4
Gravitation and the theory of relativity

4.1 Gravitational field

4.1.1 Law of gravitation

1. Gravitation

The property of bodies to interact with each other through their masses is called **gravitation**. The electric force between bodies depends on the charge but not the mass. For the gravitational force only the mass enters, and the force is always attractive as opposed to the electric force, which depends on the sign of the charge. The gravitational force is always attractive and described by the universal law of gravitation:

law of gravitation			MLT^{-2}
	Symbol	Unit	Quantity
$F_g = G\dfrac{m_1 m_2}{r_{12}^2}$	F_g	N	gravitational force
	G	N m^2/kg^2	gravitational constant
	m_1, m_2	kg	masses of bodies
	r_{12}	m	center-of-mass separation of the bodies

2. Properties of the gravitational force

The gravitational force always points towards the other body (**Fig. 4.1**). In vector notation: The force acting on the body 2 is

$$\vec{\mathbf{F}}_{g,2} = -G\frac{m_1 m_2}{r_{12}^2}\frac{\vec{\mathbf{r}}_{12}}{|\vec{\mathbf{r}}_{12}|},$$

where $\vec{\mathbf{r}}_{12}$ represents the vector from the center of mass of body 1 to the center of mass of body 2. Potential theory states that, for the calculation of the gravitational force between

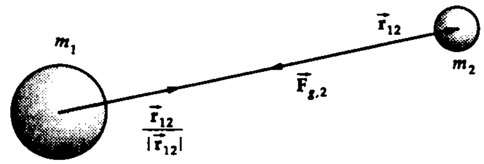

Figure 4.1: Gravitational force. The force acting on the body m_2 points opposite to the displacement vector from m_1 to m_2.

extended spherical homogeneous mass distributions, the bodies can be considered points, with the masses concentrated at the corresponding centers of mass.

➤ The expression $\vec{r}/|\vec{r}|$ (vector divided by its magnitude) represents the unit vector along the vector $\vec{r}$. The force acting on the body 2 points from body 2 to body 1 (notice the minus sign in the formula).

▲ The gravitational force is always an attractive force.

➤ The **gravitational constant** G is a natural constant. Its value is

$$G = 6.67259 \cdot 10^{-11}\ \mathrm{Nm^2/kg^2}\,.$$

➤ The formula gives both the magnitude of the force exerted by body 1 on body 2, and vice versa (2 on 1). The gravitational force always points towards the attracting body.

▲ The gravitational force between two bodies is proportional to the mass of each body and inversely proportional to the square of the distance between them.

Notice the similarity of this expression to Coulomb's law (see the section on Electricity). However, masses always attract each other, whereas the force between charges with the same sign is repulsive. The gravitational field strength is introduced by analogy to the electric field strength.

3. Gravitational field strength,

$\vec{E}_g$, a vector quantity which, for any point $\vec{r}$ in space, gives the force per unit mass that acts on a body due to gravitation:

$$\vec{E}_g = -G\,\frac{M}{r^2}\,\frac{\vec{r}}{|\vec{r}|}\,.$$

The gravitational field $\vec{E}_G$ depends only on the mass M of the attracting body, which is located at the coordinate origin and is considered to be the source of the gravitational field. The force on a test particle of mass m is $\vec{F} = m\,\vec{E}_g$. It points towards the attracting body and determines the acceleration of the test particle.

4. Gravitational potential,

Φ, potential of the gravitational field, describes the **work in the gravitational field**.

gravitational potential				L^2T^{-2}
	Symbol	Unit	Quantity	
$\Phi = -G\,\dfrac{M}{r}$	Φ G M r	J/kg = Nm/kg $\mathrm{N\,m^2/kg^2}$ kg m	potential of gravitational field gravitational constant mass of the gravitating body distance between the test body and gravitating body	

The gravitational force $\vec{\mathbf{F}}$ is calculated from the potential Φ of the gravitational field as

$$\vec{\mathbf{F}}_g(\vec{r}) = -m \operatorname{grad} \Phi(r) .$$

➤ The potential of the gravitational force is $V(r) = m\,\Phi(r)$, $\vec{\mathbf{F}}_g = -\operatorname{grad} V(r)$.
The potential energy of a test particle of mass m at the point $\vec{\mathbf{r}}$ in the gravitational field of a body of mass M is

$$E_{\text{pot}}(\vec{\mathbf{r}}) = m\,\Phi(\vec{\mathbf{r}}) .$$

The work needed to move a test particle of mass m from point $\vec{\mathbf{r}}_1$ to point $\vec{\mathbf{r}}_2$ against the gravitational force equals the difference of the potential energies at the points $\vec{\mathbf{r}}_2$ and $\vec{\mathbf{r}}_1$:

$$W_{12} = -\int_{\vec{\mathbf{r}}_1}^{\vec{\mathbf{r}}_2} \vec{\mathbf{F}}_g\, \mathrm{d}\vec{\mathbf{r}} = E_{\text{pot}}(\vec{\mathbf{r}}_2) - E_{\text{pot}}(\vec{\mathbf{r}}_1) = GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) .$$

5. Attraction to Earth,

weight, the force exerted by Earth on a body at Earth's surface due to gravitation. It is specified by the law of gravitation, the mass and radius of Earth, and the mass of the test particle.

Acceleration of gravity g, nearly constant acceleration due to the attractive force of Earth that acts on all falling bodies: $g = 9.80665$ m/s^2 for mean sea level at about 45° geographical latitude.

➤ The acceleration of gravity is not the same everywhere on Earth's surface. It depends on the geographic latitude, as a result of the non-spherical shape of Earth, and the centrifugal force of Earth's rotation, and also depends on the height at which the measurement is made. Lastly, density fluctuations in Earth's crust lead to concentrations of mass that may modify both the magnitude and direction of Earth's attraction. The latter effect is exploited in searching for raw-material deposits.

➤ According to the law of gravitation, the ratio of the acceleration of gravity g_r at a distance $r > R$ from Earth's center, and g on the Earth's surface is

$$\frac{g_r}{g} = \frac{R^2}{r^2}, \qquad R: \text{ Earth's radius.}$$

➤ The hypothesis of a "fifth force," represented by a Yukawa term, with a strength parameter α and range parameter λ, as an additional term to the potential energy of the gravitational field,

$$V(r) = -G\,\frac{Mm}{r}\,(1 + \alpha e^{-r/\lambda}) ,$$

leads to an effective gravitational constant that depends on the distance r of the test particle from the gravitating mass M. This hypothesis has not be verified by experiment.

4.1.2 Planetary motion

Besides Earth's attraction, gravitation also manifests itself in the motion of the planets. Planetary motion was described empirically in 1609 by Johannes Kepler, as formulated in **Kepler's laws**. These laws can be derived from the law of gravitation and Newton's laws.

1. Kepler's first law

All planets move in elliptic orbits, with the Sun at one focal point.

➤ An ellipse is described by specifying its major semi-axis and either its minor semi-axis or its eccentricity. In our solar system, the planetary orbits are very close to circles.

