

CHAPTER 5

THE HIDDEN VARIABLE

I will discuss the alleged “language-dependency” of truthlikeness definitions in this chapter. When I introduced the differences between content and likeness definitions in the first chapter, I briefly mentioned the possible change of a truthlikeness order after extensional substitutions. Now it is time to present and examine Miller’s objection to truthlikeness definitions.¹ This chapter has the following outline. After the introduction, in Section 5.2 I introduce Miller’s puzzle and describe the reactions it provoked. In Section 5.3 I will discuss the formal analysis of the translation puzzle; and in the fourth section I formulate the solution. I end the chapter with a summary and a list of results.

5.1. INTRODUCTION

We saw that Miller and Tichý both published their discovery of the flaw in Popper’s verisimilitude definition in 1974.² Miller also criticized Tichý’s alternative definition for being “language-dependent” in the same paper. According to Miller, a truthlikeness order that is not preserved under extensional substitutions, is “language-dependent,” and therefore “demonstrably false.”³ Miller’s criticism triggered a “language-dependency” debate. For example, according to Niiniluoto, additional requirements are needed to pass on a truthlikeness order from one language to another.⁴ He doubts whether there are semantical conditions that can guarantee the “same truthlikeness order under translation.” Pragmatic considerations are likely to enter the argument. Despite the various answers to Miller’s criticism, the problem has not been solved yet: witness Barnes (1991).

In this chapter I introduce Miller’s argument, and consider it under the strongest interpretation possible. My answer is radically different from those proposed so far. Miller’s observation about truthlikeness definitions is correct, and although he presents it as a curse, it proves to be a blessing. If one distinguishes between the language of the cognitive problem, and that of the formulation of the theories, Miller’s argument shows that truthlikeness is a relative, rather than an absolute notion.

5.2. MILLER'S OBJECTION TO THE "COUNTING METHOD"

I introduce Miller's extensional substitution argument against truthlikeness definitions as far as propositional languages are concerned in this section. First I shall introduce Miller's objection, and sketch some of its consequences. Then, I describe two intuitive foundations of the truthlikeness notion that come under fire in Miller's attack. I then deal with the solutions found in the literature, and finally, I show that the refined structuralist proposals also fall prey to Miller's argument.

5.2.1. *Miller's Argument of Extensional Substitution*

Our introduction of Miller's extensional substitution argument will be as short as possible and therefore it is presented in a slightly different manner than the original.⁵ To start with, let 'Anglo-Saxons' use a "rudimentary weather language \mathcal{L} ", to describe the weather conditions. \mathcal{L} consists of:

1. the logical machinery of propositional logic and
2. the non-logical vocabulary ' h ', ' r ', ' w ',

which designate the basic propositions 'it is hot', 'it is rainy' and 'it is windy', respectively. All Anglo-Saxons agree that the next propositions adequately represent the weather conditions on the first, second, and third day of September.

- (1) $X = \neg h \wedge \neg r \wedge \neg w$, the first day
 $Y = \neg h \wedge r \wedge w$, the second day
 $T = h \wedge r \wedge w$, the third day.

Recall that we neglect the syntactic differences of equivalent \mathcal{L} -sentences and represent propositions in the disjunctive normal form (dnf); for instance we do not distinguish between $\neg(h \vee r) \wedge \neg w$ and X .

Further, we say that the weather has changed smoothly or *continuously* if the conditions of yesterday agree with the conditions of today on more literals than the conditions of the day before yesterday do. Else, if the conditions of yesterday agree with the conditions of today on *less* literals, it is said that the weather changed *discontinuously*. In our example, the weather conditions of Y differ from those of T only in the temperature dimension whereas the conditions of X and T differ in all relevant dimensions. Consequently, all Anglo-Saxons agree that over the first three days of September the weather changed *continuously*. The same intuition underlies the naive definition of truthlikeness for propositional languages. Consider X , Y and T to be theories, and let T be the true theory. Then, it is plausible to say that Y is closer to the truth T than X for the very same reason. X differs from T in more basic dimensions than Y differs from T .

