

COMMENTS

The ten essays in this collection cover a wide range of topics. Their temporal span is also quite remarkable. Klima's essay, for instance, concerns medieval notions of reference, while Gumb's has to do with verification procedures in contemporary computer programming. The breadth of concerns in this volume, along with other (not entirely unforeseen) developments – for example the use of free logics by mechanistic mathematicians and computer scientists as foundations for partial functions – testifies to the expanding interest in free logic, and, of course, is extremely gratifying. Moreover, the novel character of the contributions contained herein supports the aptness of the title *New Essays in Free Logic*.

1. COMMENT ON BARBA ESCRIBÁ

Barba's essay is an impressive, no nonsense demonstration that supervaluations can be understood as a special case of information growth. The idea is to produce rigorous translations of "supervaluational free logic" into (1) a "very natural quantified extension of the modal system S4.1" (which van Benthem has shown, along with other extensions of S4, to be a means of modeling partial information and its growth), and into (2) Veltman's Data Logic, a logic specifically devised to capture the notion of information growth. Barba's technical maneuvers are interesting and seem straightforward (at least to these aging eyes), and his main idea seems very natural indeed. So I shall confine my comments to observations, some historical, designed to avoid misunderstandings arising from Barba's language.

As noted by Barba, supervaluations were invented by van Fraassen (who originally called them *state models*). The technique is a semantical procedure most often associated with positive free logics – logics in which some atomic statements containing only singular terms that refer to no existent are counted true – and in particular the most common version of my own proof theoretical treatment. (See, for example, the proof theory in Meyer/Lambert (1986), p.26.) van Fraassen's original proposal, however, was insensitive to changes in truth-value based on logical structure alone. For instance, in van Fraassen's original proposal there was no non-arbitrary way of counting 'Vulcan is Vulcan' true and 'Vulcan is larger than

Mercury' truth-valueless. Bencivenga's augmentation of supervaluational semantics (*via* the intuitive notion of mental experiment), on which Barba's work relies, supplied reasons in place of van Fraassen's fiats, and thus gave a more satisfactory explanation of the truth-value assigned to (especially) atomic statements containing only singular terms referring to no existent. But it must be remembered that the technique of supervaluations is only one kind of semantics for positive free logic *qua* proof theory, and it must be remembered that even if one does not like supervaluations for this purpose, there are many other purposes not necessarily having anything to do with free logic for which the technique has proved useful; for example, Fine's use of them to model the logic of vagueness, Brittan's use of them to explain the Kantian notion of presupposition, and my own use of them to explicate the Copenhagen view of quantum mechanics without giving up classical propositional logic. I say all this because, in the light of the above history, Barba's expression 'supervaluationally free logic' may be misunderstood by some. He is not talking (directly at least) about a semantics for some free logic *qua* proof theory; rather he is talking about a purely model theoretic account of free logic which is supervaluational in character (though it also happens to be positive). In such a development, the semantics consists in saying what the models are and how truth in a model is to be evaluated (recursively). The logic is contained in the definitions of logical truth and logical consequence. So Barba's translations are at the semantic level and not at the syntactic level; in effect, what is shown is that the class of supervaluationally free logic truths translates into (or "reduces" to) the class of S4.1 logical truths (and the logical truths of Data Logic). These remarks make his phrase 'intuitively appealing semantics for supervaluational free logic' less puzzling, I hope, to those who think that the technique of supervaluations *is* a semantical procedure, and hence needs no semantics itself. Finally, it should be noted that Barba's work has broader implications than free logic alone – the title of his essay to the contrary notwithstanding.

2. COMMENT ON GHILARDI

Ghilardi's essay has to do with the very powerful and sophisticated mathematical theory of categories. Though my knowledge of this subject does not extend much beyond Robert Meyer's quip that categories are like sets only bigger, I *can* see that Ghilardi's methods provide a foundation for logics whose theorems hold in all domains, including the empty domain.

But it would also be nice to have a categorial foundation for positive free logics obeying the conditions that the (perhaps empty) domain of discourse is indivisible (which excludes inner domain/outer domain semantics), that the denotation function defined on singular terms is partial (which excludes Meinongian semantics), that every statement gets a truth-value (which excludes supervaluations), and which counts certain statements, e.g., 'Vulcan (the putative planet) is Vulcan', 'Vulcan is the planet causing perturbations in the orbit of Mercury', 'Vulcan is a planet' etc. as true. The story semantics of Lambert/van Fraassen (1972) was an attempt to satisfy these conditions, but the sense in which our stories are really stories is admittedly odd because they are just sets of sentences with some logical relations imposed on them – for example, substitution of identicals. Category theory may offer a more satisfying alternative because its arrow looks like a natural formal counterpart of denotation (or reference).