Ecliptic, the plane of the Earth's orbit. It serves as an astronomical reference frame. **Perihelion**, the point of Earth's orbit with the minimum distance to the Sun. **Aphelion**, the point of the Earth's orbit with maximum distance to the Sun.

➤ The seasons on Earth are not caused by the difference of the distances to the Sun at the perihelion or aphelion, but by the inclination of Earth's equator with respect to the ecliptic. This inclination implies that sometimes the northern hemisphere is turned more towards the Sun, and at other times more away from the Sun.

2. Kepler's second law

A radius vector drawn from the Sun to a planet covers equal areas in equal time intervals (**Fig. 4.2**).

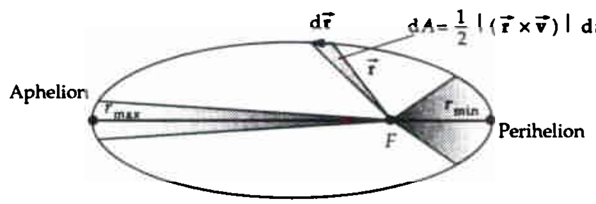

Figure 4.2: Kepler's second law. F: The focal point of the ellipse. The shadowed regions around $r_{\min}$ and $r_{\max}$ are of equal area.

➤ This statement follows from the conservation of angular momentum $\vec{l} = \vec{r} \times \vec{p}$: The areal element dA covered in the time interval dt is given by $2{\cdot}dA = |\vec{r} \times d\vec{r}|$, hence $2m{\cdot}dA/dt = |\vec{r} \times \vec{p}| = |\vec{l}|$. If the angular momentum is a conserved quantity, $|\vec{l}| =$ const., the area dA covered per time interval dt is the same for all sections of the orbit. In particular, it follows that the orbital velocity at the perihelion, v_P, is higher than at the aphelion, v_A, since $l/m = r_{\min} v_P = r_{\max} v_A \Longrightarrow v_P > v_A$.

3. Kepler's third law

The squares of the periods T_1 and T_2 of two planets are related as the cubes of the major semi-axes a_1 and a_2 of their orbits:

$$T_1^2 : T_2^2 = a_1^3 : a_2^3\,, \qquad \frac{T^2}{a^3} = \text{const}\,.$$

➤ Kepler's laws describe the planetary motion caused by the gravitational attraction by the Sun. They do not take the mutual attraction between the planets into account.

➤ According to the general theory of relativity deviations from the $\frac{1}{r^2}$-law arise near the Sun, as is manifested by the slow precession of the elliptic orbit of Mercury (rosette curve).

▲ Parabolas and hyperbolas are also possible orbits of celestial bodies. They pass, however, only once in the vicinity of the central stellar body; afterwards the celestial body leaves the planetary system (example: some comets).

4.1.3 Planetary system

4.1.3.1 Sun and planets

1. The Sun,

the central star of the **solar system** which consists of nine planets and the smaller celestial bodies (satellites, comets, asteroids). The nine planets of the solar system are partly earth-like in size and composition (Mercury, Venus, Mars), and partly much larger gaseous giants (Jupiter, Saturn, Uranus, Neptune).

Data on the Sun		
radius	696,000 km	= 109 Earth radii
mass	$1.99 \cdot 10^{30}$ kg	= 332,000 Earth masses
mean density	1,410 kg/m^3	
acceleration of gravity	273.7 m/s^2	= 27.9 times that on Earth

2. Planets and solar system

Planet, a non-self-luminous celestial body. Unlike **fixed stars**, planets are made visible by light reflected from them. Under the influence of the gravitational force of a central star the planets move in elliptic orbits around them. A star may have several planets revolving around it in different orbits (**planetary system**).

The solar system contains nine planets.

➤ It is not yet clear whether additional planets besides those currently known exist in the solar system. Since the sun light reflected by a possible further planet would be too small to be measured with present technology, one tries to determine the existence of additional planets via their gravitational force on other planets and the resulting distortions of their orbits.

➤ Indications of planets outside our solar system have been observed.

Basic data for planets of the solar system:

Planet	Major semi-axis of orbit (10^6 km)	Period of revolution (a)	Diameter (km)	Mass (in Earth masses)	Rotational period
Mercury	57.9	0.241	4,840	0.053	59 d
Venus	108.2	0.615	12,400	0.815	243 d
Earth	149.6	1.000	12,756	1.000	23 h 56 min
Mars	227.9	1.881	6,800	0.107	24 h 37 min
Jupiter	778	11.862	142,800	318.00	9 h 50 min
Saturn	1,427	29.458	120,800	95.22	10 h 14 min
Uranus	2,870	84.015	47,600	14.55	10 h 49 min
Neptune	4,496	164.79	44,600	17.23	15 h 40 min
Pluto	5,946	247.7	5,850	ca. 0.1	unknown

3. *Basic data for Earth*

Data on Earth	
equator radius	$6378.163\ \text{km} = R_E$
polar radius	$6356.777\ \text{km} = R_P$
flattening	$0.003356 = (R_E - R_P)/R_E$
mass	$5.977 \cdot 10^{24}\ \text{kg}$
mean density	$5517.0\ \text{kg/m}^3$
acceleration of gravity	$9.80665\ \text{m/s}^2$
escape velocity	11.19 km/s

Escape velocity (parabolic velocity): The minimum velocity of a planet needed to leave the gravitational field of the central body.

➤ The rotation period of Earth is not exactly 24 hours, but is about 4 minutes less. These 4 minutes correspond to the angular distance the Earth travels in one day in its orbit around the Sun.

4. *Titius-Bode relation*

The radii a_n of the planetary orbits follow a geometrical series approximately:

$$a_n \approx a_{\text{Earth}}\, k^n\,, \qquad k \approx 1.85\,,$$

($n_{\text{Earth}} = 0$, $n_{\text{Venus}} = -1$, $n_{\text{Mercury}} = -2$, $n_{\text{Mars}} = 1$, $n_{\text{Jupiter}} = 3$, $n_{\text{Saturn}} = 4, \ldots$).

➤ The missing value $n = 2$ corresponds to the belt of asteroids between Mars and Jupiter.

➤ The origin of this relation is presumed to lie in the mutual perturbations of the planets and the resulting conditions for stable orbits.

5. *Astronomical unit,*

AE, the mean distance Earth–Sun,

$$1\text{AE} = 149.6 \cdot 10^6\ \text{km}\,.$$

Pluto, the outermost known planet, is about 40 AE distant from the Sun; Mercury, the innermost, ca. 0.4 AE. Hence, the solar system is very much smaller than the distance to the nearest star (Proxima Centauri, 4.3 ly ≈ 272,265 AE).