Miller, however, rejects the preceding "counting method" because it lacks robustness.⁶ To illustrate his point, he proposes translating X , Y and T into X' , Y' and T' using another rudimentary weather language \mathcal{L}' , which also has three atomic propositions: h , m and a . The three transformation formulae (MP) of (2) fix the relation between \mathcal{L} and \mathcal{L}' :

$$(2) \quad \begin{aligned} h &:=_{tr} h, \\ m &:=_{tr} h \leftrightarrow r, \\ a &:=_{tr} h \leftrightarrow w \end{aligned}$$

Miller calls \mathcal{L} and \mathcal{L}' *intertranslatable*, and proposes to call ' m ', 'it is Minnesotian' and ' a ', 'it is Arizonian'.⁷ Formulae (2) are used to transform the propositions of (1) into

$$(3) \quad \begin{aligned} X' &= \neg h \wedge m \wedge a, \\ Y' &= \neg h \wedge \neg m \wedge \neg a, \\ T' &= h \wedge m \wedge a \end{aligned}$$

Clearly, this extensional substitution does not preserve the naive truthlikeness order. Counting the differences between Y' and T' and those between X' and T' leads to the conclusion that X' is closer to the true theory T' than Y' although $X \equiv X'$, $Y \equiv Y'$ and $T \equiv T'$.⁸ People using language \mathcal{L} conclude that the weather of the first three days of September changed gradually whereas other people, using \mathcal{L}' , conclude the opposite; the changes in the weather during the same period and at the same place, were *discontinuous*.⁹

Miller's h, r, w -argument only takes one and a half page in print. Consequently, he did not discuss all the relevant questions in depth. At least two of them are indispensable for a good understanding of the present example. The first one is: does (2) comprise object or metaformulae, and the second concerns the semantics of \mathcal{L} and \mathcal{L}' .

I will answer the object or metaformulae problem first. Miller's objection does not hold if (2) are object formulae. Since then, X' , Y' and T' are sentences of an extended h - r - w - m - a language \mathcal{L}_{ext} in which (2) are meaning postulates; and as the naive definition only takes into account basic literals, Miller's transformation does not affect the original truthlikeness order. Miller, therefore, takes (2) to be *metaformulae*: witness Tichý's (1978) note four. The metaformulae do not increase the non-logical vocabulary of \mathcal{L} , neither do they introduce m and a in \mathcal{L} . The formulae only guarantee identical truth-values of related sentences in corresponding possible worlds. According to Miller \mathcal{L} and \mathcal{L}' have the same power of expression since the transformation defines a bijection between their constituents.

The second question to be asked is whether the meanings of \mathcal{L} coincide with those of \mathcal{L}' . Having observed that the transformation formulae (MP) for \mathcal{L}' are

$$(4) \quad h :=_{tr} h$$

$$\begin{aligned} r &:=_{tr} h \leftrightarrow m \\ w &:=_{tr} h \leftrightarrow a \end{aligned}$$

Miller claims “perfect symmetry” between \mathcal{L} and \mathcal{L}' .¹⁰ Comparison of (2) and (4) supports the symmetry thesis between \mathcal{L} and \mathcal{L}' on the syntactic level. On the semantic level, however, this symmetry is compatible with two mutually exclusive interpretations of the languages.

According to the first interpretation, corresponding *possible worlds remain separate entities*, and (2) does not connect the (extensional) meanings of \mathcal{L} and \mathcal{L}' . Note that as \mathcal{L} and \mathcal{L}' are different languages, in principal they have different semantics, and the logically possible worlds depend on the vocabulary of those languages. Since the substitution, or “translation”, of $\neg h \wedge r \wedge w$ yields $\neg h \wedge \neg m \wedge \neg a$, the \mathcal{L} -possible world $\langle 0,1,1 \rangle$ relates to the \mathcal{L}' -possible world $\langle 0,0,0 \rangle$, they do not coincide.

