

CHAPTER 1

COMMUNICATION IN THE MATHEMATICS CLASSROOM

The purpose of teaching mathematics is to point out mistakes and correct them! This seems to be a common understanding of mathematics education among many students.¹² We have even seen examples of pre-school children expressing the same view in a role play about teaching mathematics.¹³ One child was playing the role of the teacher the rest were 'students'. One 'student' was supposed to do an exercise on the blackboard and wrote some serious-looking symbols in a long row. Afterwards, the 'teacher' erased a couple of those symbols and wrote some others while accusing the 'student' of being mistaken. Thus, even before having any school experiences of their own and without having an understanding of what symbols might mean, the children showed an understanding of mistakes and of the correction of mistakes as being a central parameter in mathematics education.

One reason why the notion of 'mistakes' seems so important in mathematics education can be related to the search for 'truth' in mathematics. A main task of a philosophy of mathematics has been to give an adequate explanation of 'truth'. Absolutism in epistemology is associated with the idea that the individual has the possibility to acquire absolute truth. This idea connects with the Euclidean ideal in epistemology. Relativism, though, maintains that truth is always located by someone in a certain context at a certain time. Thus, truth cannot be grasped in absolute terms. With mathematics in mind, relativism has been put forward by both radical and social constructivism.¹⁴

¹² Alrø and Lindenskov (1994). This chapter is a rewriting of Alrø and Skovsmose (1996a, 1998).

¹³ See Fosse (1996).

¹⁴ See, for instance, Glasersfeld (1995) and Ernest (1998a).

Somehow the philosophic discussion of mathematical truths, becomes reflected in a discussion of mistakes in the mathematics classroom.¹⁵ Like the concept of 'truth', the concept of 'mistake' has two extremes – one absolutist and one relativistic. The absolutist interpretation apparently has a sound basis. For instance, to think that 12 multiplied by 13 equals 155 seems a simple mistake. But the situation looks somewhat different if we come to the applications of mathematics. If we measure one side of a play ground to be (about) 12 m and the other side to be (about) 13 m, its area may well be 155 m² – the ground looks rectangular. Relativism may have a bearing when the application of mathematics is considered. Nevertheless, it often seems possible to make absolute mistakes when applications of mathematics are presented in mathematics textbooks. (We will return to this point in the section 'From exercises to landscapes of investigation' in Chapter 2.)

In the first section of this chapter we discuss mistakes and correcting of mistakes on the basis of classroom observations.¹⁶ We suggest the notion of bureaucratic absolutism to characterise the type of learning environment, where mistakes are handled in absolute terms. Mistakes are simply *mistakes* and have to be eliminated. This learning environment corresponds very well with the communication pattern: Guess What the Teacher Thinks.¹⁷ Further, referring to a non-bureaucratic classroom, we introduce the notion of perspective in order to describe student understandings and pre-understandings as resources for learning. Here the

¹⁵ Normally the (theoretical) discussion of mistakes in the mathematics classroom has concentrated on the mistakes of the students. We could as well look at teacher mistakes, teacher ways of interpreting own mistakes, student ways of interpreting teacher mistakes, teacher ways of hiding mistakes, etc. The study of mistakes can take a variety of directions. Nevertheless, we shall follow the mainstream and concentrate on student mistakes, and teacher ways of interpreting and correcting these.

¹⁶ The observed mathematics lessons we refer to in Chapter 1 and Chapter 2 were part of the normal teaching programme. Many analyses of traditional mathematics classrooms have from different theoretical perspectives pointed to the fact that communication plays an important role for the dynamics of the classroom. We are especially inspired by the microethnographic approach of the German group of symbolic interactionists and their studies of routines, relationships and patterns of communication that can be found in the traditional mathematics classroom, e.g. Bauersfeld (1980, 1988, 1995); Krummheuer (1983, 1995, 2000b) and Voigt (1984, 1985, 1989).

Other important contributions to this field of analysis are Cestari (1997); Cobb and Bauersfeld (eds.) (1995); Jungwirth, H. (1991); Lemke (1990); Pimm (1987); Sfard (2000) and Steinbring (1998, 2000). For a discussion of the culture of the mathematics classroom, see also Brown (2001); Lerman (ed.) (1994); Nickson (1992); Seeger, Voigt and Waschescio (eds.) (1998) and Wood (1994).

¹⁷ The term 'Guess What the Teacher Thinks' is used by Young (1992, 106f.).

students' guessing can be understood as their zooming-in on the classroom agenda. Finally, we discuss learning in terms of action, including the crucial notion of intention.

MISTAKES AND CORRECTIONS

As 'truth' is a key term in the philosophy of mathematics, so are 'mistakes' a key to grasp an implicit philosophy prevailing in many mathematics classrooms. Correction of mistakes opens a backdoor to the classroom philosophy of mathematics.

Philosophical absolutism maintains that some absolute truth can be obtained by the individual. Classroom absolutism comes about when (students') mistakes are treated as absolute: 'This is wrong!' 'You have to correct these calculations!' Thus, classroom absolutism seems to maintain that mistakes are absolute and can be eliminated by the teacher. Our point is not, however, that no mistakes in the mathematics classroom should be stated as real mistakes. We do not want to maintain an absolute relativism. But it seems like absolutism in the philosophy of mathematics automatically leads to absolutism in pedagogy that justifies certain forms of classroom interaction.

We can conceive of different types of mistakes found in mathematics education. In what follows we talk about 'mistakes' in the broadest way to include 'real' mistakes, other sorts of (mis)conceptions, as well as simply alternative conceptions. The mistake could concern the output of some algorithm: 'This calculation is wrong!' The mistake could concern the used algorithm: 'You should not add these numbers but do a subtraction!' The mistake could concern the sequence in which things are done: 'When drawing a graph you first have to calculate some values of the function!' The mistake could have to do with the way the text is interpreted: 'No, when the exercise is formulated like this, you first have to find the value of x !' Or it could have to do with the organisation of the tasks for the students: 'No, no, those exercises are for tomorrow!'

Although the content of these mistakes is quite different, the corrections can be expressed in the same absolute terms. The basic assumption is that the aim of a correction is to correct a mistake. The phenomenon that all sorts of mistakes are treated as absolute, i.e. as real mistakes, we refer to as classroom absolutism.



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