1 Symbol Super Colliders

Tommaso Toffoli

We argue that lattice-gas computers are the conceptual offspring of colliding-beams particle accelerators. Instead of streams of physical particles, streams of symbolic tokens are run through one another in a cross-current fashion, intersecting at pre-arranged places and times. In this way, an astronomical number of tokens continually collide and interact in a disciplined choreography.

In a particle collider, the by-products of a collision are scrutinized, occasionally subjected to new collisions, but then purged out of the system. What's new is that in a lattice gas we have a *digital* rather than an *analog* collider: our control of the nature and geometry of the interactions is such that what comes out of a collision is just as good, for the sake of arranging further interactions, as what went in. While in a particle collider one must labor to supply a steady supply of fresh projectiles, the tokens in a lattice gas are always "as good as new" and can be recirculated *ad infinitum*. This mode of operation is not an intuitive one; indeed, it took centuries of intellectual struggle to realize that this is the way nature works, and that a similar mode of operation can be employed in computation.

From the transformations and rearrangements of colliding tokens there emerges, both in nature and in "artificial" computation, a spacetime tapestry whose overall design often transcends the petty details of the specific script and acquires an identity of its own.

dedicated to Ed Fredkin, inventor of collision-based computing

Have you noticed how in recent years the term "particle accelerators" has quietly given place to "particle colliders", as in the famed Superconducting Super Collider (SSC)? There is a reason for that. We may feel exhilarated to be the first kids on the block to accelerate electrons to 1 GeV (billion electron volt), but why would we want to do that in the first place? The answer is, of course, to smash them onto each other! Acceleration is only a means to an end — *collisions* are what we want! If particle colliders are the instruments *par excellence* of experimental physics, lattice-gases are the instruments *par excellence* of experimental mathematics. In this chapter we show how close the family connections are between physical and mathematical "supercolliders".

What emerges from a collision of particles is other particles. In physics, the properties of particles are ultimately defined in terms of what happens to them *under all possible collision circumstances* — and that's why experimental physicists are so busy trying out quite exotic kinds of collisions.[1] Thus, the ostensible goal of particle physics is to determine the properties of particles; according to what we just said, that means essentially to construct a full *collision lookup table* which for every configuration of input particles will tell the corresponding configuration of output particles.[2]

Once we have the complete table, microscopic or fundamental physics is over, since the behavior of *any* system can in principle be derived by repeated use of this table. From then on, all we can do is, as it were, mere "mathematical post-processing" — not new fundamental physics. And usually we do this to pursue one of two complementary endeavors:

Direct problem To derive macroscopic, collective, average, or typical consequences of the fundamental laws, that is, to tell in coarser but more practical terms what will happen in certain (perhaps only partially specified) conditions. This is, in essence, *statistical physics*.

Inverse problem Conversely, to figure out what conditions must be set up if we want to achieve certain desired consequences. This is, of course, *engineering*.

But why stop at physics? Can't we play God and dictate a lookup table ourselves? What would happen in a world governed by *that* microphysics? Or, given certain observed or desired macroscopic properties, is there a microscopic dynamics (a.k.a., a lookup table) from which those properties would emerge merely by massive iteration? What are the *simplest* such tables? And so forth. This is where cellular automata and lattice gases enter the picture.

Cellular automata are a discrete counterpart to partial differential equations. They fulfill an obvious need — and it is not surprising that they have been reinvented innumerable times under different names and within different disciplines. The canonical attribution is to Ulam and von Neumann [23], circa 1950.[3] To this attribution we also owe that awkward name, "Cellular Automata" (I remember Richard Feynman mutter "Who'd want to pronounce *that*?"[4]). To add insult to injury, von Neumann didn't let cellular

[1] There is one catch with this definition: it is *circular*! To characterize a collision we must specify what particles went in and what particles came out; but particles do not carry a name tag, and to know who they are we must determine their properties by subjecting them to further collisions. This chicken-and-egg problem is solved in practice by using, for initial projectiles and for measuring probes, "trusted" sources of standard particles, such as a well-collimated, monochromatic proton beam.

[2] One may recognize in such a table the discrete counterpart of a partial differential equation.

[3] Though one must not forget an independent, historically isolated entry by Zuse [25].

[4] And, like with "media", how many people see "cellular automata" as a plural and know or have a use for its singular?

automata learn to speak physics on "their father's lap", even though he was a most outstanding mathematical physicist. Note that von Neumann had a special interest in the foundations of quantum mechanics, and when he thought physics he naturally viewed the world in terms of unitary transformations — a microscopically reversible continuum dynamics. But cellular automata were for him only a toy model for reductionistic arguments in *biology*, where he needed a dissipative, phenomenologically irreversible dynamics. The shortest way to achieve that was to start directly with a microscopically irreversible substrate quite unlike the physical substrate of classical or quantum mechanics. Regrettably, it took cellular automata another three decades to become expressive in the language of fundamental (rather than phenomenological) physics [16,5,12,10,9,19,24,15] and become, in this course, *lattice gas* computers.

To explain why lattice gases are what cellular automata should have been in the first place, we'll tell a story that has in it particle colliders and cellular automata machines, Aristotle and Galileo, Pascal's triangle and asteroids. First, though, we'll briefly introduce the two contenders.

1.1 Cellular Automata and Lattice Gases

Let us consider an indefinitely extended array of *cells* that can take on values 1 (denoted by a black token) and 0 (empty); for simplicity, let the array be one-dimensional; Fig. 1.1 shows a possible configuration of this array. The index x denotes the spatial position of a cell along the array.

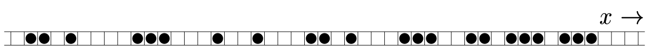


Fig. 1.1. A possible configuration for a one-dimensional cellular automaton with two-state cells; states 1 and 0 are indicated by the presence or the absence of a black token.

We shall now construct subsequent configurations in time ($t = 1, 2, \dots$) for this array by using a recurrence relation to derive each configuration from the previous one. For example, using the recurrence relation

$$q_x^{t+1} = q_{x-1}^t \oplus q_x^t \oplus q_{x+1}^t \tag{1.1}$$

(the state of a cell at time $t+1$ is the sum mod 2, or "exclusive OR", denoted by $\oplus$, of the states of the cell itself and its left and right neighbors at time t) we obtain the spacetime history of Fig. 1.2 when we start from a configuration consisting of a single token.

Generalizations to more than two states per cell, to more than one spatial dimension, and to a dependency on more than just the left and right neighbors, are straightforward.

