

# Contents

<b>1. Introduction</b>	1
<b>2. Preliminaries</b>	7
2.1 Hamiltonian dynamics	7
2.2 The role of boundary conditions in Hamiltonian field theory	10
2.3 Tangential translations as a Hamiltonian system	13
2.4 The Hamiltonian description of a mixed Cauchy – characteristic initial value problem	15
2.5 The Trautman–Bondi energy for the scalar field	20
<b>3. Hamiltonian flows for geometric field theories</b>	23
3.1 The framework	23
3.2 Hamiltonian dynamics for Lagrangian theories	29
3.3 Space-time integrals	34
3.4 Changes of $\Psi$ and of the Lagrangian	38
<b>4. Radiating scalar fields</b>	41
4.1 Preliminaries	41
4.2 Energy: convergence of integrals	45
4.3 The phase space $\mathcal{P}_{(-\infty,0]}$	49
4.4 The phase space $\mathcal{P}_{[-1,0]}$	51
4.5 The phase space $\widehat{\mathcal{P}}_{[-1,0]}$	52
4.6 The preferred Hamiltonian role of the Trautman–Bondi energy for scalar fields	54
4.7 The Poincaré group	57
4.8 “Supertranslated” hyperbolae	61
<b>5. The energy of the gravitational field</b>	65
5.1 Preliminaries	65
5.2 Moving spacelike hypersurfaces	70
5.3 Cosmological space-times	74
5.4 Space-times asymptotically flat in spacelike directions	76
5.5 Space-times with anti-de Sitter asymptotic behaviour	80

5.6	Energy in the radiation regime: convergence of integrals . . . . .	89
5.7	Phase spaces: the space $\mathcal{P}$ . . . . .	97
5.8	The phase space $\mathcal{P}_{[-1,0]}$ . . . . .	102
5.9	The phase space $\widehat{\mathcal{P}}_{[-1,0]}$ . . . . .	103
5.10	Preferred role of the Trautman–Bondi energy . . . . .	104
<b>6.</b>	<b>Hamiltonians associated with the BMS group</b> . . . . .	<b>105</b>
6.1	The Poincaré group: convergence of integrals . . . . .	105
6.2	Supertranslations (and space translations): convergence of integrals . . . . .	111
6.3	The abstract Scri . . . . .	116
6.4	Lorentz charges . . . . .	119
6.5	A Hamiltonian definition of angular momentum of sections of $\mathcal{S}$ . . . . .	121
6.6	An example: Schwarzschild space-time . . . . .	126
6.7	An example: stationary space-times . . . . .	129
6.8	Lorentz covariance of global charges . . . . .	131
6.9	BMS invariance of energy-momentum . . . . .	134
6.10	Polyhomogeneous Scri's . . . . .	136
<b>A.</b>	<b>Odd forms (densities)</b> . . . . .	<b>139</b>
<b>B.</b>	<b>Solutions of the wave equation smoothly extendable to <math>\mathcal{S}^+</math></b> . . . . .	<b>141</b>
<b>C.</b>	<b>Gravitational field: some auxiliary results</b> . . . . .	<b>143</b>
C.1	The canonical gravitational variables in Bondi coordinates . . . . .	143
C.1.1	Smooth Scri's . . . . .	143
C.1.2	Polyhomogeneous asymptotics . . . . .	145
C.2	Solutions of the vacuum Einstein equations containing hyperboloidal hypersurfaces . . . . .	146
C.3	Bondi coordinates <i>vs</i> hyperboloidal initial data . . . . .	152
C.4	The calculation of the $\mathbb{W}^{x\mu}$ 's . . . . .	157
C.5	Transformation rules of the Bondi functions under supertranslations . . . . .	159
C.6	Transformation rules of the Bondi functions under boosts . . . . .	162
C.7	Bondi coordinates in the Kerr space-time . . . . .	163
C.8	Conformal rescalings of ADM Cauchy data . . . . .	166
	<b>References</b> . . . . .	<b>167</b>

---

<sup>1</sup> **Acknowledgements** JJ wishes to thank the Région Centre for financial support, and the Department of Mathematics of the Tours University for hospitality during part of work on this paper. We are grateful to M. MacCallum for help with a symbolic algebra calculation.

Hamiltonian Field Theory in the Radiating Regime

Chrusciel, P.T.; Jezierski, J.; Kijowski, J.

2002, VI, 174 p., Hardcover

ISBN: 978-3-540-42884-8