GYULA KLIMA

EXISTENCE AND REFERENCE IN MEDIEVAL LOGIC

1. INTRODUCTION: EXISTENTIAL ASSUMPTIONS IN MODERN VS. MEDIEVAL LOGIC

"The expression 'free logic' is an abbreviation for the phrase 'free of existence assumptions with respect to its terms, general and *singular*'."[1] Classical quantification theory is not a free logic in this sense, since its standard formulations commonly assume that every singular term in every model is assigned a referent, an element of the universe of discourse. Indeed, since singular terms include not only singular constants, but also variables[2], standard quantification theory may be regarded as involving even the assumption of the existence of the values of its variables, in accordance with Quine's famous *dictum*: "to be is to be the value of a variable"[3].

But according to some modern interpretations of Aristotelian syllogistic, Aristotle's theory would involve an even stronger existential assumption, not shared by quantification theory, namely, the assumption of the non-emptiness of *common* terms[4]. Indeed, the need for such an assumption seems to be supported not only by a number of syllogistic forms, which without this assumption appear to be invalid, but also by the doctrine of Aristotle's *De Interpretatione* concerning the logical relationships between categorical propositions, commonly summarized in the *Square of Opposition.*

For example, Aristotle's theory states that universal affirmative propositions imply particular affirmative propositions. But if we formalize such propositions in quantification theory, we get formulae between which the corresponding implication does not hold. So, evidently, there is some discrepancy between the ways Aristotle and quantification theory interpret these categoricals. One possible suggestion concerning the nature of this discrepancy could indeed be that Aristotelian logic contained the (tacit) existential assumption as to the non-emptiness of common terms. On the other hand, we know that this suggestion would definitely be rejected by the most ardent followers and expounders of Aristotle's logic, namely, medieval logicians[5]. Indeed, they would reject not only this suggestion (and for good reasons, too)[6], but also the existential assumptions of standard quantification theory mentioned above.

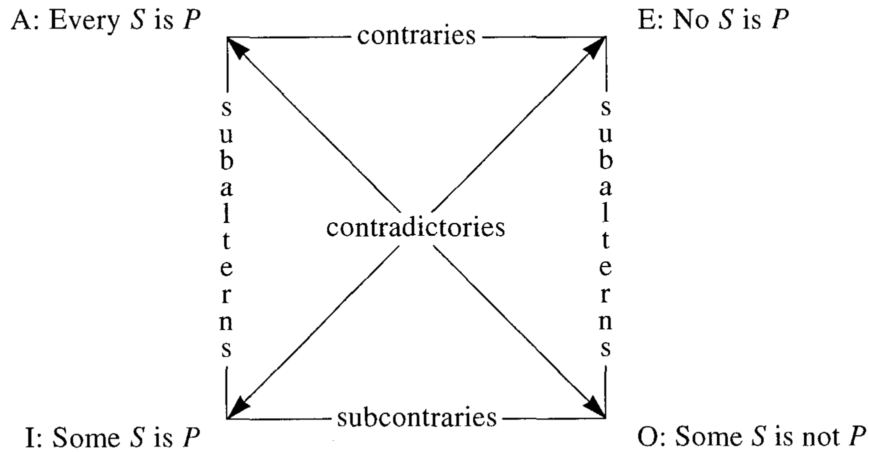
E. Morscher and A. Hieke (eds.), New Essays in Free Logic, 197-226.

In this paper I am going to give a brief, primarily systematic (as opposed to primarily historical) account of how it was possible for medieval logicians to maintain Aristotle's theory of the four categoricals and to dispense with these existential assumptions in the framework of their theory of reference, the *theory of supposition*[7]. Besides the informal account I will also indicate how these informal ideas can be put to work in a formal semantic system[8]. By this I hope to show that the ideas of medieval logicians can provide us with valuable insights even in the apparently modern field of free logic.

2. Supposition Theory and the Square of Opposition

The main reason why medieval logicians did not need any extra (tacit or otherwise) assumptions "to save" the logical relationships of the *Square of Opposition* is that these logical relationships are direct consequences of their semantic analysis of the four types of categorical propositions. According to this analysis, in contrast with the analysis of quantification theory, the subject terms of these propositions have a referring function: they stand for (*supponunt pro*) the particulars falling under them, provided there are any such particulars. If, however, these subject terms are "empty"[9], then they simply refer to nothing (*pro nullo supponunt*). But then the affirmative categoricals are false, whence their contradictories are true, that is to say, affirmative categoricals have existential import, while negative ones do not, which "automatically" yields the relationships of the *Square*. Namely, if A: universal affirmative, E: universal negative, I: particular affirmative, O: particular negative; then A $\Rightarrow$ I, E $\Rightarrow$ O, A $\Leftrightarrow$ '~O', E $\Leftrightarrow$ '~I', A $\Rightarrow$ '~E', '~I' $\Rightarrow$ O. Notice that from A $\Rightarrow$ I, E $\Leftrightarrow$ '~I' and A $\Leftrightarrow$ '~O' the remaining are derivable, that is, if A is assigned existential import and the diametrically opposed propositions are construed as contradictories, then all relationships of the *Square* (see next page) are valid.

