke upholds the principle that the quantifiers retain their “existential import”, and are thereby restricted at each world to the domain of actual objects.

This combination of features incorporated in Kripke’s semantical structures, *viz.*, existential import of the quantifiers, and inclusion of locally non-existent objects in the extensions (and anti-extensions) of predicate expressions, is characteristic of the “outer domain” formulations of free logic, partly inspired by Meinong’s pioneering semantical work. For example, in the outer domain semantics of Leblanc and Thomason (1968) a model consists of an ordered triple $\langle D, D', I \rangle$, where D is the inner domain of existent objects, and D' is the outer domain of “subsistent but non-existent” objects. D and D' are disjoint, and the interpretation function I is *total* with respect to the set $D \cup D'$. This allows an essentially standard definition of truth conditions for formulas involving “non-denoting” singular

terms, since set membership is still the foundation of truth for atomic predications. The interpretation function is (potentially) partial on the set of singular terms only with respect to the *inner* domain D as its range, which is also the range of the quantifiers.

In order to make the connection between Kripke's modal semantics and the outer domain formulations of free logic more explicit, let D^* be the union of all the individual domains of worlds w in a given Kripke structure. Then the set D_w of objects which exist in w constitutes the inner domain over which the quantifiers range, while $D^* - D_w$ is the Meinongian outer domain. The (binary) interpretation function I assigns extensions to n -ary predicates P^n relative to a world w , subject only to the constraint that $I(w, P^n) \subseteq (D^*)^n$. Thus the extension of P^n at w may include both objects $d \in D_w$ and objects i such that *not* $i \in D_w$. In some sense these extensions then resemble complex numbers with both real and "imaginary" components. With these features of outer domain semantics in place, and equipped with a few salient restrictions on terms, Kripke is able to accomplish two major goals. First, his approach preserves the classical rules of FOL extended to modal systems, and thereby expands the realm of application of the customary logical patterns to cover the "complex" values supplied by combining both actual and possible objects. Second, the distinctively "free" aspect of Kripke structures leads to the refutation of both the Barcan formula and its converse, so that his system allows the maximum degree of articulation with respect to scope interactions between the quantifiers and the modal operators.

Of course, the feat of preserving classical logic within a generalized modal framework of outer domains, requires that Kripke's (1963) system essentially omits singular terms. There are no individual constants, and open formulas are treated as universal closures, so that in effect there are no free variables. However, when singular terms are introduced within this formal setting then the submerged foundation of free logic becomes visible on the surface. The law of Existential Generalization fails, since the truth of $\psi(t)$ at w does not imply the truth of $\exists x\psi(x)$ at w , because it may be the case that $I(w, t) \in I(w, \psi)$, even though it is not the case that $I(w, t) \in D_w$. It is then convenient to introduce an "existence predicate" E which is co-extensive in w with D_w . In systems with identity, E can stand as an abbreviation for $\exists x(x = x)$. The existence predicate can then serve to articulate the distinctive additional premise of free logic now required in this modal setting, *viz.*, that t exists. In terms of Universal Specification the form required in a Kripke structure with singular terms is the standard schema for free logic:

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