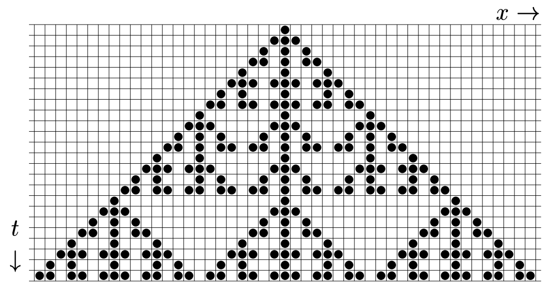


Fig. 1.2. Spacetime history obtained by running recurrence relation (1.1) starting from a configuration consisting of a single token.

A dynamical system of this kind is called a *cellular automaton* if:

- The number of possible cell states is *finite*.
- The recurrence relation is *finite*, i.e., the new state of a cell depends only on the state of a finite number of cells at the previous instant.
- The recurrence relation is space- and time-translation *invariant*, i.e., does not make use of the absolute position of a cell in space and time.

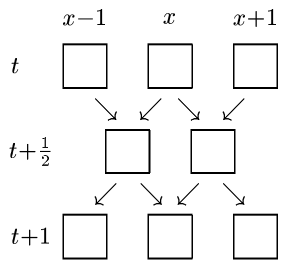
It is easy to verify that the system of Fig. 1.2 is indeed a cellular automaton.

The indexing used in recurrence relation (1.1) suggests the interpretation that there is a physical cell for each spatial position x, and that this cell updates its state at each successive instant of time t; thus, q_x^t can be read as "the state of cell x at time t". In the following spacetime diagram (1.2), each column of boxes indicates the successive states — and thus the "lifeline" — of the cell at a particular spatial position.

$$\begin{array}{cccc} & x-1 & x & x+1 \\ t & \square & \boxed{} & \square \\ & \searrow & \downarrow & \swarrow \\ t+1 & \square & \boxed{} & \square \end{array} \quad . \tag{1.2}$$

We have used a thicker outline for the successive states of cell x. According to the recurrence relation, this cell constructs its "new state" by gathering "current state" information from its left neighbor, its right neighbor, and "itself".

Though cellular automata are traditionally conceived in this way, that is, with the identity of a cell persisting from one step to the next, this feature is by no means obligatory. In fact, one can think of a spacetime arrangement whereby cell states do not line up in time columnwise, as in diagram (1.2), but form an alternating or *quincunx* pattern:



(1.3)

On this spacetime lattice, the recurrence relation

$$q_x^{t+\frac{1}{2}} = q_{x-\frac{1}{2}}^t \oplus q_{x+\frac{1}{2}}^t, \tag{1.4}$$

defines a cellular automaton whose evolution, from an initial configuration consisting of a single token, is, in effect, "Pascal's triangle mod 2" (Fig. 1.3).

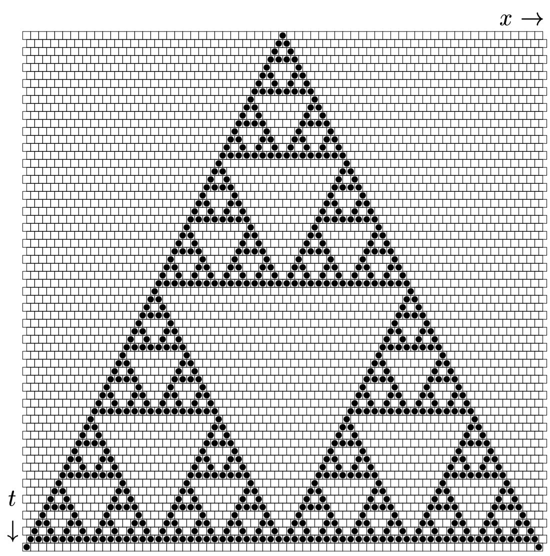


Fig. 1.3. When started with a single token, the cellular automaton defined by recurrence relation (1.4) (which makes use of spacetime lattice (1.3)) yields "Pascal's triangle mod 2".

It turns out that "spacetime crystallographic groups" of the latter kind (1.3), analogous to a body-centered cubic arrangement (of course what we have shown here is only the simplest one-dimensional variety; but see Fig. 1.6) are almost invariably used in the context of lattice gases, while cellular automata almost invariably use the former kind (1.2), analogous to a plain cubic arrangement. The point we wanted to make by introducing the cellular automaton of Fig. 1.3 was that this distinction may reflect convenience, tradition, or accident, but *is not fundamental* and should not distract us when we want to contrast cellular automata with lattice gases on the basis of their *essential* differences.

A *lattice gas*, illustrated in its simplest form in Fig. 1.4, may be viewed a *dual* structure to a cellular automaton. In a cellular automaton, state information resides on the nodes (the squares of (1.3)), while in a lattice gas it resides on the *arcs* — which play the role of *signals* — while nodes represent *events* — that is, stateless functions through which signals interact. In Fig. 1.4 we have two kinds of signals, which travel at the "speed of light" respectively rightwards and leftwards.

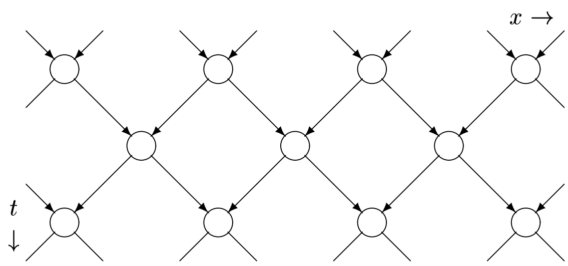


Fig. 1.4. Spacetime lattice of a simple, one-dimensional lattice gas. Arcs represent *signals*, which carry state information, while nodes represent *events* — stateless functions through which signals interact. Here we have two kinds of signals, which travel "at the speed of light" respectively rightwards and leftwards.

In the slightly more complex lattice of Fig. 1.5 we have introduced a third signal which is "at rest" (zero spatial velocity) in the given reference frame and thus represents static information — "data storage" rather than "data communication". Thus, even though data in lattice gases are are always imagined to be "in transit" between events, nonetheless they can still be used to implement *local state*, as in cellular automata.

In physics, the term "lattice gas" originally denoted simply a stylized gas model in which space and time were discretized so that the underlying continuum spacetime effectively turned into a lattice. The canonical example of a lattice gas in this sense is the so-called "HPP gas", invented by Yves Pomeau [9], which runs on the two-dimensional lattice of Fig. 1.6 (the third dimension in the figure is of course for *time*). Signals are binary and travel in the four compass directions; each signal represents the presence ("1") or absence ("0") of a particle on the corresponding spacetime track.

