

CHAPTER 2

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FRAMING STUDENTS' MATHEMATICS-RELATED BELIEFS

A Quest For Conceptual Clarity And A Comprehensive Categorization

Abstract. Despite the general agreement among researchers today that students' beliefs have an important influence on mathematical problem solving there is still a lack of clarity from a conceptual viewpoint. In this chapter we present a literature review of available categorizations or models of students' beliefs related to mathematics learning and problem solving. These reveal that although they all cover a broad spectrum of relevant beliefs, there appears to be no consensus on the structure and the content of the relevant categories of students' beliefs. A philosophical and psychological analysis of the nature and the structure of beliefs enables us to come to a deeper understanding of the development and the functioning of students' beliefs and to clarify the relation between beliefs and knowledge. The insights developed through this analysis result in an elaborated and concrete definition of students' mathematics-related beliefs and allow us to develop a theoretical framework that coherently integrates the major components of prevalent models of students' beliefs. We differentiate between students' beliefs about mathematics education, students' beliefs about the self, and students' beliefs about the social context, i.e., the class context.

1. INTRODUCTION

Recent theories on cognition and learning (e.g., Greeno, Collins, & Resnick, 1996; Salomon & Perkins, 1998) point to the social-historical embeddedness and the constructive nature of thinking and problem solving. According to these theories, each form of knowing and thinking is constituted by the meanings and rules that function in the specific communities in which they are situated (e.g., the scientific community, the class, the group). Acquiring knowledge or learning, therefore, consists of getting acquainted with the concepts and rules that characterize the activities in the different contexts. As such, learning becomes fundamentally a social activity.

From such a perspective, learning is primarily defined as a form of engagement that implies the active use of certain cognitive and metacognitive knowledge and strategies, but cannot be reduced to it. Indeed, more and more researchers (e.g., Bereiter & Scardamalia, 1993) are convinced that referring only to cognitive and metacognitive factors does not capture the heart of learning. Several studies (e.g.,

Connell & Wellborn, 1990; Schiefele & Csikszentmihalyi, 1995) point to the key role conative and affective factors play as constituting elements of the learning process, as well as and in close interaction with (meta)cognitive factors. Motivation and volition (i.e., the conative factors) are no longer seen as just the fuel or the engine of the learning process, but are perceived as fundamentally determining the quality of learning. In a similar way, self-confidence and positive emotions (affective factors) are no longer considered as just positive side effects of learning, but become important constituent elements of learning and problem solving.

Recent developments in the field of research on mathematical problem solving tend to illustrate this change in perspective. Studies on students' beliefs about mathematics (e.g., Garofalo, 1989; Kouba & McDonald, 1986; Schoenfeld, 1985a) and on their motivational beliefs (e.g., Pintrich & Schrauben, 1992; Seegers & Boekaerts, 1993), as well as research on the influence of emotions (Cobb, Yackel, & Wood, 1989; DeBellis, 1996) and on other affective factors such as "students' perceived confidence" (Vermeer, 1997) aim at unraveling the role of conative and affective factors in mathematical problem solving. On a conceptual level researchers try to capture the interrelated influence of (meta)cognitive, conative and affective factors on mathematical learning and problem solving in line with the notion of a "mathematical disposition". Such a disposition refers to the integrated mastery of five categories of aptitude (De Corte, Verschaffel, & Op 't Eynde, 2000):

1. A well-organized and flexibly accessible knowledge base involving the facts, symbols, algorithms, concepts, and rules that constitute the contents of mathematics as a subject-matter field.
2. Heuristics methods, i.e., search strategies for problem solving which do not guarantee, but significantly increase, the probability of finding the correct solution because they induce a systematic approach to the task.
3. Metaknowledge, which involves knowledge about one's cognitive functioning (metacognitive knowledge), on the one hand, and knowledge about one's motivation and emotions that can be used to deliberately improve volitional efficiency (metavolitional knowledge), on the other hand.
4. Mathematics-related beliefs, which include the implicitly and explicitly held subjective conceptions about mathematics education, the self as a mathematician, and the social context, i.e., the class-context.
5. Self-regulatory skills, which embrace skills relating to the self-regulation of one's cognitive processes (metacognitive skills or cognitive self-regulation), on the one hand, and of one's volitional processes (metavolitional skills or volitional self-regulation), on the other hand.

Acquiring such a mathematical disposition is necessary for students to become competent problem solvers, equipped to recognize and tackle mathematical problems in different contexts, and, as such, able to overcome the well-known phenomenon of inert knowledge (see the National Council of Teachers of Mathematics [NCTM], 2000). After all, according to Perkins (1995), the integrated mastery of these different kinds of knowledge (i.e., domain-specific, metacognitive, metavolitional), skills and beliefs results in a sensitivity to the occasions when it is

appropriate to use them and an inclination to do so. This sensitivity to situations and contexts, and the inclination to follow through, are both determined by the concepts and beliefs a person holds. A person's beliefs about what counts as a mathematical context and what (s)he finds interesting or important will, as such, have a strong influence on the situations (s)he will be sensitive to, and whether or not (s)he will engage in them. The relevance of beliefs as a component of a mathematical disposition and their impact on mathematics learning is echoed in the *Curriculum and evaluation standards for school mathematics* (NCTM, 1989) in the U.S.A.: "These beliefs exert a powerful influence on students' evaluation of their own ability, on their willingness to engage in mathematical tasks, and on their ultimate mathematical disposition" (p. 233).

It is nowadays generally assumed that the impact of students' mathematics-related beliefs on their learning and problem-solving behavior is mediated through cognitive as well as conative and affective processes. First, several researchers have shown how students' "beliefs about mathematics" determine how they choose to approach a problem and which techniques and cognitive strategies will be used (e.g., Garofalo, 1989; Schoenfeld, 1985a). Secondly, others have pointed to the implications of students' "mathematically related beliefs" for their motivational decisions in mathematics learning and problem solving (e.g., Kloosterman, 1996). Finally, it is argued that "students' beliefs related to mathematics education" provide an important part of the context within which emotional responses to mathematics develop (e.g., Isoda & Nakagoshi, 2000; McLeod, 1992). Moreover, this relationship between beliefs and emotions seems to be reciprocal. It is explained that local emotional experiences over time provide the context for the development and strengthening of more stable "global" affect as attitudes and beliefs (Goldin, this volume).

Notwithstanding the general agreement among researchers that students' beliefs have an important influence on mathematical learning and problem solving, there is still a lack of clarity from a conceptual viewpoint (see also Furinghetti & Pehkonen, this volume). The diversity in the terms used to describe relevant beliefs, sometimes referring to the same, at other times to different beliefs, is symptomatic of the actual state of the research domain. Despite, or maybe precisely because of the attention paid to the multiple ways in which different student beliefs influence mathematical learning and problem solving, research on this topic has not yet resulted in a comprehensive model of, or theory on, students' mathematics-related beliefs. As a matter of fact, most of the studies are situated in, respectively, cognitive, motivational or affective research traditions and in many cases they operate in relative isolation from each other. This results in a conceptual and theoretical confusion that is not at all advantageous for the development of a comprehensive theory or model of mathematics-related beliefs and the impact of these beliefs on students' learning and problem solving. We are well aware that the state of the art of the research field does not allow the development of a comprehensive theory at the moment. Nevertheless an attempt will be made in this chapter to clarify the conceptual discussion and to introduce a framework for students' mathematics-related beliefs that might be a further step in that direction.

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