3. COMMENT ON GUMB

Gumb, a computer scientist, and the first to extend Craig's interpolation lemma and Beth's definability methods to free logic, has crafted a splendid and detailed introduction to the use of free logic in computer programming, especially in program verification. These are techniques to establish the adequacy of computer programs in all sorts of enterprises both theoretical and practical. It is also rich in motivational and historical notes. In fact, free logic has been adopted as the logical foundation in many actual computer programming languages, and has also become the preferred foundation for the treatment of partial functions on which many systems of mechanistic mathematics are based. Dana Scott's COLD is one such system and William Farmer's IMPS another, the former based on a positive free logic and the latter on a negative free logic. Gumb's treatment itself relies on a positive free logic because he believes that only in such a treatment can non-strict functions be adequately treated. These functions, which many computer scientists think are indispensable to any workable program, are functions which receive values even when their arguments are "undefined". For instance, consider the (assumed) partial function designated by 'the referent of the singular term t '. If ' t ' is replaced by an irreferential singular term like '(the planet) Vulcan', the function in question might nevertheless assign it a value, perhaps, in Frege-like fashion, an "error object". Indeed, Gumb reports, this is the preferred treatment in computer science.

I wonder, however, about Gumb's reason for thinking that only a positive free logic is appropriate for dealing with non-strict functions. Gumb seems to believe that since something like an inner domain/outer domain model structure is needed to accommodate error objects – they fall outside the range of the quantifiers and hence in the outer domain – a positive free logic is required because such model structures are perhaps most commonly used in developing positive free logics. Nevertheless, such a model structure does not by itself guarantee that one's free logic will be positive. In fact, the most general treatment of negative free logic, the treatment of Ronald Scales in Scales (1969), utilizes inner domain/outer domain model structures. Of course, if one wishes statements such as ' $1/0 = 1/0$ ' to turn out true, then a positive free logic is required. But that depends on how one defines truth, and not on the kind of model structure employed. Thus, in a positive free logic the clause in the truth-definition for identity will be ' $a = b$ ' is true when and only when the referent of ' a ' is the same as the referent of ' b ', where the referents of the singular terms are chosen from the union of the inner and outer domains. But in a negative free logic, the clause in question will have the restriction that the referents of the singular terms must be in the inner domain (intuitively, the universe of existents) and thus exclude error objects. Hence ' $1/0 = 1/0$ ' will be true given the unrestricted clause for identity in the truth-condition, but false in the restricted reading despite the fact that the model structure is of the inner domain/outer domain stripe and that the referent of 'Vulcan' is an error object. This reinforces the fact that whether a free logic is positive or negative has more to do with the definition of truth in a model rather than with the character of the model structure itself.

4. COMMENT ON KLIMA

Klima's essay is a model of erudition and provocative argument. Contra a view of the relationship between free logic, classical predicate logic and medieval logic holding free logic to be but the completion of a task begun by the tradition beginning with Frege and Russell, namely, the ridding of logic of existence assumptions with respect to its terms both singular and general – indeed, a view I have promoted from time to time – Klima argues that the majority of medieval logicians espoused, *via* the doctrine of supposition, something like free logic, and only a minority sponsored the doctrine of existential import with respect to terms. The picture which evolves is that classical logic undid the balance which prevailed in medieval logic

by supporting only the admission of general terms free of existence assumptions, and that free logic goes at least part way in restoring the medieval balance by permitting singular terms that also may fail to refer to existents. Klima's expertise – historical, philosophical and technical – is hard to resist, and I shall make no such attempt here. But I do want to make a remark concerning his defense of the medieval logic against the charge that it undermines the affirmation of universal laws concerning unactualized entities, for example, Newton's law of inertia.

According to Klima, the dominant account in medieval logic can be preserved in the face of objections that it cannot accommodate laws like Newton's law of inertia, but only at the expense of allowing non-existents in the range of the variables. He stoutly defends this position against those who might characterize it as relentlessly Meinongian, that is, as requiring there to be non-actual entities "out there" awaiting reference to, and quantification over, them. His view is that such objects are intentional objects, that such objects are what we think of, and as such are things to which we refer. Nevertheless, I still feel uncomfortable. What disturbs non-Meinongians is not the ontological independence of non-existent entities, like bodies on which there are no external forces acting or round squares, but rather that there are any such objects at all serving as the objects of reference, even if always tethered to some mind. Nor is it any real resolution to say that what is referred to is the thought of round square or the thought of body on which no external force is acting because these objects exist whereas round squares and bodies upon which there is no external force acting do not. If the effort to accommodate Newton's law of inertia requires the variables to range over non-existents, they had better be non-existent.

5. COMMENT ON LEHMANN

Lehmann's Fregeian inspired semantics for free logic is new, and is a departure from the similarly inspired essays of Smiley and Skyrms, both in semantical details and in the treatment of statements of the form 'Vulcan exists'. Indeed, I find much of what Lehmann says appealing and clever. It is no secret that despite my own use of supervaluations, and despite a deep appreciation of the ingenuity explicit in their various manifestations by van Fraassen and Bencivenga, I am less than enamored of them as a foundation for positive free logic. So, I find Lehmann's view that interpretations should represent the possibilities given by reference failure intuitive, his treatment of existence as an object language notion to be just

New Essays in Free Logic
In Honour of Karel Lambert
Morscher, E.; Hieke, A. (Eds.)
2001, VII, 255 p., Hardcover
ISBN: 978-1-4020-0216-8