As sketched in Fig. 1.7, particles that come together at a node go through and continue in the original direction, with one exception: if two particles

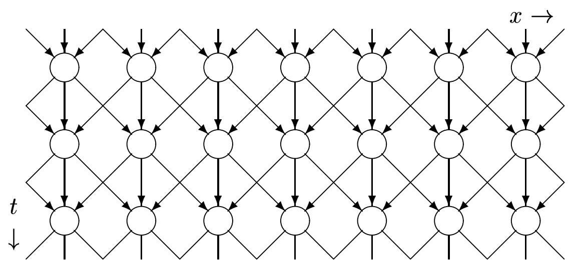


Fig. 1.5. Here we add a third kind of "signal", which, having zero spatial velocity, is better thought of as a form of *memory* rather than data transmission.

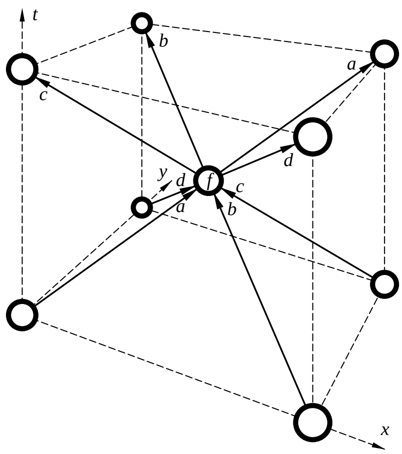


Fig. 1.6. Spacetime layout for a simple two-dimensional lattice gas; signals come together to an event from four directions and similarly depart from in four directions.

meet head on and the other two tracks are empty, then the two particles will "bounce" off one another and come out of the node on these two tracks, at right angles to the original direction of travel.

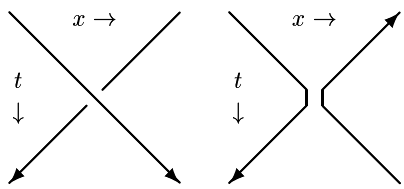
(a)  (b)

Fig. 1.7. In the HPP lattice gas, particles crossing one another's paths (a) go through a site unaffected, while particles colliding head-on (b) are scattered at right angles.

In the HPP gas, as soon as the numbers involved become large enough for averages to be meaningful — say, averages over spacetime volume elements containing thousands of particles and involving thousands of collisions — a definite continuum dynamics emerges. And, in the present example, it is a rudimentary *fluid* dynamics, with quantities recognizably playing the roles of density, pressure, flow velocity, viscosity, speed of sound, etc. Fig. 1.8 illustrates sound-wave propagation in this model. Note that, even though the microscopic interactions only display a limited form of rotational symmetry (namely, invariance under quarter-turn rotations), the "speed of sound" in the HPP gas is fully isotropic.

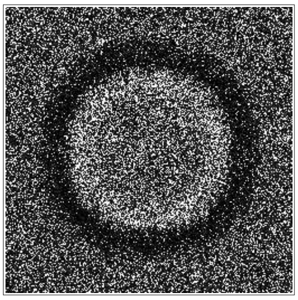


Fig. 1.8. Wave propagation in the HPP lattice gas. Note the emergence of circular symmetry.

When it was proposed, the HPP gas was really a piece of *conceptual art*, designed by a professional hydrodynamicist to impress fellow professional hydrodynamicists (who else?): "Imagine, we've been spending all our lives knee-deep in differential equation, continuous variables, Banach spaces, and all that, and here is something that in principle can get the same results with the most incredibly simple machinery!" (The catch of course is that while the components are simple you need *lots of them* — billions of collisions loci to see a picture with some detail like vortices or wakes.) The joke was appreciated by an inner circle, but for the moment nothing came out of it.

One must give to Edward Fredkin the credit of envisaging *and taking very seriously* the possibility of doing general-purpose computation by purely *ballistic* means — that is, collision-based computing. Ed played with many variations of this idea until he came up first with "conservative logic" (where the two Boolean states, 0s and 1s, were independently conserved by the logic gates — and thus could be conceived as balls or atoms which preserved their identity through a collision. Eventually he came up with the *Billiard-Ball Model of Computation*, which showed that, in spite of widely spread doubts to the contrary, general-purpose computation could be achieved entirely by a "Galilean" mechanism consisting of elastic balls and flat mirrors. These developments are reported in [5].[5]

Meanwhile, Ed Fredkin's *Information Mechanics* group had started developing high-performance, low-cost hardware engines dedicated to cellular automata and similar fine-grained, massively-parallel computation schemes [19]. To implement the Billiard Ball model of computation on one of these machines, Norman Margolus managed to cajole a cellular automaton into thinking it was a collision-based scheme.[6] While experimenting with variations of Ed's billiard-ball rule, I stumbled on a rule from which fluid behavior — the laws of perfect gases, viscous damping, and all that — naturally emerged. Gérard Vichniac located a precedent for that in Pomeau's HPP gas, and we invited its author to one of our Information Mechanics workshops. According to Pomeau, seeing this gas actually running on one of our machines made him realize that what he had conceived primarily as a *conceptual* model could indeed be turned, once suitable hardware was available, into a *computationally accessible* model; this stimulated his interest in finding lattice-gas rules which would provide better models of fluids. A landmark was reached with the slightly more complicated FHP model [7] (it uses six rather than four particle directions), which gives, in an appropriate macroscopic limit, a fluid obeying the well-known *Navier–Stokes* equation — and thus suitable for modeling actual hydrodynamics. Soon after, analogous results for three-dimensions were obtained by a number of researchers [6].

As we said, the structure of a cellular automaton seems designed for the convenience of an accountant — or of a spreadsheet. The nodes of the spacetime lattice contain static *data*; in fact, cell states sit snugly in their spacetime boxes, as in (1.2), and their location does not convey any directional information. The arcs only denote the functional dependency paths between old and new states: a *transition function* sweeps over the array and fills the boxes of one row by looking at the contents of the neighboring boxes in the previous row. Several of these "old state" boxes will have to be looked at in order to construct a single "new state"; conversely, the same "old state" will be looked at several times, by different "neighbors" in the new row.