Light year, ly, the distance traversed by light in one year:

$$1\ \text{ly} = 9.4605 \cdot 10^{12}\ \text{km} = 63{,}240\ \text{AE}\,.$$

Parsec, pc (parallax second), the distance at which the radius of Earth's orbit around the Sun is observed to subtend an angle of 1 arc second:

$$1\ \text{pc} = 3.262\ \text{ly} = 30.857 \cdot 10^{12}\ \text{km}\,.$$

6. *Measurement of astronomical quantities*

[M] **Parallax,** the virtual displacement of a star (e.g., with respect to other, more remote stars) in the sky in the course of one year, due to the motion of Earth on its orbit. The nearer a star, the larger its parallax.

Parallax range finding, measurement of the distance to a star by comparison of photographs taken in the course of one year. A star at a distance of 1 pc performs a parallax motion of 1 arc second. The method is applicable up to about 100 ly. For larger distances, indirect methods (luminosity, Doppler shift, ...) are used.

7. Moon,

stellar body orbiting a planet. The diameter of **Earth's moon** is about one fourth of Earth's diameter. Many planets, in particular the larger planets Jupiter, Saturn and Uranus, have several moons with nearly the dimension of planets. The **rings of Saturn,** which consist of rocks and dust orbiting the planet, resemble moons.

Data on Earth's Moon		
diameter	3476.0 km	= 27 % of Earth's diameter
mass	$7.350 \cdot 10^{22}$ kg	= 1.2 % of Earth's mass
mean density	3 342 kg/m^3	= 61 % of Earth's density
acceleration of gravity	1.620 m/s^2	= 16.6 % of g on Earth
escape velocity	2.37 km/s	

8. Planet rotation

Planets (and moons) rotate about their own axes; Earth once in 24 hours, Earth's Moon once per month (ca. 28 days). Hence, Earth's Moon always turns the same face towards Earth; the other half of its surface remains permanently out of sight of Earth.

Equator, great circle in the plane of rotation of the planet. The inclination of this equatorial plane against the orbital plane determines the length of the day in the course of the year and is responsible for the occurrence of seasons.

9. Asteroids and comets

Asteroids, small planets, significantly smaller than any of the nine planets. Most of the asteroids are found in an **asteroid belt** between Mars and Jupiter. Their diameters range from a few kilometers up to 740 km (Ceres).

Comet, an object on a hyperbolic or highly eccentric elliptic orbit. The hyperbolic orbit approaches the Sun (or Earth) only once, the elliptic orbit in periodic intervals that may reach 200 years. The most famous comet is **Halley's comet** with a period of 76 years. When comets are remote from the Sun (i.e., not within the orbits of the nine planets) they are not observable. Comets typically have sizes between 1 km and 100 km. Frozen gases on the surface of the comets evaporate when they approach the Sun and become visible as a **comet tail.**

Meteor, a luminous phenomenon caused by **meteorites** that enter Earth's atmosphere and burn out due to the air friction. Their often metallic residues sometimes reach Earth's surface.

4.1.3.2 Satellites

Satellite, a body moving on an orbit in the gravitational field of another body, in general a planet. Originally, the term referred to moons; nowadays **artificial satellites** are also included.

▲ For satellites, Kepler's first law may be modified as follows: satellites move along conic sections, i.e., on circular, elliptic, parabolic or hyperbolic curves, depending on the satellite's initial velocity.

Satellites on parabolic and hyperbolic orbits escape the gravitational field of the central object.

1. *First critical velocity*

Circular orbit velocity, v_K, first critical velocity, the velocity that a body must have to move on a circular orbit near Earth's surface. It is the minimum velocity of a satellite to avoid impact on the surface of Earth. The circular orbit velocity follows from the balance between the centrifugal force and the gravitational force of Earth that provides the centripetal force to maintain the circular motion.

2. *Second critical velocity*

Parabolic orbit velocity, v_P, second critical velocity or **escape velocity**, the minimum velocity that a body must have to leave the gravitational field of Earth. The body then moves on a parabolic orbit arbitrarily far away from Earth.

For Earth, the critical velocities are (**Fig. 4.3**):

critical velocities			LT^{-1}
	Symbol	Unit	Quantity
$v_K = \sqrt{\frac{GM}{R}} = 7912$ m/s	v_K	m/s	circular orbit velocity
$v_P = \sqrt{2}\, v_K = \sqrt{\frac{2GM}{R}}$	v_P	m/s	parabolic orbit velocity
$= 11190$ m/s	G	$N\,m^2/kg^2$	gravitational constant
	M	kg	Earth's mass
	R	m	Earth's radius

➤ For velocities $v_K < v < v_P$, elliptic orbits result. Hyperbolic orbits arise for $v > v_P$.

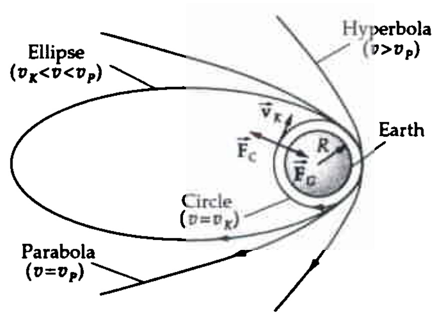

Figure 4.3: Satellite orbits. $\vec{F}_c$: centrifugal force, $\vec{F}_G$: weight (centripetal force), R: Earth's radius, v_K: first critical velocity, v_P: second critical velocity.

3. *Third critical velocity*

Third critical velocity, the minimum velocity that a body on Earth must have to leave the solar system. It follows from the same formula as the second critical velocity, but now the Sun's mass and the distance from the Sun have to be inserted:

$$v = \sqrt{\frac{2GM_{\text{Sun}}}{r_{\text{Sun-Earth}}}} = 42.1 \text{ km/s}.$$

➤ Using the relation $g = GM/R^2$, v_K and v_P can also be expressed in terms of the acceleration of gravity g at Earth's surface.

4.2 Special theory of relativity

1. Special theory of relativity,

developed by Albert Einstein (1905) to explain phenomena in motion at velocities near the speed of light.

The central concept of the special theory of relativity is the postulate that the laws of physics are the same in any uniformly moving reference frame, and the postulate of the **constancy of the speed of light in vacuum** in all inertial systems. This postulate leads to a new definition of the concepts of time and space in the framework of a **space–time continuum**.

2. General theory of relativity,

extension of the special theory of relativity, also developed by Einstein (1916), that also includes arbitrarily accelerated reference frames in the **relativity principle**.

■ The general theory of relativity leads to an equal treatment of gravitation and inertial forces by means of a curved space–time continuum, and constitutes the basis of modern **cosmology**.

3. Relativistic effects

Differences between the ordinary, non-relativistic physics and the special or general theory of relativity become important only for velocities close to the speed of light, and for motions in the vicinity of extremely massive objects, respectively. They are in general not observable in everyday life.

■ Physical applications of the theory of relativity are found in elementary-particle physics (particle accelerators), in atomic physics, and in astronomy and astronautics. Because of the increasing sensitivity of precision measurements, relativistic effects may also be demonstrated using highly sensitive instruments in macroscopic processes on Earth (time dilatation in airplanes).

