CHAPTER 3

TRUTHLIKENESS

We encountered the distinction between verisimilitude and truthlikeness definitions in Chapter 1. Verisimilitude definitions define distance to the truth using truth-value and logical strength, and were presented in Chapter 2. Truthlikeness definitions establish distance to the truth using similarity between the possible worlds, and are the subject of the present chapter.

Truthlikeness definitions originate from two different sources. The first root is Hilpinen's address to the Warsaw conference in 1974.[1] In Section 3.1, we shall see how Hilpinen uses D. Lewis's (1973) counterfactuals to specify his ideas about an approximate logic and truthlikeness.[2] Subsequently, it was Niiniluoto who took up Hilpinen's comparative approach, and converted it into a quantitative truthlikeness proposal. His approach is the subject of Section 3.2. Kuipers published his first version of comparative truthlikeness in 1987. His refined comparative definition is the subject of Section 3.5.

Pavel Tichý's 1974 paper forms the second root of the truthlikeness strategy. In this publication, he revealed the major flaw of Popper's original proposal, and initiated a new truthlikeness approach based on Hintikka's constituents. Oddie adjusted and elaborated Tichý's quantitative method; this forms the subject of the Section 3.3.[3] More recently, Heidema in cooperation with Brink and Burger have published qualitative truthlikeness definitions.[4] They order the propositions of a two-propositional language (almost) in the same way as Tichý and Oddie. Hence, as the quantitative-comparative distinction seems to be subsidiary, and I preferred a historical line of presentation to the systematic one, I shall introduce the proposal of Heidema and others in Section 3.4. The last section concerns the conclusions of this chapter.

3.1. HILPINEN

Hilpinen's *Approximate Truth and Truthlikeness* includes the main ingredients of his address to the Warsaw conference in 1974. It is remarkable in two respects. In the first place, the conference was held before Tichy's and Miller's commentary on the flaw in Popper's definition was published and generally acknowledged. In the second place, it was the first publication that stressed the difference between likeness and content approaches, and introduced the *truthlikeness* notion. Our introduction summarizes Hilpinen (1976).

3.1.1. The Definition

Hilpinen fixes the similarity between theories, and more specifically the similarity between a theory and the truth, by setting up an approximate *A-logic*. This approximate logic is the standard propositional logic enriched with a new approximate operator **A**. The basic idea is that a statement 'P' is *approximately true* at world u, if and only if, P is true in some world similar (or close to) to u. U is the class of all possible worlds of the language. Suppose that N_u refers to the set of possible worlds close (or similar) to u, and let P be a proposition of the language, then,

$$\vDash_u \mathbf{A}P \text{ if and only if } \vDash_v P \text{ for some } v \in N_u$$

No N_u is empty since $u \in N_u$, and the similarity relation is symmetric. The A-logic turns out to be at least as strong as the Brouwerian system of modal logic.[5]

Next, Hilpinen chooses *characteristics* C^i fixing the mutual likeness of the possible worlds. It is important to note that Hilpinen *is the only one* who explicitly mentions the relation between the *independently* chosen characteristics C^i and the likeness between possible worlds. This will prove to be the most important part of the solution to the problem of alleged 'language-dependency' of likeness definitions.[6] Following David Lewis, Hilpinen defines a *system of nested spheres* $\mathcal{N}_u^i$ related to a world $u \in U$ and to a characteristic C^i. It is a family of subsets of U such that

For every $K, L \in \mathcal{N}_u^i$, $K \subseteq L$ or $L \subseteq K$ (nesting), and
$u \in K$ for all $K \in \mathcal{N}_u^i$ (weak centering).

Consequently, $u \in U$, $K \in \wp(U)$, and $\mathcal{N}_u^i \in \wp(\wp(U))$. Hilpinen assumes that $\mathcal{N}_u^i$ is closed under taking (nonempty) intersections and unions, and therefore that $\cap\mathcal{N}_u^i$ is the smallest, and $\cup\mathcal{N}_u^i$ is the largest sphere around u. $\mathrm{Mod}(\mathcal{L}) - \cup\mathcal{N}_u^i \neq \varnothing$ if there are worlds not similar to u at all. To simplify the comparison with the other definitions, I assume that, in addition to being centering, $\mathcal{N}_u^i$ *is centered*—i.e. $\{u\} \in \mathcal{N}_u^i$. Furthermore, from now on, I assume that C^i is fixed, and shall drop the index i. Nevertheless, Hilpinen's truthlikeness definition remains depending on the independent choice of the specific set C^i of characteristics.