A lattice gas, by contrast, is more like a pinball machine. State information resides in the inertially-traveling tokens, that is, in the spacetime signals, while nodes act as stateless transducers — n-input, n-output functions. Each signal derives its entire contents from a single node and delivers its entire contents to a single node; no duplication of neighbor data occurs *en route* as in cellular automata. By its very location in the spacetime lattice, a signal associates dynamical information — a certain direction and velocity — with

[5] Signals and events (and, specifically, *reversible* events) as a basis for a physically-minded model of computation had been proposed in [16], but reduction to *really elementary* interactions was still lacking there.

[6] Born as a clever trick, the "Margolus neighborhood" was incorporated as an architectural primitive in subsequent machines.

the data it carries. It is impossible to specify *where* certain data are without simultaneously specifying *whence* and *whither*: as in physics, the two aspects of *position* and *velocity* are intimately joined; this is seen with particular clarity in Fig. 1.6.

The advantages of lattice gases over cellular automata from the viewpoint of *information mechanics* (i.e., information theory applied to a dynamical context) are amply discussed in [20]. Hardware and software engines are also discussed elsewhere [21,18]. Here we are going to present a broader intuitive picture, placing more emphasis on complementary themes.

1.2 Heat, Ice, and Waves

To better appreciate the contrast between cellular automata and lattice gases, let's recast it in the familiar setting of Pascal's triangle. We may think of this triangle,

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ \cdot & & \cdot & & \cdot & & \cdot & & \cdot & & \cdot \end{array},$$

as a particular history of a discrete, one-dimensional dynamical system having spacetime variables (here all set to 0 for definiteness) in the following positions

$$t\Big\downarrow \quad \begin{array}{ccccccccccc} \cdot & & \cdot & & \cdot & & \cdot & & \cdot & & \cdot \\ & 0 & & 0 & & 0 & & 0 & & 0 & \\ \cdot & & 0 & & 0 & & 0 & & 0 & & \cdot \\ & 0 & & 0 & & 0 & & 0 & & 0 & \\ \cdot & & 0 & & 0 & & 0 & & 0 & & \cdot \\ & 0 & & 0 & & 0 & & 0 & & 0 & \\ \cdot & & \cdot & & \cdot & & \cdot & & \cdot & & \cdot \end{array} \quad . \tag{1.5}$$

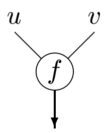
As a dynamics for this system, let each variable be constructed as the sum of the two variables standing above-right and above-left of it (the above assignment of 0 to all variables trivially satisfies this dynamics).

Suppose now that variable values are arbitrarily assigned on the $t = 0$ row, while the values for $t = 1, 2, \dots$ are obtained by iterating the dynamics just described. If we start with a 1 in the center position and 0s in all others we obtain the following history

$$
\begin{array}{c|ccccccccccc}
t & & & & & & & & & & & \\
\hline
0 & & 0 & & 0 & & 1 & & 0 & & 0 & \\
1 & \cdot & & 0 & & 1 & & 1 & & 0 & & \cdot \\
2 & & 0 & & 1 & & 2 & & 1 & & 0 & \\
3 & \cdot & & 1 & & 3 & & 3 & & 1 & & \cdot \\
4 & & 1 & & 4 & & 6 & & 4 & & 1 & \\
\cdots & \cdot & & \cdot & & \cdot & & \cdot & & \cdot & & \cdot
\end{array}
\quad , \tag{1.6}
$$

which, as anticipated, reproduces Pascal's triangle.

Note that each element in (1.6) is computed as the output of a two-argument function of the form



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with, in our case, $f(u,v) = u + v$. In order to normalize to 1 the sum of each row of Pascal's triangle, such as we would get for the coefficients of the binomial distribution $(\frac{1}{2}a + \frac{1}{2}b)^n$, we may replace the above function by $f(u,v) = \frac{1}{2}(u+v)$. That is, each spacetime variable is now obtained as the *average*, rather than the sum, of the two variables above it, and we get

$$
\begin{array}{ccccccccccc}
 & 0 & & 0 & & 1 & & 0 & & 0 & \\
\cdot & & 0 & & \frac{1}{2} & & \frac{1}{2} & & 0 & & \cdot \\
 & 0 & & \frac{1}{4} & & \frac{1}{2} & & \frac{1}{4} & & 0 & \\
\cdot & & \frac{1}{8} & & \frac{3}{8} & & \frac{3}{8} & & \frac{1}{8} & & \cdot \\
 & \frac{1}{16} & & \frac{1}{4} & & \frac{3}{8} & & \frac{1}{4} & & \frac{1}{16} & \\
\cdot & & \cdot & & \cdot & & \cdot & & \cdot & & \cdot
\end{array}
\quad . \tag{1.7}
$$

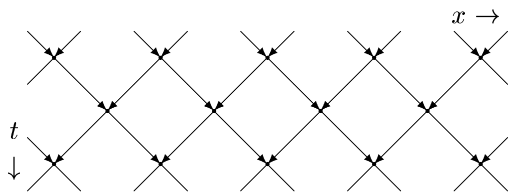
This recurrence relation, where each element is the mean of the two neighboring values from the previous time instant, is of course a discrete counterpart of the *partial differential equation*

$$\frac{du}{dt} = \frac{d^2u}{dx^2},$$

that is, the well-known *diffusion* (or "heat") equation.

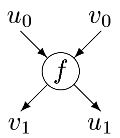
Now, we would like to look at essentially the same process but from the viewpoint of a *physicist* rather than of an accountant.

Let us imagine spacetime to be uniformly criss-crossed by quantities that travel inertially rightwards and leftwards at unit speed (we may think of this as the "speed of light"). Maintaining the same orientation of the t axis as in (1.5), we have the following spacetime diagram



. (1.8)

This diagram consists of *signals* and *events* much as explained in the previous section. In general, an event will represent an arbitrary, two-input, two-output function of the form



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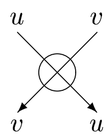
through which the two signals u and v interact; an event is thus the *locus of interaction* of signals.

Note that here and in what follows we identify signals not according to whether they lie on the left or the right of an event, but according to their direction of travel, rightwards or leftwards. Thus, if we call u the signal that enters a node from the left, we'll also call u the signal that exits that node from the *right*. Accordingly, in (1.9), where the arrow representing an event is preceded by the ordered pair of arguments and followed by the ordered pair of results, in *both* pairs the first element denotes the right-traveling signal and the second, the left-traveling one.