4.2.1 Principle of relativity

1. Inertial system,

a frame in which Newton's laws hold, in particular the law of inertia. In such a frame, a body that is free of forces remains in its state of motion. Therefore, inertial systems are those frames that move with uniform speed relative to each other.

➤ The velocity of a system cannot be specified without reference to a system relative to which the velocity is being measured. Hence, an inertial system cannot be defined as a system that moves with uniform velocity without referring to another frame that is also an inertial system.

▲ A system that moves with uniform velocity $v = \text{const.}$ relative to an inertial system is also an inertial system.

Event, an incident that is fixed in a coordinate system by specifying its time coordinate t and its spatial coordinate x. Therefore, any physical event in a given reference frame is assigned to a **coordinate** (x, t) in the space–time continuum.

2. Galilean transformation,

transformation of the coordinates when changing from one inertial system to another inertial system *without accounting for the special theory of relativity.* Let x and x' denote the space coordinate, t and t' the time coordinate in the two frames, respectively. If the coordinate origins of both systems coincide at time $t = 0$, and their relative motion is in the x-direction with velocity v (**Fig. 4.4**), the Galilean transformation is then:

Galilean transformation			
	Symbol	Unit	Quantity
$x' = x - vt$	x, x'	m	space coordinates
$t' = t$	t, t'	s	time coordinates
	v	m/s	relative velocity of the reference frames

The second relation, $t' = t$, says that the time measurement (motion of a watch, pendulum motion, etc.) does not depend on the velocity of the spatial motion of the chronometer.

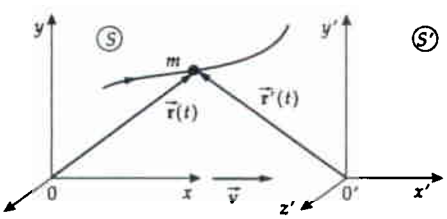

Figure 4.4: Galilean transformation. The coordinate origin O' of the frame S' moves relative to the coordinate origin O of the frame S on a straight line and uniformly with the velocity $\vec{v}$ along the x-axis. Trajectory in S: $\vec{r}(t) = \vec{r}\,'(t) + \vec{v}t$. For both frames, the same time scale is assumed.

➤ A system S' is denoted as a moving frame if it moves relative to the frame S of the observer with the velocity $\vec{v} \neq 0$. And vice versa, for an observer who is at rest in S', the frame S is moving with the velocity $-\vec{v}$.

3. *Trajectory,*

$x(t)$, characterizes the motion of a body m in a given frame. Its trajectory in S:

$$\vec{r}(t) = \vec{r}\,'(t) + \vec{v}t\,.$$

■ According to the Galilean transformation, the trajectory in a frame S' that moves with velocity v along the x-direction is given by

$$x'(t') = x'(t) = x(t) - vt\,.$$

■ A body moving uniformly with velocity u has the trajectory

$$x(t) = x_0 + ut\,, \qquad x_0\text{: coordinate at time } t = 0\,.$$

In a coordinate system moving with the velocity v the trajectory is given by

$$x'(t) = x(t) - vt = x_0 + ut - vt = x_0 + (u - v)t.$$

Under a Galilean transformation, the velocity u' in the moving frame S' is thus obtained by subtracting the original velocity u of the body and the relative velocity v of the moving system S':

$$u' = u - v\,, \quad u = u' + v\,.$$

4. *Relativity principle in classical, non-relativistic mechanics*

The laws of classical mechanics have the same form in any inertial system.

■ Transformation of Newton's second law:

Observer in S: $\vec{\mathbf{F}} = m\,\ddot{\vec{\mathbf{r}}}$.
Observer in S': $\dot{\vec{\mathbf{r}}} = \dot{\vec{\mathbf{r}}}' + \vec{\mathbf{v}}, \quad \dot{\vec{\mathbf{v}}} = 0,$
$\ddot{\vec{\mathbf{r}}} = \ddot{\vec{\mathbf{r}}}', \quad \vec{\mathbf{F}}' = m\,\ddot{\vec{\mathbf{r}}}'.$

The force law has the same mathematical form for both observers.

5. *Maxwell's equations,*

describe the propagation of electromagnetic waves, do **not** follow this relativity principle:

▲ Electromagnetic waves (light) propagate in vacuum with the speed

$$c = 2.997\,924\,58 \cdot 10^8 \text{ m/s}.$$

If this velocity were to transform according to the Galilean transformation, the above value would be valid only in a unique, and hence distinguished, reference frame. This contradicts experimental experience.

For the propagation of sound in gases, the sound velocity quoted in the literature holds for the reference frame in which the gas is at rest. A very rapidly moving source of sound may actually be faster than the sound emitted by the source, and in this way it may generate a shock wave.

This leads to the question of whether a source moving faster than the speed of light can pass the light emitted by itself.

6. *Ether hypothesis,*

analogy between light and sound propagation. According to this hypothesis, electromagnetic waves are carried by a medium called the **ether**. The reference frame in which the ether is at rest would constitute an absolute coordinate system.

■ The value of the speed of light would then hold just in the reference frame in which the ether is at rest.

[M] In particular, the existence of an ether would imply that electromagnetic waves in a moving reference system propagate (analogous to sound propagation) with distinct velocities forward (i.e., direction of motion of the source) and sideways. This hypothesis was tested for the first time in the **Michelson-Morley experiment** (1887) by means of a **Michelson interferometer**. Here one observes with an interference setup whether the speed of light changes because of Earth's motion. The moving system in which the experiment was performed is Earth itself on its path around the Sun. The experiment proves that light propagates with equal velocity c along Earth's orbit and in the perpendicular direction, disproving the ether hypothesis.

7. *Special relativity principle*

All inertial systems are equivalent. In a vacuum, light propagates in any inertial system and in all directions with the same speed: the speed of light in vacuum c.

➤ Contrary to the ether hypothesis (which presupposes an absolute motion), according to the relativity principle there exists only **relative motion** in the selected reference frame; hence, the term **theory of relativity**.