Hilpinen's truthlikeness proposal combines two elements. The first one has to do with the *distance* between the truth u and the most u-similar possible world of the theory. The second element concerns the amount of *information of the theory*, or its 'verbosity'. Informally, the definition of Hilpinen says that the theory P has a higher degree of truthlikeness than Q if and only if the minimum distance of P to the truth is smaller than the distance of Q to the truth, and P is more informative about the truth than Q. Hilpinen uses the systems of nested spheres to give an exact definition of $E_u(P)$, the distance between P and u, and of $I(P)$, the amount of information of P. He defines $E_u(P)$, by way of the set of elements of $\mathcal{N}_u$ that do not contain models for P:

(1) $$E_u(P) :=_{def} \{K \mid K \in \mathcal{N}_u \text{ and } K \cap \mathrm{Mod}(P) = \varnothing\}$$

$E_u(P)$ is the set of spheres around u that do not intersect with Mod(P). Thus, $E_u(P)$ is the set of spheres "between" $\cap\mathcal{N}_u$ and Mod(P) (see Figure 1). The definition of $E_u(P)$ yields the following propositions.

$$E_u(\top) = \varnothing$$
$$E_u(\bot) = \mathcal{N}_u$$
$$E_u(P \vee Q) = E_u(P) \cap E_u(Q)$$
$$E_u(P) \subseteq E_u(P \wedge Q)$$
$$E_u(P) = \varnothing \text{ or } E_u(\neg P) = \varnothing$$

Hilpinen also uses his system of nested spheres to define the amount of information of hypothesis P. The more distant worlds P excludes, the more information P provides about the truth u. Formally, he defines:

(2) $$I_u(P) :=_{def} \{K \mid K \in \mathcal{N}_u \text{ and } \varnothing \neq \mathrm{Mod}(P) \subseteq K\}$$

If the definition had allowed Mod(P) to be empty, then $P = \bot$ would have implied $I_u(\bot) = \mathcal{N}_u$, and the contradiction would have been maximally informative about the truth. Hilpinen avoids this counter-intuitive consequence, with the constraint Mod(P) $\neq \varnothing$; hence the contradiction is the worst element. Intuitively, $I_u(P)$ is the set of spheres 'between' Mod(P) and the largest sphere $\cup\mathcal{N}^i_u$. Hilpinen"s definition implies the following propositions.

$I_u(\top) = \varnothing$ (in case there are uncomparable worlds)
$I_u(\bot) = \varnothing$
if Mod(P) $\neq \varnothing$ and Mod(Q) $\neq \varnothing$, then: $I_u(P \vee Q) = I_u(P) \cap I_u(Q)$
if Mod($P \wedge Q$) $\neq \varnothing$, then: $I_u(P) \subseteq I(P \wedge Q)$
$I_u(P) = \varnothing$ or $I_u(\neg P) = \varnothing$

Hilpinen defines truthlikeness by combing $E_u(P)$ and $I_u(P)$. Let P and Q be propositions of an A-logic $\mathcal{L}$, and let model u (or τ) represent the true model. Then,

DEFINITION 3.1: The *truthlikeness of Q does not exceed that of P* if and only if
$E_u(P) \subseteq E_u(Q)$ and $I_u(Q) \subseteq I_u(P)$
Notation: $P \leq^H_u Q$ (or $\psi \leq^H_\tau \varphi$)

Since $\leq^H_u$ is reflexive and transitive, but not antisymmetric, it is a preordering of the $\mathcal{L}$-propositions. Intuitively, definition 3.1 boils down to: Mod(P) is at least as close to u as Mod(Q) is, and, by excluding more distant worlds, Mod(P) reveals at least as much information about u as Mod(Q). Figure 1 depicts this situation.

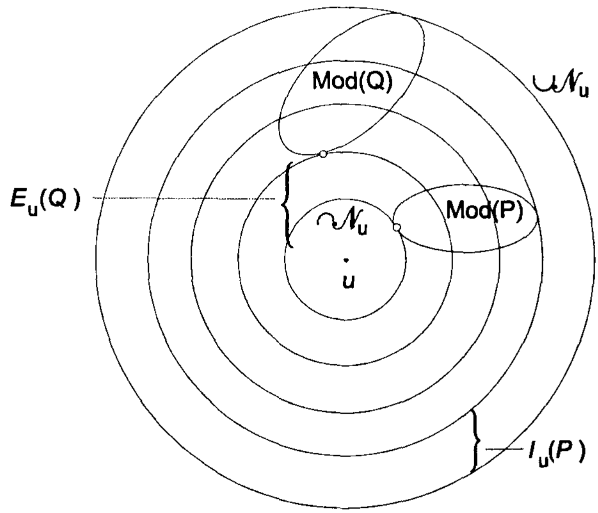

Fig. 1. The nested spheres $\mathbf{N}_u$

Note that, strictly speaking Hilpinen orders *sentences*, such as P and Q, by their similarity to a *possible world* u. No other proposal presented here defines similarity between sentences on the one hand, and models on the other. In addition to definition 3.1, Hilpinen defines (I use my symbols):