Secondly, we may interpret (2) such that the extensions of the \mathcal{L} -sentences are *identical* to the extensions of the corresponding \mathcal{L}' -sentences. According to this interpretation, the \mathcal{L} -possible world $\langle 0,1,1 \rangle$ is identical to the \mathcal{L}' -possible world $\langle 0,0,0 \rangle$ since the “translation” of $\neg h \wedge r \wedge w$ is $\neg h \wedge \neg m \wedge \neg a$. Identification of \mathcal{L} - and \mathcal{L}' -extensions means that a translation from \mathcal{L} to \mathcal{L}' does not change the original meanings. Later publications imply that Miller favours the second interpretation; he acknowledges: “there is a problem of explaining under what conditions this [that is: sentences in different languages having the same assertive power (SZ)] obtains.”¹¹

Finally, it should be noted that the influence of Miller’s example goes beyond the scope of truthlikeness. It is also relevant for comparison and assessment of empirical success of a theory. Hence, empiricists should also take notice of Miller’s example. Moreover, the argument applies to forms of logic in which the likeness between possible worlds plays an important role. Among others, many forms of non-monotonic reasoning; conditional logic (counterfactuals) and preferential reasoning are important examples.

To summarize my introduction of Miller’s objection, I tentatively conclude that, as far as extensional interpretations are concerned, (2) comprises *metaformulae identifying* the corresponding possible worlds of \mathcal{L} and \mathcal{L}' .

5.2.2. Two Intuitions

Miller’s argument affects two ideas underlying the approach-to-the-truth project. I will discuss the notion of an *independent truth* first and then I will say something about the idea of an *objective approach* to the truth.

In the pre-Tarskian era, the borderline between *truth* and *reality* was vague. C.S. Peirce, one of the first to write about the modern idea of approach to the truth,

stressed the difference between the two. The following quotation shows, however, that, according to Peirce, besides its property-like character, truth is autonomous and language independent. Moreover, he seemed to hold, following Kant, that Truth is a *regulative* idea. After many years of experimenting, science will ultimately succeed in approaching the Truth.

"Truth is a character which attaches to an abstract proposition, such as a person might utter. It essentially depends upon that proposition's not professing to be exactly true. But we hope that in the progress of science its error will indefinitely diminish, just as the error of 3.14159, the value given for π , will indefinitely diminish as the calculation is carried to more and more places of decimals." (C.S. Peirce (1965, 5.565))

This intuitive idea of Truth I shall call the *citadel conception* of Truth. It has the following characteristics. Truth has an independent character, and is only obtainable by hard labour. There are many roads leading to the same Truth possibly formulated in different languages. It is a rather vague metaphysical notion primarily ascribed to propositions. No wonder adherents of this conception of truth needed to emphasize the difference between reality and truth.

The next quotation demonstrates Popper's insight that Tarski's truth definition may replace the preceding metaphysical notion of truth.

"Yet whenever I used to write, or say, something about science getting nearer to the truth, or as a kind of approach to the truth, I felt that I really ought to be writing 'Truth', with a capital 'T', in order to make quite clear that a vague and highly metaphysical notion was involved here, in contradiction to Tarski's 'truth' which we can with a clear conscience write in the ordinary way with small letters. It was only quite recently that I set myself to consider whether the idea of truth involved here was really so dangerously vague and metaphysical after all. Almost at once I found that it was not, and that there was no particular difficulty in applying Tarski's fundamental idea to it." (Popper (1972, p 231-232))

In contrast with the citadel conception of Truth, I shall call Tarski's systematic definition of truth, *truth-in- \mathcal{L}* . Its characteristics are: truth-in- \mathcal{L} can only be defined on the basis of one specific formal language \mathcal{L} ; if $\text{voc}(\mathcal{L})$ consists of observable notions, such as found in Miller's example, the truth-value of a sentence is relatively easy to obtain; reformulation of the same truth must use the same formal language \mathcal{L} ; truth is an exact notion and it is a property of sentences.

Popper proposed to replace the *independent* truth notion in Peirce's approach-to-the-truth with Tarski's *dependent* truth-in- \mathcal{L} , and as adequate translations preserve truth-values and deduction relations, translation of the truth does not affect a verisimilitude, content, order.¹² As under the same translation truthlikeness, likeness, orders may change, Miller's argument shows that truthlikeness proposals break with the tradition of an independent, absolute, truth.

The second idea that increases the impact of Miller's objection is that approach to the truth must be an *objective* notion. Adequate translations must preserve truth-values, and Tarski's truth definition provides an *objective* notion of truth. Similarly,



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