4.2.2 Lorentz transformation

1. Introduction of the Lorentz transformation

The validity of the relativity principle is maintained only if the Galilean transformation is replaced by another transformation, the **Lorentz transformation**. Let the coordinates of an event in 3D space relative to a reference frame S be given by x, y, z and the time t. The coordinates x', y', z', t' of the same event in a coordinate system S' that moves uniformly with the speed v along the x-axis relative to the first system, are (**Fig. 4.5**):

Lorentz transformation

	Symbol	Unit	Quantity
$x' = \dfrac{x - vt}{\sqrt{1 - v^2/c^2}}$	x, y, z	m	space coordinates in frame S
$y' = y$	t	s	time coordinate in frame S
$z' = z$	x', y', z'	m	space coordinates in frame S'
$t' = \dfrac{\left(t - \dfrac{v}{c^2}x\right)}{\sqrt{1 - v^2/c^2}}$	t'	s	time coordinate in frame S'
	v	m/s	relative velocity of S' against S
	c	m/s	speed of light

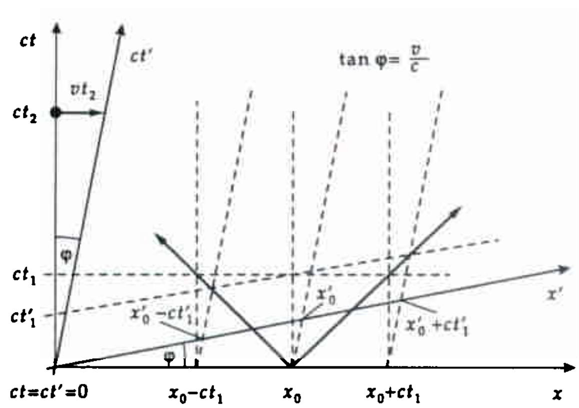

Figure 4.5: Lorentz transformation in the Minkowski graph. Besides the axes (x, ct), (x', ct') of the two frames, the world line (= trajectory in Minkowski space) of a light pulse emitted at $(x = x_0, t = 0)$ is plotted. The scale on the axes of system S' may be determined by recognizing that the light pulse propagates in both systems with the speed of light c.

➤ The inverse of the Lorentz transformation is obtained by changing the sign of velocity. The frame S moves with velocity $-v$ relative to the frame S'.

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}, \quad t = \frac{\left(t' + \dfrac{v}{c^2}x'\right)}{\sqrt{1 - v^2/c^2}}.$$

2. Relativistic factor,

γ, characteristic parameter of the Lorentz transformation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} .$$

For velocities much below the speed of light,

$$v \ll c \implies \gamma \approx 1 .$$

▲ For $v \ll c$, the Lorentz transformation becomes the Galilean transformation.

■ This guarantees that the Lorentz transformation does not contradict common experience, since relativistic effects become measurable only for large speeds beyond our everyday range of experience.

3. Minkowski diagram and world point,

serve for visualization of the Lorentz transformation. The position x and on the abscissa the time t (or ct) are plotted on the ordinate, so that to any event a **world point** (t,x) may be assigned in the graph (**Fig. 4.6**).

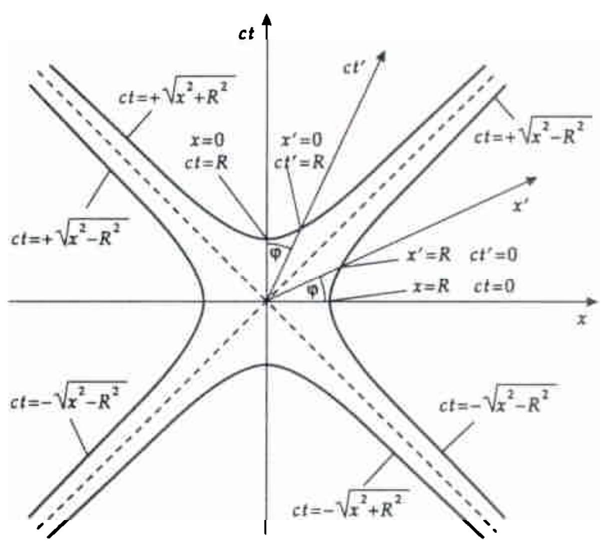

Figure 4.6: Lorentz transformation in a Minkowski diagram. The axes x, ct and x', ct' of the two frames, and the hyperbolas $ct = \pm\sqrt{x^2 \pm R^2}$ are plotted.

World line, the trajectory of a particle in a Minkowski diagram. For convenience, the units on the axes are taken so that a motion with the speed of light, $x(t) = ct$, appears as a straight line with a slope of 45°, and therefore the distance is plotted in light seconds (ls) and the time in seconds. A **light second** is the distance passed by light in 1 second: $1 \text{ ls} \approx 3 \cdot 10^8$ m.

When making a Lorentz transformation, the coordinate axes of the moving frame are plotted in a Minkowski diagram. The coordinates of the origin ($t' = 0,\ x' = 0$) are ($t = 0,\ x = 0$), i.e., the origins of both coordinate systems lie at the same world point. The x'-axis of the frame S' is given by

$$t' = 0 \quad \Longrightarrow \quad \gamma\left(t - \frac{v}{c^2}x\right) = 0 \quad \Longrightarrow \quad t = \frac{v}{c^2}x\,.$$

This corresponds to a straight line enclosing the angle φ with the x-axis, with

$$\tan\varphi = \frac{v}{c}.$$

Correspondingly, one gets the same value, although counted in the opposite direction, for the angle between the ct'-axis and the ct-axis. Finally, the scales on the axes of frame S' have to be wider by the factor γ (> 1) than in the frame S (see p. 145).

➤ For an observer in the system S', the system S moves with the speed $-v$.

4. Comparison with the Galilean transformation

The most radical change in the Lorentz transformation as compared with the Galilean transformation is the statement that the time coordinate cannot be the same in both systems. This follows directly from the postulate of the constancy of the speed of light, and this consequence cannot be avoided.

■ Two events that occur simultaneously at distinct space points in one reference frame are not simultaneous in another reference frame. This **relativity of simultaneity** is a general phenomenon and is connected with the fact that the information on an event cannot propagate faster than the speed of light from one space point to another one.

▲ The largest propagation velocity of a physical phenomenon is the speed of light.

The relativistic factor γ is not defined for the velocity $v = c$ (division by zero), and becomes imaginary for velocities $v > c$. Therefore, a massive body cannot reach a velocity $v \geq c$ in vacuum. This experience is expressed by the **addition theorem of velocities**.

5. Tachyons,

hypothetical particles that move at or faster than the speed of light, but cannot go below it.

M Tachyons would emit light in vacuum. Radiation arises if a massive particle moves in an optical medium with refractive index n faster than $c_{gr} = c/n$ (c: vacuum speed of light, c_{gr}: group velocity).

4.2.2.1 Addition of velocities

1. Addition of velocities under Lorentz transformation

Let a body move with the velocity $\vec{\mathbf{u}}'$ in a reference frame S' that has a relative velocity $\vec{\mathbf{v}}$ against the frame S. The velocity $\vec{\mathbf{u}}$ of the body relative to the frame S does not follow by simple vector addition of $\vec{\mathbf{u}}'$ and $\vec{\mathbf{v}}$. According to the Lorentz transformation, it is given by the

addition theorem of velocities			
$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x}$$ $$u_y = \frac{u'_y}{\gamma\left(1 + \frac{v}{c^2}u'_x\right)}$$ $$u_z = \frac{u'_z}{\gamma\left(1 + \frac{v}{c^2}u'_x\right)}$$	Symbol	Unit	Quantity
	u_x, u_y, u_z	m/s	velocity in S
	u'_x, u'_y, u'_z	m/s	velocity in S'
	v	m/s	relative velocity of S' along the x-axis of S
	c	m/s	speed of light
	γ	1	relativistic factor

➤ Inversion by changing the sign of the relative velocity, $v \to -v$.