In the *identity* (or "null") event,

$$f_{\text{iden}} : (u, v) \mapsto (u, v), \tag{1.9}$$

the two signals cross one another's paths without interacting:



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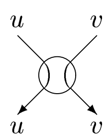
Similarly, one may consider the *reflecting* event,

$$f_{\text{reflect}} : (u, v) \mapsto (v, u), \tag{1.10}$$

where the leftwards outgoing signal carries the value of the rightwards ingoing signal, and vice versa (the passengers "trade ships" as the two vessels cross one another's paths):



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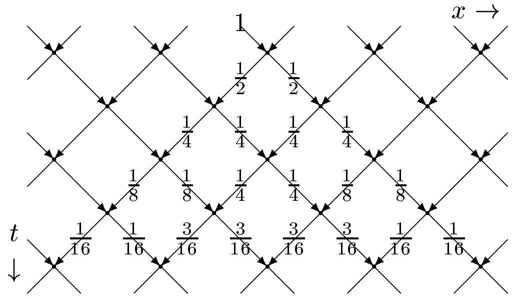
Finally, we may want to consider the *equalizing* event, which adds together the two incoming signals and then redistributes this sum evenly between the two outputs:

$$f_{\text{mean}} : (u, v) \mapsto \left(\tfrac{1}{2}(u+v), \tfrac{1}{2}(u+v)\right) . \tag{1.11}$$

We are now ready to translate the recurrence relation of (1.7) into a signal/event diagram. If we put an equalizing event (1.11) at each node and inject a unit signal into the “origin” node we obtain the following cascade



, (1.12)

which is *identical* to the binomial distribution of (1.7) if at each node one chooses to be blind to how much comes from the left and how much from the right, so that the two incoming signals are lumped into a single entity. Thus, events of type (1.11) (equalizing events) yield the *diffusion equation*. With events of type (1.10) (reflecting events) each piece of data moves back and forth on alternating steps, yielding no net progress in the x direction; the dynamics is effectively *frozen*. At the other extreme, events of type (1.9) (identity events) let rightwards and leftwards signals travel through one another undisturbed; but this linear superposition of left- and right-traveling trains of values is nothing less than the general solution of the *wave equation* (cf. [22]), and we obtain genuine, perfectly undamped *harmonic* motion — a vibrating string! Anything in between (1.9) and (1.11) will yield a version of the *telegrapher's equation* [13] — which displays a mixture of both inertial (wave equation) and dissipative (heat equation) traits.

In conclusion, while the cellular-automata-like recurrence scheme of (1.7) loses valuable information (the momentum of a signal) at every step, a comparable lattice-gas-like scheme, such as (1.8), takes better care of the information that is entrusted to it — and in the end can let it do more interesting things. Here we can tune an extremely simple system all the way from a frozen dynamics to diffusive transport to perfect harmonic motion, effortlessly yielding a wide range of basic types of physical dynamics.[7] What we've done is move from a game of *checkers* to a game of *billiards* [5]!

[7] And note that we've only been considering those dynamics that are linear combinations of the two extreme cases f_{iden} and f_{reflect}; this is a but a small subset of all the possible dynamics.

1.3 Colliding-Beams Particle Accelerators

My first doctoral thesis (I started my scientific career as an experimental physicist) entailed developing a wide-angle muon detector [14] to be used in one of the first experiments planned for ADONE.

At that time, conventional particle accelerators hurled a beam of particles at a stationary target. The resulting debris would fly more or less in all directions, but retained an overall momentum in the forward direction equal to the momentum of the incoming particle. Thus, much of the laboriously built up beam energy escaped in the form of collective forward motion of the reaction by-products, rather than being used for probing deeper into the internal structure of the target (see [17] for a "computer-science" counterpart to this effect). With a more energetic beam the probing would get deeper, but the fraction of energy wasted would grow even larger.

Italy, I was told by my professor, did not have the resources to compete in this expensive race by brute force and was exploring a more subtle approach. The idea was to have head-on collisions between two beams of particles of the same mass traveling at the same speed in opposite directions; in this case the center of mass of the collision was stationary and the entire energy of the two beams would be spent in actual smashing work — a 100% "payload". After feasibility experiments with a small working model called ADA[8] (1961–64), in 1965 construction was afoot for a full-size colliding-beams accelerator to be called ADONE.[9] Ironically, even though ADA had been the first working electron-positron colliding-beams accelerator, Italy's scientific cleverness was not matched by its ability to resolve labor disputes, and, after a long series of strikes, ADONE only started operating in 1969 — when other laboratories had already had the first pick of the technique, and, incidentally, I had moved to the United States to pursue my interest in physics-motivated computer science.

The economics of colliding beams required new beam management practices. When a beam of electrons strikes a solid target, each electron is practically sure to undergo a collision and the beam is totally spent by the encounter. On the other hand, when the "target" is another beam, the sparse flocks of particles constituting the two beams stream through one another with only rare collisions. Thus the beams are only very slowly depleted, and it is expedient to recirculate them so as to give the unaffected particles new chances to collide. Thence the concept of *storage ring*: two beams, one of, say, electrons (e^-) and the other of positrons (e^+), are gradually built up (using an external injector) in an evacuated circular pipe or "ring", where they travel in opposite directions on essentially the same circular trajectory,[10] as

[8] "Anello Di Accumulazione", or storage ring.

[9] "Big ADA".

[10] The particles are kept on a circular course by deflection magnets, which impart the same curvature to e^- traveling one way and e^+ traveling the opposite way.

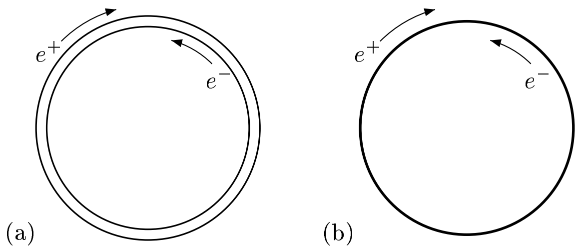


Fig. 1.9. Colliding-beams accelerator. (a) A beam of electrons (e^-) and one of positrons (e^+) are accumulated in the same evacuated ring, where they travel in opposite directions on parallel but slightly separated circular paths; the particles stream past one another without interacting. (b) When the spacing between the circular paths is reduced to zero so that the paths merge, the two streams collide head on.

in Fig. 1.9a. Initially the beams' paths are kept slightly separated so that the particles do not interact. When the beams have attained the desired particle density and energy, the spacing between the paths is reduced to zero so that the two paths overlap and the particles start colliding (Fig. 1.9b).

