

## CHAPTER 2

### INQUIRY CO-OPERATION

What counts as traditional mathematics education will naturally vary during time, and also from country to country. Thus, it is difficult to provide any general characteristic of 'tradition'. We shall, however, suggest that the school mathematics tradition is characterised by certain ways of organising the classroom. For instance, a mathematics lesson can be divided into two parts: First, the teacher presents some mathematical ideas and techniques. This presentation is normally closely related to the presentation in the given textbook. Secondly the students work with selected exercises. These exercises can be solved by using the just presented techniques. The solutions are checked by the teacher. An essential part of the students' homework is to solve exercises from the textbook. The time spent on teacher presentation and on students doing exercises can naturally vary. Other elements can be included as for instance students' presentations of selected topics and solutions.<sup>38</sup>

In the school mathematics tradition the patterns of teacher-students communication can also become a routine, and much research has tried to identify the communicative patterns that dominate this tradition. We are interested in possible causes for such communicative patterns, as for instance the quizzing pattern of communication we described in Chapter 1, and here we shall pay attention to one particular aspect of the school mathematics tradition, the *exercise paradigm*. This paradigm has a deep influence on mathematics education, concerning the organisation of the individual lessons, the patterns of communication between teacher and students, as well as the social role that mathematics may play in society, for instance operating as a gatekeeper (the mathematical exercises fit nicely into processes of exams and tests). Normally, exercises in mathematics are formulated by an authority from outside the classroom. It is neither the teacher nor the students who have formulated the

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<sup>38</sup> See Blomhøj (1995) for a similar characteristic of traditional mathematics education.

exercises. They are set by an author of a textbook. This means that the justification of the relevance of the exercises is not part of the mathematics lesson itself. Most often, the mathematical texts and exercises represent a 'given' for the classroom practice, including the classroom communication.

The exercise paradigm has been challenged in many ways: by problem solving, problem posing, thematic approach, project work, etc. To put it more generally, the exercise paradigm can be contrasted by investigative approaches.<sup>39</sup> We see the activities of solving exercises as being much more restrictive for the students than being involved in investigations. We want to elaborate on learning as action and not as a forced activity, and this makes us pay special attention to students being part of an investigative approach. In order to create possibilities for making investigations, it is important to consider possibilities outside the exercise paradigm. 'Openness from the start', illustrated by the project 'How much does a newspaper fill?' shows what it could mean to leave the well known frame of the exercise paradigm.

In this chapter we try to characterise more generally challenges to the exercise paradigm in terms of *landscapes of investigation*. We will discuss what it would mean to enter such a landscape. And by discussing an episode from the project 'What does the Danish flag look like?' we try to clarify the notion of *inquiry co-operation* as a particular form of student-teacher interaction when exploring a landscape of investigation. This co-operation we will specify into an *Inquiry-Co-operation Model* (IC-Model) that designates a significant pattern of communication. Such a pattern cannot easily be identified within a classroom practice located in the exercise paradigm.

## FROM EXERCISES TO LANDSCAPES OF INVESTIGATION

Let us look at an example of an exercise in mathematics education: Shopkeeper A sells dates for 85p per kilogram. B sells them at 1.2 kg for

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<sup>39</sup> An investigative approach can take many forms. One example is project work, as exemplified for primary and secondary school education in Nielsen, Patronis and Skovsmose (1999); Skovsmose (1994) and for university studies in Vithal, Christiansen and Skovsmose (1995). See also Cobb and Yackel (1998).

£1. (a) Which shop is cheaper? (b) What is the difference between the prices charged by the two shopkeepers for 15 kg of dates?<sup>40</sup>

Clearly we are dealing with dates, shops and prices. But most likely the person who constructed this exercise neither made any empirical investigation of how dates are sold, nor interviewed anyone to find out under what circumstances it would be relevant to buy 15 kg of dates. The situation is artificial. The exercise is located in a semi-reality. Solving exercises with reference to a semi-reality is an elaborated competence in mathematics education, based on a well specified (although implicit) agreement between teacher and students.<sup>41</sup>

Some of the principles in the agreement are the following: The semi-reality is fully described by the text of the exercise. No other information concerning the semi-reality is relevant in order to solve the exercise, and accordingly not relevant at all. The whole purpose of presenting the exercise is to solve the exercise. Asking any other questions about the specific nature of the semi-reality is similar to any form of disturbance of the mathematics lesson. A semi-reality is a world without sense impressions (to ask for the taste of the dates is out of the question), only the measured quantities are relevant. Furthermore, all the quantitative information is exact, as the semi-reality is defined completely in terms of these measures. For instance, the question whether it is OK to negotiate the prices or to buy somewhat less than 15 kg of dates is non-existing. The exactness of the measurements combined with the assumption that the semi-world is fully described by the provided information makes it possible to maintain the one-and-only-one-answer-is-correct assumption. The metaphysics of the semi-reality makes sure that this assumption gets a validity, not only when references are made exclusively to numbers and geometric figures, but also when references are made to 'shops', 'dates', 'kilograms', 'prices', 'distances', etc.<sup>42</sup>

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<sup>40</sup> The example is taken from Dowling (1998), where he describes the 'the myth of references'. The following presentation and discussion of landscapes of investigation is based on Skovsmose (2000b, 2000c, 2001a, 2001b). The notion of 'virtual reality' referring to the world set by the mathematical exercises has been used by Christiansen (1994, 1997).

<sup>41</sup> See Brousseau (1997) and Christiansen (1995) for a discussion of 'the didactical contract'.

<sup>42</sup> If it is not realised that the way mathematics fits the semi-reality has nothing to do with the relationship between mathematics and reality, then the ideology of certainty finds a place for growing. For a discussion of the ideology of certainty, see Borba and Skovsmose (1997).



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