2. *Derivation of the addition theorem*

The above expressions are obtained if the uniform motion of a particle in a moving coordinate frame S',

$$x' = u'_x t, \quad y' = u'_y t, \quad z' = u'_z t,$$

undergoes a Lorentz transformation, and one then considers the resulting expressions for $x(t)$, $y(t)$ and $z(t)$ in the (rest) frame S of the observer. For this purpose, it is suitable to consider the distance $(\mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$ passed during a short time $\mathrm{d}t$. According to the differentiation rules,

$$\mathrm{d}x = \gamma \mathrm{d}x' + \gamma v \mathrm{d}t', \quad \mathrm{d}y = \mathrm{d}y', \quad \mathrm{d}z = \mathrm{d}z', \quad \mathrm{d}t = \gamma \mathrm{d}t' + \frac{v}{c^2}\mathrm{d}x'.$$

In the moving frame S', another time interval $\mathrm{d}t'$ elapses as compared with the interval in the rest system S.

Velocity in the frame S:

$$u_x = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\gamma \mathrm{d}x' + \gamma v \mathrm{d}t'}{\gamma \mathrm{d}t' + \gamma \frac{v}{c^2}\mathrm{d}x'} = \frac{\frac{\mathrm{d}x'}{\mathrm{d}t'} + v}{1 + \frac{v}{c^2}\frac{\mathrm{d}x'}{\mathrm{d}t'}}.$$

Similarly, one finds the velocities u_y and u_z.

3. *Conclusions from the addition theorem*

▲ For low velocities $v \ll c$, the relativistic addition of velocities reduces to the ordinary, non-relativistic vector addition of velocities, $u = u' + v$.

▲ For velocities close to the speed of light, one finds, however, $u < u' + v$, i.e., the velocity is smaller than the simple vector sum.

In particular, for $u'_x \approx c$ and $v \approx c$, the relativistic addition theorem leads to

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x} \approx \frac{c + c}{1 + \frac{c}{c^2}c} \approx c.$$

▲ The velocity of a body cannot exceed the speed of light.

4.2.3 Relativistic effects

Relativistic effects, effects predicted by means of the Lorentz transformation.

4.2.3.1 Length contraction

1. Distance in the moving system

The distance between two points on the x'-axis in the frame S' is given by

$$l' = x_2' - x_1'.$$

In the frame S, the length l is measured by determining the coordinates of the initial point and the endpoint x_1, x_2 **at the same time** t, $l = x_2 - x_1$. The Lorentz transformation then yields

$$x_1' = \gamma(x_1 - vt), \quad x_2' = \gamma(x_2 - vt),$$

or

$$l = \frac{1}{\gamma} l'.$$

In the frame S the **length** of the same distance appears to be **shortened by the factor** $1/\gamma$.

2. Length contraction

The length of a distance in a moving frame appears to an observer in his own rest frame to be contracted by the factor

$$\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}.$$

➤ The relativity principle leads to the seeming paradox that, for an observer in the frame S', the length of a distance in the frame S appears **also to be contracted:** $l' = (1/\gamma)l$. This paradox is resolved by the relativity of simultaneity of the measurement in both systems.

4.2.3.2 Time dilatation

1. Time interval in a moving system

If in the moving frame S' two events occur at the positions x_1' and x_2' at the times t_1' and t_2', the time distance Δt between the events in the rest frame S is given by

$$\begin{aligned}\Delta t = t_2 - t_1 &= \gamma \left[\left(t_2' + \frac{v x_2'}{c^2} \right) - \left(t_1' + \frac{v x_1'}{c^2} \right) \right], \\ &= \gamma \left(\Delta t' + \frac{v}{c^2}(x_2' - x_1') \right).\end{aligned}$$

If both events happen in the moving frame S' **at the same position** ($x_2' = x_1'$), then

$$\Delta t = \gamma\, \Delta t'.$$

2. Time dilation

The **time** between two events in a moving frame appears to an observer in the rest frame to be **increased** by the factor

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\,.$$

➤ This statement holds also for an observer in the frame S': $\Delta t' = \gamma\,\Delta t$. The time interval in the other frame appears **to any** observer to **be increased.**

It further follows that two events that occur simultaneously ($\Delta t' = 0$) in a moving frame do not appear as simultaneous events in a rest frame if the events do not occur at the same position:

$$\Delta t = \gamma \frac{v}{c^2}(x_2' - x_1')\,.$$

3. Example: Cosmic radiation

Upon entering Earth's atmosphere, the primary cosmic radiation generates (by collisions with air molecules) a hard secondary radiation that consists of energetic particles. Muons created at a height of about 30 km have a lifetime of $2 \cdot 10^{-6}$ s in their rest frame. At a velocity of $v = 0.9995\,c$ ($\gamma \approx 32$), these fast muons could (without relativistic effects) traverse a distance of only ≈ 600 m. Hence, they would not be observed at Earth's surface. When taking the time dilatation into account, a lifetime of $32 \cdot 2 \cdot 10^{-6}\ \mathrm{s} \approx 6 \cdot 10^{-5}$ s results. This time interval is sufficiently long to let the particles traverse the path from where they were created to Earth's surface. Hence, the muons created by cosmic radiation can be detected in laboratories on the ground.

4.2.4 Relativistic dynamics

Relativistic dynamics, generalization of dynamics for velocities that are not small compared with the speed of light. It takes the relativistic increase of mass into account and leads to the concept of the equivalence of mass and energy.

4.2.4.1 Relativistic increase of mass

1. Increase of mass

Because of the addition theorem of velocities, the law of momentum conservation, $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$, can hold in relativistic dynamics only if the mass becomes velocity-dependent (**Fig. 4.7**).

relativistic increase of mass			M
	Symbol	Unit	Quantity
$m(v) = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_0$	$m(v)$	kg	mass at velocity v
	m_0	kg	rest mass
	v	m/s	velocity of the body
	c	m/s	speed of light
	γ	1	relativistic factor

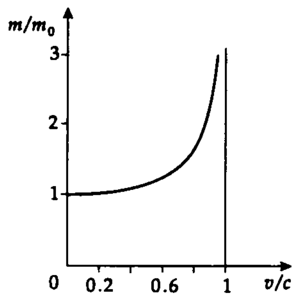

Figure 4.7: Relativistic increase of mass.

➤ The relativistic mass may become arbitrarily large as the velocity of the body approaches the speed of light. Therefore, it is impossible to accelerate a body by a force or by collisions to the speed of light, since this would require an infinite expense of energy.

2. *Relativistic momentum,*

$$\vec{\mathbf{p}} = m(v)\vec{\mathbf{v}} = \frac{m_0\vec{\mathbf{v}}}{\sqrt{1-\dfrac{v^2}{c^2}}} = \gamma m_0\vec{\mathbf{v}}\,.$$

When this expression is inserted into the momentum-balance equation, the law of momentum conservation, and all relations derived from it, continue to hold without modification.