We shall draw a spacetime diagram in which x denotes position along the ring (the right and left ends of the x axis coincide) while t denotes elapsed time (Fig. 1.10). On this diagram, the lifelines of all of one beam's particles run parallel to one another with a constant spacetime slope (typically very close to 1 — the speed of light); those of the other beam have the opposite slope. If we superpose the two families of lifelines we obtain a grid in which every crossing is a potential interaction locus. When the beams are slightly separated, as in Fig. 1.9a, the two families of lines live, as it were, on different planes, and interactions are switched off. When the two beams are brought together as in Fig. 1.9b, the places were spacetime lines visually cross, in Fig. 1.10, become actual intersection points (cf. **crossover** ⤫ vs **tie** × in conventional electric schematics) and interactions between particles are enabled.

To continue with our conceptual motivation of lattice gases, let us observe that, in physics experiments, detectors are arranged around the ring to intercept and study the debris that scatter off these collisions. With the geometry described above, collisions can take place anywhere along the ring. However, sophisticated detectors tend to be bulky and expensive, and it is not feasible to strew a lot of them along the whole length of the ring. Besides, detectors can be made to work more efficiently if they are focused on a very small volume of space. Thus, it is desirable to have any collisions occur only at precisely appointed places. By slightly distorting the two beam trajectories, one can make them intersect at only a few discrete places along the ring, as shown in Fig. 1.11a.

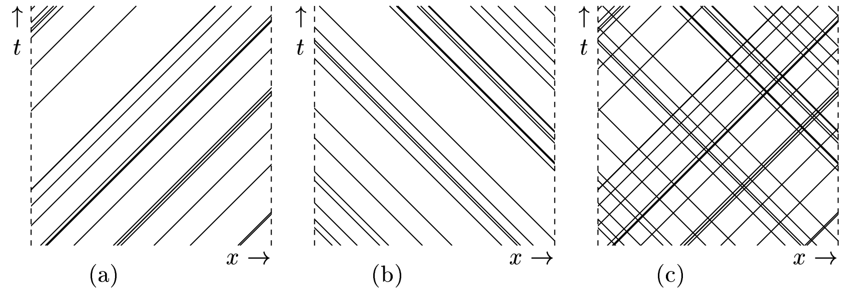


Fig. 1.10. Spacetime lifelines of individual particles in the storage ring. The x axis represents length along the ring; its right and left ends of course coincide. (a) Lifelines of positrons. (b) Lifelines of electrons. (c) Superposition of the two families of lifelines. A grid crossing represents two particles passing by one another; they will actually collide if both families of lines run in the same plane rather than on slightly separated planes (cf. Fig. 1.9).

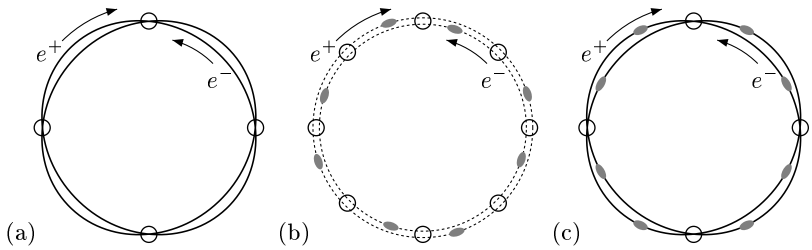


Fig. 1.11. (a) Beam routing. Small distortions to the circular trajectories make the two beams overlap only at a few selected places. The small circles denote the collision sites, at which the detectors will be aimed. (b) Beam bunching. Here particles come in concentrated volleys or "bunches" (the grey blobs); being evenly spaced, the bunches from the two beams will meet only at certain regular intervals. (c) By combining routing and bunching, one can further customize where and when collisions may take place.

Furthermore, detectors can be made to yield richer data about the collisions if these happen only at precisely appointed times as well. To this purpose, the particles are "bunched" so that instead of a continuous beam one has "beads on a necklace" — a regular succession of concentrated volleys that are well separated from one another (Fig. 1.11b). In this way, particles pass by one another only at certain discrete moments in time. Finally, by combining and coordinating the two approaches (namely, routing and bunching), one can further customize when and where collisions may take place (Fig. 1.11c).

The spacetime diagrams corresponding to the routing and bunching of Fig. 1.11 are illustrated in Fig. 1.12. Note that a bunch is shown as having a lifeline that persists through a number of collision sites. In fact, even though an individual bunch particle may undergo a destructive interaction at a site, the bunch itself, which consists of very many particles, as a whole retains its identity through an experiment (and may even be "replenished" on a regular basis).

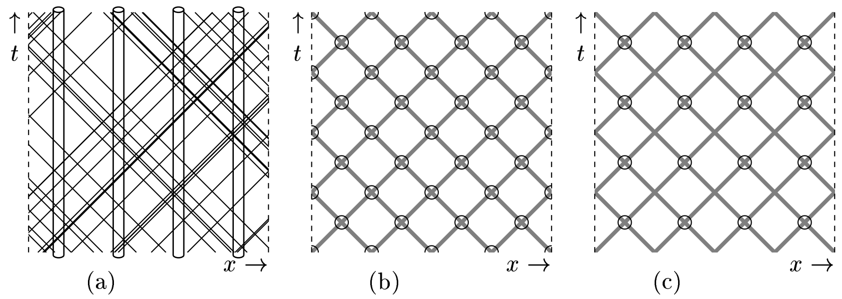


Fig. 1.12. Spacetime diagrams for the beam geometries of Fig. 1.11. (a) Lifelines of particles in beams routed as in Fig. 1.11a. The small cylinders are the spacetime representation of the collision sites; the two families of particle lifelines only intersect within these cylinders; outside of the cylinders, what graphically looks like an intersection is in reality a passive crossover. (b) Lifelines of particle bunches, as in Fig. 1.11b. Now it is the timing constraints that limit collisions to discrete spatial sites. (c) Adding routing to bunching further restricts the possibilities for collision, as at some of the grid intersections the beams cross over without interacting.