There can be basically two types of justification for attributing this kind of existential import to affirmative categoricals (regardless of their quantity, i.e., whether they are singular, indefinite, universal or particular), on the basis of the two fundamental types of predication theories endorsed by various medieval authors[10].



According to what historians of medieval logic dubbed the *inherence theory of predication*, an affirmative categorical proposition (in the present tense with no ampliation, cf. n.9 above) is true if and only if an individualized property (form, or nature) signified by the predicate term actually inheres in the thing(s) referred to by the subject term. So, for example, the proposition 'Socrates is wise' is true if and only if wisdom actually inheres in Socrates, that is, Socrates has wisdom, or, which is the same, Socrates' wisdom actually exists. But of course Socrates' wisdom, or for that matter any other inherent property of Socrates, can actually exist only if Socrates himself exists. Thus it follows that if Socrates does not exist then the proposition 'Socrates is wise' is false, and so are all affirmative categoricals the predicate term of which signifies some inherent property of Socrates.

On the other hand, according to the other basic type of medieval predication theories, the so-called *identity theory*, an affirmative categorical proposition is true only if its subject and predicate terms refer to the same thing or things. For example, on this analysis 'Socrates is wise' is true if and only if Socrates, the referent of 'Socrates', is one of the wise persons, the referents of the term 'wise'. If any of the two terms of an affirmative categorical is "empty", then the term in question refers to nothing. But then, since "nothing is identical with or diverse from a non-being", as Buridan (the "arch-identity-theorist" of the 14th century) put it, "every affirmative proposition whose subject or predicate refers to nothing is false"[11].

In any case, as we can see, from the point of view of the doctrine of the *Square* it does not matter which predication theory a medieval author endorsed, as both of these theories imply that affirmative categoricals in general, including universal affirmatives, have existential import[12].

3. Two objections to the medieval analysis

Anyone trained in the modern Frege-Russell tradition in logic may have at least two immediate misgivings concerning attributing existential import to all affirmative categoricals, regardless of further philosophical worries concerning the above-mentioned theories of predication[13].

First, if universal affirmatives have existential import then their contradictories must be true when their subject terms are empty. But the contradictory of, say, 'Every winged horse is a horse' is 'Some winged horse is not a horse'. The latter, however, cannot be true, both because it is contradictory and because it implies the existence of winged horses, while there are no winged horses.

Second, this position seems to undermine the very idea of the affirmation of universal laws concerning hypothetical, never actualized situations. For example, Newton's law of inertia, referring as it does to bodies not acted upon by external forces, would not be true on this analysis as a categorical statement.

Of course, our medieval colleagues were quite aware of these possible objections and worked out their theories accordingly.

4. Replies

4.1. Reply #1: Reference and negation

The first type of objection was easily dismissed by a distinction between *negating* (what we would call *propositional* or *external*) and *infinitizing* (what we would call *term-* or *internal*) negation[14]. To use Russell's famous example, the intended contrast is between

[1] The King of France is not bald
[$\Leftrightarrow$ It is not true that the King of France is bald]

which is true, because France presently has no king, and so it is not the case that the King of France is bald [*negating negation*], and

[2] The King of France is non-bald

which is true when the King of France is a non-bald person, i.e., a person who is both King of France and has hair, whence the proposition is actually false, precisely because there is no such a person [*infinitizing negation*].

The regimented Latin syntax of medieval logic could systematically express this distinction by placing the negation ['*non*'] either before [*negating negation*] or after [*infinitizing negation*] the copula, yielding

[1_L] *Rex Franciae non est calvus*

and

[2_L] *Rex Franciae est non calvus*

respectively[15].

Of course, anyone familiar with Russell's treatment of this example would recognize the distinction between the scopes of the negation in [1] and [2] (or, sticking with Russell's original terminology, the distinction between *primary* and *secondary* occurrences of the description[16]), but they would reject in the same breath that this scope-distinction has anything to do with the strange claim concerning the truth of

[3] Some winged horse is not a horse

implied by the medieval analysis. After all, Russell's distinction is based on the elimination of the *merely apparent reference* to the King of France in both [1] and [2] by paraphrases in which there is not even an appearence of such a reference. This is immediately evident if we consider the corresponding formulae of quantification theory:

[1'] $\sim(\exists y)(Ky \mathbin{\&} \forall x(Kx \supset x = y) \mathbin{\&} By)$
[2'] $(\exists y)(Ky \mathbin{\&} \forall x(Kx \supset x = y) \mathbin{\&} \sim By)$

In these formulae (where 'K' represents '... is present King of France', and 'B' represents '... is bald') there is not even a trace of the apparent referring phrase 'the King of France', and this is why there is not even an apparent reference here to a person who is presently King of France. So Russell's distinction boils down to the difference in the position of the negation in the logical form of [1] and [2], whereas in the case of [3] no such distinction seems to make sense. Indeed, [3] can be formalized *only in one way* with respect to the position of negation in it, namely:

http://www.springer.com/978-1-4020-0216-8

New Essays in Free Logic
In Honour of Karel Lambert
Morscher, E.; Hieke, A. (Eds.)
2001, VII, 255 p., Hardcover
ISBN: 978-1-4020-0216-8