3. *Relativistic force*

For the **relativistic force**:

$$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m_0\vec{\mathbf{v}}}{\sqrt{1-\dfrac{v^2}{c^2}}}\right).$$

There is a distinction made between forces acting parallel or perpendicular to the motion. Let $\vec{\mathbf{v}}$ be parallel to the x-axis,

$$F_x = \frac{m_0 a_x}{\left(1-v^2/c^2\right)^{3/2}} = m_0\gamma^3 a_x\,,$$

$$F_y = \frac{m_0 a_y}{\sqrt{1-v^2/c^2}} = m_0\gamma a_y\,,$$

$$F_z = \frac{m_0 a_z}{\sqrt{1-v^2/c^2}} = m_0\gamma a_z\,.$$

$\vec{\mathbf{a}}$ is the acceleration vector.

▲ To accelerate a body farther along its direction of motion, a force increased by a factor γ^3 is required as compared with the non-relativistic case. For an acceleration perpendicular to the motion, the corresponding factor is only γ.

4.2.4.2 Relativistic kinetic energy

1. Relativistic work,

the work performed on accelerating a body,

$$\Delta W = F\Delta s = m_0\gamma^3 a\Delta s = m_0\gamma^3 \frac{\Delta v}{\Delta t} v\Delta t = m_0\gamma^3 v\Delta v\,,$$

F acting force, Δs distance covered, Δv velocity increase, Δt time interval.
For acceleration from rest, $u = 0$, up to a velocity $u = v$ the integration yields

$$W = \int_0^v \frac{m_0 u}{\left(1 - \frac{u^2}{c^2}\right)^{3/2}}\, du = m_0 c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right),$$

the expression for the relativistic kinetic energy.

2. Relativistic kinetic energy

relativistic kinetic energy			ML^2T^{-2}
	Symbol	Unit	Quantity
$E_{\text{kin}} = m_0 c^2 \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right]$ $= m_0 c^2(\gamma - 1)$	E_{kin}	J	kinetic energy
	m_0	kg	rest mass
	v	m/s	velocity
	c	m/s	speed of light
	γ	1	relativistic factor

➤ In the non-relativistic case,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}\,, \qquad E_{\text{kin}} \approx \frac{m_0}{2} v^2\,.$$

This is the non-relativistic expression for the kinetic energy.

3. Equivalence of mass and energy

Since the zero level of energy can be set arbitrarily, one assigns to any body a relativistic total energy $E = mc^2$, with a **velocity-dependent mass** $m = \gamma m_0$.

▲ **Equivalence of mass and energy**:
A body with mass m has **relativistic total energy** E,

$$E = mc^2\,.$$

A body at rest has **rest energy (mass energy)**

$$E_0 = m_0 c^2\,.$$

■ The mass energy can be released only by converting it to another form of energy.

Application of the theory of relativity to elementary particles (relativistic quantum field theory) leads to just such processes.

If particles and antiparticles closely approach each other, the mass energy $2m_0c^2$ of both particles may be converted to other kinds of energy, in particular to electromagnetic radiation (**pair annihilation**). Conversely, particle-antiparticle pairs may be created from radiation energy (**pair creation**).

4. Energy-momentum relation for relativistic particles

energy-momentum relation	Symbol	Unit	Quantity
$\frac{E^2}{c^2} = p^2 + m_0^2 c^2$	E	J	relativistic total energy
	p	kg m/s	momentum
	m_0	kg	rest mass
	c	m/s	speed of light

where for E the relativistic total energy mc^2 has to be inserted.

5. Center-of-mass energy,

E_{cm} (cm = center of mass), in a collision of two particles the total energy of both particles, measured in the center-of-mass system is

$$E_{\mathrm{cm}} = \sqrt{m_1^2 c^4 + m_2^2 c^4 + 2E_1 E_2 \left(1 - \frac{v_1}{c}\frac{v_2}{c}\right)\cos\theta}$$

(E_1, E_2, relativistic energy of the particles 1 and 2 in an arbitrary system; v_1, v_2, their velocities in this system; θ, angle between the particles). If the particle 2 is at rest in the **laboratory system**, then

$$E_{\mathrm{cm}} = \sqrt{m_1^2 c^4 + m_2^2 c^4 + 2E_{1\mathrm{lab}}\, m_2 c^2}\,.$$

The center-of-mass energy characterizes the total energy available in collisions of elementary particles. The velocity in the center-of-mass system is

$$\frac{\vec{v}_{\mathrm{cm}}}{c} = \frac{\vec{p}_{1\mathrm{lab}}\, c}{E_{1\mathrm{lab}} + m_2 c^2}$$

($\vec{p}_{1\mathrm{lab}}$—momentum in the laboratory system). The relativistic factor is

$$\gamma_{\mathrm{cm}} = \frac{E_{1\mathrm{lab}} + m_2 c^2}{E_{\mathrm{cm}}}\,.$$

➤ In thermodynamics, the variables pressure and entropy are invariant against Lorentz transformations, whereas the temperature and the amount of heat depend on the state of motion of the system.

4.3 General theory of relativity and cosmology

General theory of relativity, extension of the special theory of relativity to arbitrary (non-inertial) systems. It deals in particular with **gravitation**, using the mathematical tool of a **curved four-dimensional space-time continuum**.

1. General relativity principle

An inertial system in a gravitational field is equivalent to a reference frame in a gravitation-free space that is uniformly accelerated (relative to an inertial frame). This means that an observer cannot distinguish by any experiment which of the systems he is in.

■ An astronaut in a falling elevator, slowed only by air friction, falls with 5/6 of the gravitational acceleration at Earth's surface. He feels only the remaining sixth part of the gravitational force and may therefore believe to be on the Moon, where the weight force is only 1/6 of that on Earth.

Curvature of space, arises as a consequence of the presence of masses and manifests itself by the gravitational force.

2. Test of the general theory of relativity (GTR)

- **Light deflection** in the gravitational field of the Sun. A beam of light from a remote star that passes close to the surface of the Sun is deflected by the space curvature by an angle of $1.75''$. The star then seems to change its position relative to neighboring stars. The phenomenon can be demonstrated during a solar eclipse. Light is also deflected according to Newton's theory, but only by half of the value predicted by GTR. Light deflection is thus no test of the GTR on its own, but the precise experimental value is such a test.
- **Rotation of the apse line** (the line connecting aphelion and perihelion) of the inner planets, due to a modification of Newton's law of gravitation in strong gravitational fields. After accounting for the influence of the other planets, GTR has predicted for Mercury an excessive rotation of $43''$ per century, which has been confirmed by experiment.
- **Red shift** of star light. According to GTR, light is affected by gravitation. The energy spent by the light to leave the gravitational field of a star causes a reduction of the radiation energy, i.e., a shift of the spectral lines towards the long-wave (infra-red) region. The red-shift of spectral lines is also predicted by Newton's theory (combined with the quantum-mechanical rule $E = h \cdot f$).

Black hole, a star with a very strong gravitational field, so that light cannot leave the space region.