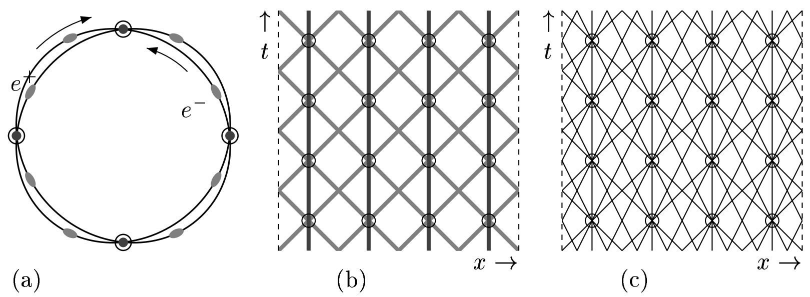


Fig. 1.13. (a) To the circulating beams of Fig. 1.11c we have added bunches of stationary particles (zero speed) hovering at the collision sites. (b) Overall spacetime diagram for the three kinds of bunches, traveling at speeds -1, 0, $+1$. In (c) we have added two more circulating bunched beams traveling at velocities $+1/2$ and $-1/2$, yielding 5-body interactions at each site.

We may even imagine that, in addition to the two circulating beams, bunches of *stationary* particles are located at each of the collision sites of Fig. 1.11, as indicated by the darker dots in Fig. 1.13a. The lifelines of these bunches, for which x is constant, are indicated in Fig. 1.13b. We now have three families of bunches, with respective velocities -1, 0, and $+1$, that cross one another's lifelines at certain regularly spaced interaction loci (Fig. 1.13b). We can further enliven the collider by adding two more bunched beams of particles that travel at velocities $+1/2$ and $-1/2$, yielding five-body interactions at each site, as in Fig. 1.13c.

What we've been doing may properly be termed *spacetime crystallography*; that is, in Fig. 1.12 and Fig. 1.13 we have constructed regular diagrams of causal dependencies expressing the spacetime ordering with which collisions have been arranged within the particle collider. We will note for the moment that these diagrams are identical to those of *lattice gases* (see Sect. 1.1), once we replace the different types of particle bunches with different types of lattice-gas signals.

We'll leave off for a moment particle colliders, and turn to the issue of ballistic dynamics.

1.4 Why Aristotle Didn't Discover Universal Gravitation

You cannot park in space! If you have engine troubles on the expressway you can pull over, stop, leave your car on the emergency lane, and come pick it up with a tow truck an hour later. But if you are going to Mars and your ion drive runs out of fuel, there is just no way you can "leave your spaceship there". Your ship will go hurtling through space in an orbit around the Sun. If you actually insisted on stopping it, say, by firing the retro rockets (and that may cost you *a lot* of energy), your ship will start falling into the Sun. In neither case would it make sense to say, "Oh, I just left it last night at celestial coordinates such and such. Here are the keys and a gallon of fuel — will you please go get it for me?" In sum, the natural "rest" state for a spaceship or an asteroid is not "motionlessness" but travel along an inertial trajectory.

If things are so, where in the world did Aristotle ($\overline{1}616$–$\overline{1}678$)[11] get his idea that an object will grind to a halt as soon as you stop pushing it? Well, that is simply what *everybody* knows, because that's what you can see with your own eyes. A ship won't move without wind or oars. Kill a bird with a stone and it will stop in midair and fall to the ground. And have you ever tried to push a cart? It is evident that to make it go faster you have to push harder. When Aristotle said that the "the velocity of an object is proportional to the applied force" he was just stating in a formal way what everybody intuitively knew all along.

As a corollary of this belief, if an object moves on a certain course you should look *along* its trajectory — right behind the object or right ahead of

[11] Aristotle's dates are, of course, 384–322 B.C.; by expressing them as $-1000+616$ and $-1000+678$ (that's what our notation means) we facilitate comparison between B.C. and A.D. dates. Specifically, it becomes obvious that Galileo (1564–1642) lived about two thousand years after Aristotle ($\overline{1}616$–$\overline{1}678$).

it — to find out who's pushing or pulling it. More formally, to find out what makes a body move you must look in the direction of its *velocity* vector.

Aristotle's prescription works reasonably well with earthly — or "sublunar" — things, but when we look at the planets' orbits we can't see behind or before the planets anything of any account. Are planets being pushed by invisible angels? Or is each planet bolted to an invisible crystal sphere that carries the planet with it and that somebody (presumably hidden below the horizon) keeps pushing around like a donkey a grain mill? Or are celestial things governed by laws quite unlike those in force on earth?

The issue remained stuck at that point for two thousand years, until Galileo (1564–1642) started playing harmless games with "bocce" balls on gently sloping tracks; the latter "diluted" gravity, and thus velocity and thence air friction. From this Galileo concluded, "In ideal conditions, velocity comes for free; it is *change* in velocity that one has to pay for!" More formally, to find out what makes a body go faster, slow down, or turn away from a straight line, you must look in the direction of its *acceleration* vector.

Though it had been Galileo that first had turned the telescope to the planets, it didn't occur to him to turn to the planets his own conclusions on forces and accelerations. But Newton (1643–1727) did; and when he looked along a planet's acceleration vector rather than its velocity vector what did he see? Lo and behold, the Sun! — a *very big angel* indeed.[12]

After that observation, it didn't take much imagination to guess that it was the Sun that, by somehow pulling on the planets, steered them into their quasi-circular orbits, and that similarly it was the Earth that was making the Moon go circles.[13] Thus, a seemingly minor shift of attention, from velocities to accelerations, opened the door to one of the most powerful insights in the history of science.

The contrast between cellular automata and lattice gases is essentially one between the Aristotelian and the Galilean approach to dynamics, between a *dissipative* or "damped" philosophy of history and a *conservative* or "ballistic" one. In the first view, the natural state of an object is rest and we have to "do something" just to make it move. In the second, the natural state is inertial motion and a dynamics only has to specify what is new *with respect to that*. Moreover, as explained at some length in [5], in the first approach variables are thrown away at every step and are replaced with freshly fabricated ones, while in the second approach variables retain their identity

[12] Kepler (1571–1630) had shown that, if one wanted to be precise, planets moved on ellipses of low eccentricity rather than circles (or circles upon circles upon circles as per Ptolemy's model) as Galileo thought, and with different velocities on different points of the ellipse. Though the Sun was not at the center of these ellipses but a bit off-center, at one of the two foci, still the acceleration of the planets always kept pointing exactly toward the Sun!

[13] This whole argument paraphrases in part Feynman [4, Lecture 2].

through a step of the dynamics — even though they may thereby undergo a change of state (cf. Fig. 1.7).

1.5 "On The Nature of the Universe"

Let's return to colliding-beams accelerators. The by-products of a collision are scrutinized, occasionally subjected to new collisions, but then as a rule purged out of the system. In most cases, these particles will no longer have the desired energy or momentum (or, even worse, they may no longer be the desired *kinds* of particles). At any rate, since some noise is unavoidably picked up at each collision, the dynamical characteristics of the particles become less predictable and thus less useful for studying further interactions. For all practical purposes, particles become, after a few collisions, "tired" and "worn out" and must be replaced by fresh ones.

