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Computational Learning Theory

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Erratum

Fig. 1 on page 381 in the printed version has unfortunately been replaced by something completely unrelated due to an error during the data conversion.

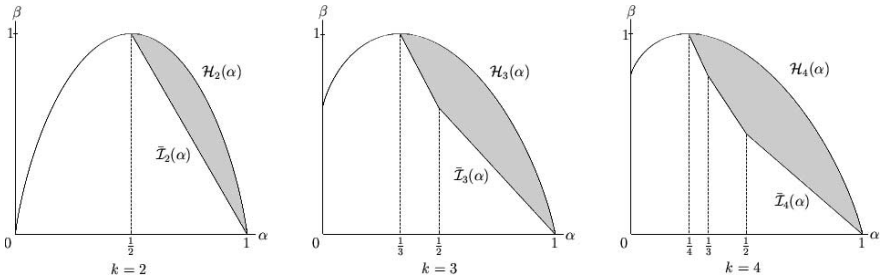

Fig. 1. Prediction-dimension diagrams for $k = 2, 3, 4$

Kolmogorov-Loveland stochastic sequences that are not random, i.e., sequences that are unpredictable but on which a constructive gambling strategy can accrue unbounded winnings. This result gave a clear qualitative separation between unpredictability and randomness, and hence between prediction and gambling. However, the precise quantitative relationship between these processes has not been elucidated. Given the obvious significance of prediction and gambling for computational learning [2,3,25] and information theory [7,8] this situation should be remedied.

Recently, Lutz [13,14] has defined computational effectivizations of classical Hausdorff dimension ("fractal dimension") and used these to investigate questions in computational complexity and algorithmic information theory. These effectivizations are based not on Hausdorff's 1919 definition of dimension [9,6], but rather on an equivalent formulation in terms of gambling strategies called *gales* [13]. These gales (defined precisely in section 4 below) give a convenient way of quantifying the discount rate against which a gambling strategy can succeed. (Ryabko [16,17,18] and Staiger [23,24]) have conducted related investigations of classical Hausdorff dimension in equivalent terms of the rate at which a gambling strategy can succeed in the absence of discounting.) The *feasible dimension* $\dim_{\mathrm{p}}(X)$ of a set X of sequences is then defined in terms of the maximum discount rate against which a feasible gambling strategy can succeed.

In this paper we use feasible dimension as a model of feasible gambling, and we compare $\dim_{\mathrm{p}}(X)$ quantitatively with the *feasible predictability* $\mathrm{pred}_{\mathrm{p}}(X)$ of X, which is the highest success ratio that a polynomial-time randomized predictor (defined precisely in section 3 below) can achieve on all sequences in X. Our main theorem, described after this paragraph, gives precise bounds on the relationship between $\mathrm{pred}_{\mathrm{p}}(X)$ and $\dim_{\mathrm{p}}(X)$. We also investigate the *deterministic feasible predictability* $\mathrm{dpred}_{\mathrm{p}}(X)$, in which the predictor is required to commit to a single outcome. We use the probabilistic method to prove that $\mathrm{dpred}_{\mathrm{p}}(X) = \mathrm{pred}_{\mathrm{p}}(X)$ whenever X consists of a single sequence, and we show

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