3. Properties of the universe

GTR predicts that the universe is either infinite or finite, depending on the total mass of the universe. A finite universe can be compared with the surface of a sphere: it has no boundary, but nevertheless is finite.

Hubble effect, proof of the expansion of the universe. The spectra of very remote stars show a shift to the infra-red, the radiating objects thus move away from the observer. This Hubble shift (cosmologic red-shift) is to be interpreted only by imperfect analogy to the optical Doppler effect.

Hubble constant H specifies the increase of expansion velocity:

$$H = 50 \text{ to } 100 \text{ km/s per Mpc}$$

(1 Mpc = 1 Megaparsec = 3.26 Mill. light years). In a curved space, any observer may believe that all other points move away from him (like the points of the surface of a balloon being blown up).

It depends on the mass available in the universe whether the universe reaches a maximum extension and then collapses (**closed universe**), or whether it continues to expand (**open universe**). The majority of the mass of the universe seems to exist as **dark matter**, invisible to all types of telescopes and other devices. The investigation of the rotation of galaxies suggests that galaxies are enclosed by halos of dark matter.

Big bang, hypothesis that the universe developed ca. $1\text{–}2 \cdot 10^{10}$ years ago from **one** point (**singularity**) of extremely high energy density. It then quickly expanded, and was cooled by that expansion.

3-Kelvin-background radiation, the observed strongly cooled, nearly isotropic thermal radiation in the universe, the remainder of the radiation from the first seconds after the big bang.

4.3.1 Stars and galaxies

1. Stars and their classification

Star, self-luminous stellar object. A star releases energy by a **nuclear-fusion process** that proceeds at very high temperature ($\approx 10^6$ K) in its interior.

Classification:

Stars are classified according to the wavelengths (colors) of the emitted light, and by their magnitude. The typical distances between stars in galaxies are light years, the distances between galaxies are millions of light years. About 5,000 to 10,000 stars are visible to the naked eye, with a small telescope, 100,000. In total, about 10 billion individual stars are accessible by astronomic instruments.

2. Star catalogs

Stars are classified according to **Sky maps** (star catalogs). The brightest stars have proper names from Arabic or Greek. Most of the stars visible with an unaided eye are denoted according to the sky mapping of Bayer (1603); the names consist of a Greek letter specifying the luminosity of the star in its constellation, and the name of the constellation. If the Greek alphabet is not sufficient, the name continues with Latin letters and numbers. Weaker stars are classified by catalog numbers.

■ The brightest star in the constellation Cassiopeia:

Proper name	Schedir
Bayer's Name	α Cassiopeiae (short: α Cas)
Bonn sky mapping	BD +55°139

3. Stellar brightnesses and spectral classes

Stellar brightness, specifies the apparent brightness of a star. Originally from 1^m to 6^m (m, *magnitudo*, Latin for size), today it ranges from the brightness of the Sun, -27^m, to the weakest recordable stars, 23^m. Smaller (more negative) numbers mean brighter stars; each class is $10^{0.4} = 2.512$ times brighter than the next following class.

stellar brightness	example
-27^m	Sun
-13^m	full Moon
-11^m	half Moon
-5^m to -1^m	close planets
up to -2^m	brightest stars (Sirius, Vega)
$+6^m$	observation limit of eye
$+14^m$	Pluto
$+23^m$	photographic observation limit

Spectral class, classifies the type of spectrum of the light emitted by a star.

Spectrum of the light from a star, consists of broad **emission bands**, overlayed by **absorption lines**. Spectral classes are denoted by a Latin capital letter and a number.

■ The Sun has the spectral type G 2.

▲ The spectral class of a star is closely related to its surface temperature.

4. Galaxy,

disk- or spiral-shaped ensemble (diameter 30,000 parsec) of stars. **Milky Way**, spiral-shaped galaxy with a total mass of about 200 billion Sun masses, the Sun being located in one spiral arm. The Milky Way is visible in the sky as a dim band of light. It is surrounded by spherical **stellar clusters**. Galaxies are combined into nebula groups and **nebula clusters** (with diameters of several million light years).

4.3.1.1 Star evolution

1. Energy source of the stars

Stars get their energy from nuclear-fusion processes that take place in the star interior at several million degrees Celsius. In these reactions, hydrogen fuses to helium, catalyzed by carbon and nitrogen (**Bethe-Weizsaecker cycle** or **carbon-nitrogen cycle**). This "hydrogen burning" proceeds relatively slowly.

■ The Sun has consumed only about 3 parts per thousand of its mass over the 4.5 billion years of its existence. In stars with larger mass the energy conversion proceeds very much faster.

When the hydrogen is burned, the energy production in the star decreases. As a consequence, the star contracts since the gravitational force dominates. During the contraction process, the pressure and temperature in the central region increase, so that higher-mass fusion processes up to carbon become feasible. The total energy production again rises steeply, and the contraction due to gravitation is stopped. Ultimately, a **red giant star** develops: the star explodes and reaches temperatures of up to 1 billion degrees Celsius in its interior.

■ The Sun will reach this stage probably in 3.5 billion years. Stars with large mass finally become unstable after consuming their fuel and first form pulsating stars, later novae and supernovae, and finally white dwarfs, neutron stars or black holes.

2. Special states of stars

Double star, a system of two stars rotating about each other due to gravitation.

Variable stars, stars with varying brightness. Periodic variables arise by shadowing of double stars, or by periodic instabilities of the fusion process.

Novae (exploding variable stars), stars which have an explosively expanding gas shell and grow in brightness within about one day by 7 to 10 stellar magnitudes, and then fade away again over months or years. Thereby, only a minor part of the star mass is expelled. Several novae occur periodically. In our Milky Way system, 166 novae have been observed so far.

Supernovae, explosive final stages in the evolution of massive stars. Supernovae occur much more rarely than novae but reach increases of brightness of up to 20 stellar magnitudes (increase of brightness by a factor 10^8). About 7 to 10 supernova explosions are supposed to have happened in the Milky Way system in the past two millennia; several of them have been recorded by ancient historians. After a supernova, the remnants of the star are mostly only expanding gas shells (**gas nebulae**), and possibly white dwarves.

Pulsar, radio source with periodically varying intensity. The periods are in the range of milliseconds to seconds. The pulse length is about 5 % of the period. Pulsars are most likely rapidly rotating neutron stars with extraordinarily strong magnetic fields.

Neutron star, remnants of a star after the supernova stage. Stars release the major part of their energy in a supernova and then collapse so strongly under their own gravitational force that they no longer consist of common matter (atomic nuclei + electron shells). They now consist of tightly packed neutrons, after absorption of the shell electrons by the nuclear protons (see p. 885). Neutron stars have masses of the order of the Sun's mass. Typical radii are ca. 10 km, densities ca. $3 \cdot 10^{17}$ kg/m^3 (density of nuclear matter). The radio radiation arises from plasma clouds accelerated in the gravitational field; the periodicity arises because of the rotation of the system. During a further contraction of a neutron star of sufficient mass, a **black hole** may arise.

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