

THERE IS MORE TO DISCOURSE THAN MEETS THE EARS:
LOOKING AT THINKING AS COMMUNICATING TO LEARN
MORE ABOUT MATHEMATICAL LEARNING

ABSTRACT. Traditional approaches to research into mathematical thinking, such as the study of misconceptions and tacit models, have brought significant insight into the teaching and learning of mathematics, but have also left many important problems unresolved. In this paper, after taking a close look at two episodes that give rise to a number of difficult questions, I propose to base research on a metaphor of *thinking-as-communicating*. This conceptualization entails viewing learning mathematics as an initiation to a certain well defined *discourse*. Mathematical discourse is made special by two main factors: first, by its exceptional reliance on symbolic artifacts as its *communication-mediating* tools, and second, by the particular *meta-rules* that regulate this type of communication. The meta-rules are the observer's construct and they usually remain tacit for the participants of the discourse. In this paper I argue that by eliciting these special elements of mathematical communication, one has a better chance of accounting for at least some of the still puzzling phenomena. To show how it works, I revisit the episodes presented at the beginning of the paper, reformulate the ensuing questions in the language of thinking-as-communication, and re-address the old quandaries with the help of special analytic tools that help in combining analysis of mathematical content of classroom interaction with attention to meta-level concerns of the participants.

In the domain of mathematics education, the term *discourse* seems these days to be on everyone's lips. It features prominently in research papers, it can be heard in teacher preparation courses, and it appears time and again in a variety of programmatic documents that purport to establish instructional policies (see e.g. NCTM, 2000). All this could be interpreted as showing merely that we became as aware as ever of the importance of mathematical conversation for the success of mathematical learning. In this paper, I will try to show that there is more to discourse than meets the ears, and that putting communication in the heart of mathematics education is likely to change not only the way we teach but also the way we think about learning and about what is being learned. Above all, I will be arguing that communication should be viewed not as a mere aid to thinking, but as almost tantamount to the thinking itself. The *communicational approach to cognition*, which is under scrutiny in this paper, is built around this basic theoretical principle.

In what follows, I present the resulting vision of learning and explain why this conceptualization can be expected to make a significant con-



tribution to both theory and practice of mathematics education. I begin with taking a close look at two episodes that give rise to a number of difficult questions. The intricacy of the problems serves as the immediate motivation for a critical look at traditional cognitive research, based on the metaphor of learning-as-acquisition, and for the introduction of an additional conceptual framework, grounded in the metaphor of learning-as-participation. In the last part of this article, in order to show how the proposed conceptualization works, I revisit the episodes presented at the beginning of the paper, reformulate the longstanding questions in the new language, and re-address the old quandaries with the help of specially designed analytic tools.

1. QUESTIONS WE HAVE ALWAYS BEEN ASKING ABOUT MATHEMATICAL THINKING AND ARE STILL WONDERING ABOUT

In spite of its being a relatively young discipline, the study of mathematical thinking has a rich and eventful history. Since its birth in the first half of the 20th century, it has been subject to quite a number of major shifts (Kilpatrick, 1992; Sfard, 1997). These days it may well be on its way toward yet another reincarnation. What is it that makes this new field of research so prone to change? Why is it that mathematics education researchers never seem truly satisfied with their own past achievements?

There is certainly more than one reason, and I shall deal with some of them later. For now, let me give a commonsensical answer, likely to be heard from anybody concerned with mathematics education – teachers, students, parents, mathematicians, and just ordinary citizens concerned about the well-being of their children and their society. The immediate suspect, it seems, is the visible gulf between research and practice, expressing itself in the lack of significant, lasting improvement in teaching and learning that the research is supposed to bring. It seems that there is little correlation between the intensity of research and research-based development in a given country and the average level of performance of mathematics students in this country (see e.g. Macnab, 2000; Schmidt et al., 1999; Stigler and Hiebert, 1999). This, in turn, means that as researchers we may have yet a long way to go before our solutions to the most basic problems asked by frustrated mathematics teachers and by desperate students become effective in the long run. The issues we are still puzzled about vary from most general questions regarding our basic assumptions about mathematical learning, to specific everyday queries occasioned by concrete classroom situations. Let me limit myself to just two brief examples of teachers' and researchers' dilemmas.

A function $g(x)$ is partly represented by the table below. Answer the questions in the

x	$g(x)$
0	-5
1	0
2	5
3	10
4	15
5	20

- (1) What is $g(6)$? _____
 (2) What is $g(10)$? _____
 (3) The students in grade 7 were asked to write an expression for the function $g(x)$.
 Evan wrote $g(x) = 5(x - 1)$
 Amy wrote $g(x) = 3(x - 3) + 2(x - 2)$
 Stuart wrote $g(x) = 5x - 5$
 Who is right? Why?

Figure 1. Slope episode – The activity sheet.

Example 1: Why do children succeed or fail in mathematical tasks? What is the nature and the mechanism of the success and of the failure?

Or, better still, why does mathematics seem so very difficult to learn and why is this learning so prone to failure? This is probably the most obvious among the frequently asked questions, and it can be formulated at many different levels. The example that follows provides an opportunity to observe a ‘failure in the making’ – an unsuccessful attempt at learning that looks like a rather common everyday occurrence.

Figure 2 shows an excerpt from a conversation between two twelve year old boys, Ari and Gur, grappling together with one of a long series of problems supposed to usher them into algebraic thinking and to help them in learning the notion of function.¹ The boys are dealing with the first question on the worksheet presented in Figure 1. The question requires finding the value of the function $g(x)$, represented by a partial table, for the value of x that does not appear in the table ($g(6)$). Before proceeding, the reader is advised to take a good look at Ari and Gur’s exchange and try to answer the most natural questions that come to mind in situation like this: What can be said about the boys’ understanding from the way they go about the problem? Does the collaboration contribute in any visible way to their learning? If either of the students experiences difficulty, what is the nature of the problem? How could he be helped? What would be an effective way of overcoming – or preventing altogether – the difficulty he is facing?

While it is not too hard to answer some of these questions, some others seem surprisingly elusive. Indeed, a cursory glance at the transcript is enough to see that while Ari proceeds smoothly and effectively, Gur is unable to cope with the task. Moreover, in spite of Ari’s apparently adequate algebraic skills, the conversation that accompanies the process of solving does not seem to help Gur. We can conclude by saying that while Ari’s performance is fully satisfactory, Gur does not ‘pass the test’.

Learning Discourse

Discursive approaches to research in mathematics
education

Kieran, C.; Forman, E.A.; Sfard, A. (Eds.)

2002, IV, 302 p., Hardcover

ISBN: 978-1-4020-1024-8