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FREE LOGIC: A FIFTY-YEAR PAST AND AN OPEN FUTURE

1. DEFINITION AND TYPES OF FREE LOGIC

The term 'free logic', as introduced by Karel Lambert in 1960[1], is short for 'presupposition-free logic' or, more explicitly, for 'logic free of existential presuppositions'. Since existential presuppositions are connected with terms, this paraphrase was understood as 'logic free of existential presuppositions with respect to its singular and general terms'. Modern logic in the sense of standard first order predicate or quantificational logic with identity ($QL^=$, for short) is almost fully free with respect to its general terms or predicates, with one exception, namely universal terms or predicates like '$Px \vee \neg Px$' or '$x = x$'. In standard systems of $QL^=$, '$\exists x(Px \vee \neg Px)$' and '$\exists x(x = x)$' are theorems. If we read such a formula as 'something exists' this seems to express a matter of ontology rather than logic; taking it as a law of logic is therefore "a defect in logical purity" as already noted by Russell[2]. Giving up this presupposition results in a logic called *inclusive logic* by Quine[3] or *empty logic* by some free logicians. The main concern of free logic was and is therefore existential presuppositions with respect to singular terms. In standard systems of $QL^=$ we usually have for every singular term t and every variable v the theorem $\exists v(v = t)$. And in standard quantification theory without identity (QL) we still have for every formula A the theorem $A(t/v) \rightarrow \exists vA$ and its dual $\forall vA \rightarrow A(t/v)$.

If we allow t to be a singular term not referring to an existent object and we take A to be the predicate or general term 'x does not exist', these theorems are instantiated by obvious falsehoods. This, however, is the case only if we interpret the quantifiers as having existential import and being read as 'for every existing thing' and 'for at least one existing thing' (or 'there exists something'), so this understanding of quantification becomes part of the definition of free logic.

Even if the main aim of free logic is to eliminate existential presuppositions with respect to singular terms, a definition of free logic has to take into account also general terms, not only because otherwise it would be incomplete, but also because otherwise it would be incoherent: existential presuppositions with respect to singular terms could return through the back door via general terms containing singular ones like '$x = t$'. An ad-

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equate definition of free logic therefore has to include three components:

A logical system L is a *free logic* iff
(1) L is free of existential presuppositions with respect to the singular terms of L,
(2) L is free of existential presuppositions with respect to the general terms of L, and
(3) the quantifiers of L have existential import.

A logical system L is a *universally free logic* iff
(1) L is a free logic, and
(2) L is an inclusive (or empty) logic.

According to this definition, free logic is not a particular logical system but rather a whole family of systems so that we can also use the plural form and speak of "free logics". Free logics in this sense are logical systems which allow singular terms to be empty or non-denoting insofar as they do not refer to existent things, and at the same time the theorems of such a system remain logically true even if the singular terms occurring in them are empty. Here it is important to notice that in some versions of free logic a singular term may be empty, in not referring to any existing object, and yet referential, in referring to a non-existing object.

There are three types of free logic to be distinguished in this context depending on whether or not elementary sentences containing empty singular terms do or do not have certain truth-values. An elementary or logically simple sentence is a sentence containing no logical operator (i.e. no truth-functional connective and no quantifier). Now we can define:

A logical system L is a *negative free logic* iff L is a free logic and every elementary sentence of L containing at least one empty singular term is false.

A logical system L is a *positive free logic* iff L is a free logic and there is at least one true elementary sentence of L containing at least one empty singular term.

Among the systems which are free but neither negative nor positive only one type is attractive enough to get a name of its own, and is usually defined as follows:

A logical system L is a *neutral free logic* iff L is a free logic and every elementary sentence of L containing at least one empty singular term (with the only exception perhaps being 't exists') has no truth-value at all.

2. Some motivations behind free logic

The interest in problems of free logic increased and the first works in the field were published at the same time as the first steps were taken towards the development of a semantics for modal logics which later became well known by the title 'Possible World Semantics'. This seems to be no mere coincidence since there is an overlapping area of common motivation behind the two developments. Also in different fields of modal logic (like alethic, epistemic and deontic logic) we very often deal with things, persons, actions, situations etc. which do not actually exist, whether they do not yet exist or no longer exist or never exist. In modal logic therefore it is common to use singular terms which do not denote an object existing in every possible world.