Of course, nothing of that kind has really happened to the particles; it is only our *information* about them that has become "tired" and "worn out;" the particles themselves are just fine, thank you!

Yet, since everything in our coarse world seems to run down, it takes a veritable *leap of faith* to believe that the particles with their agitation keep a piston from settling at the bottom of a cylinder never get tired, and will keep supporting the piston even when all batteries in the world will have run out and the universe will be approaching thermal death. The "spring of air", as they called it in the seventeenth century [2], is, to make a bad pun, an *eternal* spring; it's among the few entities that have nothing to fear from the second principle of thermodynamics.

This leap of faith — which many still find hard to make today — would have been unthinkable before Galileo. Two thousand years before, it is true, Democritus ($\bar{1}540$–$\bar{1}630$) had proposed an atomistic theory, which Epicurus ($\approx \bar{1}650$–$\bar{1}729$) had then developed into an all-encompassing "philosophical" (that is, physical) system. But neither Epicurus nor his best literary agent, Lucretius ($\bar{1}902$–$\bar{1}945$), could figure out how the whole thing could possibly work, and had to make recourse to an *ad hoc* addition to their theory to make it believable not only to others but to themselves. I will explain what I mean, using mostly Lucretius' words [11, *passim*].

"You must know, my Memmius, that *nothing can ever be created by divine power out of nothing* [conservation of matter and energy]. ... Furthermore, *nature resolves everything into its component atoms and never reduces anything to nothing* [universal atomic hypothesis]. ... From the action of the wind etc. you have evidence of *bodies whose existence you must acknowledge though they cannot be seen* [you may not see the atoms, but the spring of air is enough witness to them]. ... *Material objects are of two kinds, atoms and compounds of those atoms. The atoms themselves cannot be swamped by any force, for they are preserved indefinitely by their absolute solidity* [if you don't see the atoms themselves, you may at least see aggregates of atoms;

though these may change shape and be eroded, the atoms themselves *never wear out*]."

So far, so good! Everything looks strictly kosher even when seen with post-Newtonian eyes. But then, with as little advertisement as possible, comes most embarrassing admission: "In this connection, there is another fact that I want you to grasp. *When atoms are travelling straight down through empty space by their own weight* [so, after all, *some force* must be constantly pulling the atoms lest they slow down] *at quite indeterminate times and places they ever swerve so little from their course*, just so much that you can call it a change of direction. If it were not for this swerve, everything would fall downwards like rain-drops through the abyss of space."

Ah, poor Lucian, you had practically managed to walk on water across the whole lake, but your faith failed you at the last moment! In the end, you couldn't believe that atoms can keep colliding forever, on their own, without some external intervention, and you had to invent this swerve, this infamous *clinamen*! It's the Aristotelian fallacy again: no force, no velocity. If you swirl a handful of sand in the air, the sand grains may collide for a while, but eventually they will asymptote to a downward, *parallel* fall. Their weight will still make them travel vertically, but their horizontal motion relative to one another, not refreshed by anything, will soon be spent, and they will stop colliding.

Critics have made much of Epicurus' *clinamen*, but I think they've missed a most important point. I'll quote from Ronald Latham [11]:

> In one particular, again, Epicurus indulged in a metaphysical subtlety foreign to the spirit of his materialistic doctrine. As a moralist, he believed in free will. If the movements of the atoms were absolutely determined, as Democritus had taught, it seemed to him that all human actions must equally be determined. Therefore the atoms must swerve, very rarely and very little, from the paths ordained for them by nature. To contemporaries this seemed an absurd notion. We may doubt whether it was really relevant to the moral question at issue. But it was one concession in a dogmatic system to that element of the inexplicable and unpredictable in nature which some modern physicists have been driven to acknowledge by a somewhat similar concession.

Note the attempt to bring in free will, morality, and even quantum mechanics! But if we reread Lucretius, we find no attempt to introduce free will under the name of "random will" or whatever. His words are clear: *"If it were not for this swerve, everything would fall downwards like rain-drops through the abyss of space."* This is Aristotle plain and simple! If we really wanted to read in between Lucretius' lines, we could construe an argument as follows. "Collisions are sufficient to account for all the interesting things that happen in the world; thence we need not postulate the Gods in order to keep matters being interesting. However, in order to have collisions, atoms must move toward one another; but all things stop moving if they are not pushed, and thus atoms will eventually stop colliding. Now, it is a fact that interesting things keep happening, thence collisions must be taking place all along; therefore things must be being pushed somehow. Who can possibly push them? The man in the street thinks it can only be other *animate* beings, namely, the Gods; thence superstitious fears and all that. I [Epicurus] think that this "God postulate" is more expensive than the theorem we wanted to obtain. Why introduce the Gods, if all we need from them is a bit of stirring? Any mindless stirring will do! Thence my recipe: Add a little *clinamen* — never mind what that might be — to the inertial (but ultimately damped) motion, and here we go!"

Lattice gases solve Epicurus' dilemma by *postulating away* friction instead of introducing external stirring. What's original with the lattice gas idea is that we have a *digital* rather than an *analog* collider: our control over the nature and the geometry of the interactions is such that what comes out of a collision is just as good, for the sake of arranging further interactions, as what went in. While in a particle collider one must labor to supply a steady supply of fresh projectiles, the tokens in a lattice gas are always "as good as new" and can be recirculated *ad infinitum*. Not only the tokens themselves, but also their directions and velocities are discrete, and therefore always "as good as new". The magnetization of an analog audio tape may imperceptibly decay, but a bit is a bit is a bit — always a brand new "1" or a brand new "0". Similarly, a token on a lattice like (1.8) can only go right or left, never in a direction in between.

Given that our perceptual world is still not much different from Aristotle's, the lattice-gas model of computation is not an intuitive one; it takes courage and faith to entertain it. But, once we have grasped it, we can dream Democritus' dream in peace. We no longer have to worry about having to explain to our disciples that, yes, it was supposed to be atoms all the way down, but actually without a God or a random swerve it won't work.

1.6 Conclusions

It took centuries of intellectual struggle to realize that the physical world can get by with collisions *and nothing else*. Fredkin's intuition — that this idea applies to computation as well, has given rise to Symbol Super Colliders, both as conceptual and experimental tools.

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