$P <_u Q :=_{def} P \leq_u Q$ and $Q \nleq_u P$, P is more truthlike than Q
$P =_u Q :=_{def} P \leq_u Q$ and $Q \leq_u P$, P and Q are equal truthlike;
$P \parallel Q :=_{def} P \nleq_u Q$ and $Q \nleq_u P$, P and Q are uncomparable regarding u (and C^i).

Two theories P and Q are uncomparable if the distance factor and the information factor point in different directions.[7] This obtains if P is closer to the truth but Q is more informative, or the reverse is true:

$E(P) \subset(\supset) E(Q)$ and $I(P) \subset(\supset) I(Q)$

Finally, we show how Hilpinen's approach fares in the bi propositional case. Let $K_1 := \{\langle 1,1\rangle\}$, $K_2 := \{\langle 1,1\rangle, \langle 1,0\rangle, \langle 0,1\rangle\}$ and $K_3 :=$ Mod($\mathcal{L}$) and $\mathcal{N}_u := \{K_1, K_2, K_3\}$. Since $P =_{p\wedge q} Q$ does not imply that P and Q are logically equivalent, we order sets of $\mathcal{L}[p,q]$ propositions.

Figure 2 shows the $\leq^H_{p\wedge q}$-ordering of the sets of $\mathcal{L}[p,q]$-propositions with the same truthlikeness (the truth is $p \wedge q$). It demonstrates, at least in this elementary case, that Hilpinen's definition is truth-value dependent. There is even no false theory that is more similar to $p \wedge q$ than the tautology. Let us turn to some remarks regarding Hilpinen's proposals and examine how it fares regarding the metatheoretical properties mentioned in the first chapter.

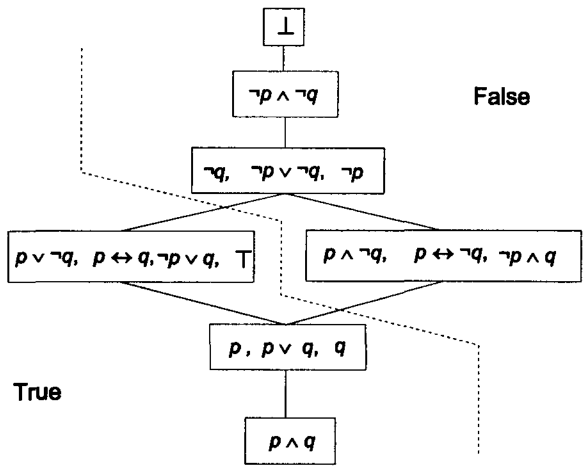

Fig. 2. Hilpinen's ordering of Prop($\mathscr{L}$[p,q])

3.1.2. Remarks and Comments

In the first part of this subsection, I shall characterize the difference between the content approach and Hilpinen's likeness strategy regarding propositional languages. In the second part, we shall discuss the metatheoretical properties of the approximate logic definition of truthlikeness.

The following example illustrates the difference between Hilpinen's truthlikeness and Miller's content definition (the truth is complete in both cases). As Miller's definition is weaker than Hilpinen's proposal, the best way to compare them is to see under which conditions $\psi <^{\Delta}_{\tau} \varphi$ does not imply $\psi \leq^{H}_{\tau} \varphi$. Suppose that $\psi <^{\Delta}_{\tau} \varphi$; then there are four situations to be considered in which Hilpinen's definition establishes a strict ordering ($\mathfrak{W}$ represents the relevant structure of our world):

1. φ and ψ are true; then $E_{\mathfrak{W}}(\varphi) = E_{\mathfrak{W}}(\psi) = \varnothing$; (and $I(\varphi) \subset I(\psi)$)
2. ψ is true and φ is false; $E_{\mathfrak{W}}(\varphi) \supset E_{\mathfrak{W}}(\psi)$; (and $I(\varphi) \subseteq I(\psi)$)
3. ψ and φ are false and $E_{\mathfrak{W}}(\varphi) = E_{\mathfrak{W}}(\psi)$; (and $I(\varphi) \subset I(\psi)$)
4. ψ and φ are false and $E_{\mathfrak{W}}(\varphi) \subset E_{\mathfrak{W}}(\psi)$; (and $I(\varphi) \subseteq I(\psi)$)

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