The reasons for developing systems of free logic are, however, largely independent of such modal considerations. They also concern the occurrence of empty singular terms in non-modal contexts and some of these sentences nevertheless seem to be obviously true. Introducing a singular term for an allegedly existing planet, chemical substance, particle, number etc. should not in any way depend on our knowledge that the purported object allegedly denoted by the singular term in fact exists. This motive depends on the common aim of keeping our scientific language, in particular its vocabulary and its formation rules, independent of the facts we want to describe by it and independent of our knowledge of these facts. If so, we need a language containing singular terms t for which $\exists v(v = t)$ is not a theorem and which therefore can occur in a sentence $A(t/v)$ without $\exists vA$ being deducible from it. But this is just a free logic as defined above. Similar arguments apply to a language which serves the purpose of expressing fictions, myths and fairy tales in which (empty) names of non-existing persons occur without turning all these sentences into truth-valueless expressions (as Frege did) or into falsehoods. Another solution for this problem was offered by Russell who eliminated most singular terms from a language and replaced them by definite descriptions. This, however, seems to be a quite unnatural solution.

There are also philosophical motives which led to the development of free logic. An adequate discussion of traditional arguments for the existence of God or of Descartes' "Cogito ergo sum" is only possible in a language allowing empty singular terms. Otherwise these problems are turned into trivialities and their alleged solutions into *petitiones principii*. The same holds for evaluative or normative contexts of ethics in which very often non-existing persons, actions and situations are taken into account.

3. SYNTACTICAL SYSTEMS OF FREE LOGIC

3.1 THE LANGUAGES FL, FL$^-$, FL$^=$ AND FL$^+$ OF FREE LOGIC

We describe the formal language FL in the usual way by listing its vocabulary and specifying its formation rules, thereby defining its formulas.

The *vocabulary* of FL is the same as that of QL$^=$, augmented by the symbol 'E!' for existence. It therefore consists of the following signs:

(1) the descriptive symbols of FL
 (1a) the n-place ($n = 1, 2, \ldots$) predicates of FL: P, Q, R,...
 (1b) the individual constants of FL: a, b, c,...
(2) the logical symbols of FL
 (2a) the (individual) variables of FL: x, y, z,...
 (2b) the connectives of FL: $\neg$, $\rightarrow$
 (2c) the (universal) quantifier of FL: $\forall$
 (2d) the logical predicates of FL: $=$, E!
(3) the auxiliary signs of FL: (,)

By dropping the existence predicate E! we get the vocabulary of a restricted language FL$^=$, and by dropping also the identity symbol and the individual constants (i.e. by omitting all symbols of (1b) and (2d)) we get the vocabulary of an even more restricted language FL$^-$. On the other hand, we get the vocabulary of an enlarged language FL$^+$ by adding the following clause (2e):

 (2e) the description operator of FL$^+$: ι

A symbol of FL, FL$^-$, FL$^=$ or FL$^+$ which is either an individual constant or an individual variable will be called an *individual symbol*. A single sign or a finite sequence (string) of such signs will be called an *expression* of one of these languages. Expressions which are either individual constants or definite descriptions (which will be introduced below) are called *indi-*

vidual names and correspond to what we usually call *singular terms* when we do not refer to a particular language. Expressions which are either individual variables or individual names will be called *individual terms*. As usual, we will call not only a single sign $\forall$ or ι standing alone a *quantifier* or *description operator*, respectively, but also an expression of the form $\forall v$ or ιv where $\forall$ or ι, respectively, is prefixed to a variable v.

In the metalanguages of our languages we will use the connectives, the universal quantifier, the logical predicates and the auxiliary signs autonymously, whereas we introduce the following kinds of metavariables for signs and expressions of FL^+ (and accordingly for the other languages which are less rich than FL^+):

(1) for n-place predicates: P^n
(2) for individual names: t, t_1, t_2,...
(3) for (individual) variables: v, v_1, v_2,...
(4) for individual terms (i.e. individual names or variables): s, s_1, s_2,...
(5) for expressions, in particular for formulas: A, B, B_1, B_2...
(6) for classes of formulas: C

We can now define a *(well-formed) formula* of FL recursively by the following formation rules:

(1) If P^n is an n-place predicate of FL
and $s_1, s_2, \ldots, s_n$ are individual terms of FL then
$P^n s_1 s_2 \ldots s_n$ is a formula of FL.
(2) If s_1 and s_2 are individual terms of FL then
$s_1 = s_2$ is a formula of FL.
(3) If s is an individual term of FL then
$\mathrm{E!}s$ is a formula of FL.
(4) If A and B are formulas of FL then
$\neg A$ and $(A \rightarrow B)$ are also formulas of FL.
(5) If A is a formula of FL and v is an (individual) variable of FL then
$\forall v A$ is a formula of FL.
(6) Nothing is a formula of FL except by virtue of (1)–(5).

A formula A of FL is an *elementary formula* of FL iff A contains no connective and no quantifier, i.e. iff A is of one of the forms (1), (2) or (3). And a formula A of FL is an *atomic formula* of FL iff A is elementary and does not contain a logical predicate, i.e. iff A is of the form (1). The definition of a formula of $\mathrm{FL}^=$ results from the definition above by omitting

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