

# Introduction

The progress of the string theory in the last decade strongly influenced the development of many branches of geometry. In particular, new directions of researches in the enumerative geometry and symplectic topology have been created as a joint venture of physicists and mathematicians. Among the most striking achievements of this period we mention the description of the intersection theory on moduli spaces of Riemann surfaces in terms of the Korteweg - de Vries integrable hierarchy of PDEs, and the proof of mirror conjecture for Calabi - Yau complete intersections.

One of the essential ingredients of these beautiful mathematical theories is a bunch of new approaches to the problem of constructing invariants of algebraic varieties and of compact symplectic manifolds known under the name *quantum cohomology*. Physical ideas from topological gravity brought into the problem of invariants new structures of the theory of integrable systems of differential equations. The discovery of dualities between different physical theories suggested existence of deep and often unexpected relationships between different types of invariants.

In order to present, by both mathematicians and physicists, these new ideas to young researchers, we have decided to organize a CIME Summer School under the general title "Quantum Cohomology". The School took place at Calabrian sea resort Cetraro from June 30 to July 8, 1997. It was organized in four courses covering various aspects of these new mathematical theories. These Lecture Notes contain the extended text of the lecture courses.

In the course of Kai Behrend "Localization and Gromov - Witten Invariants" the approach to enumerative invariants of algebraic varieties based on the Bott residue formula has been developed. Behrend gave essentially self-consistent exposition of this approach for the important particular case of Gromov - Witten invariants of projective spaces.

The lecture course of course "Fields, Strings and Branes" by César Gómez, written in collaboration with Rafael Hernández, collect some ideas of duality in string theories important for the development of quantum cohomology. The design of the presentation looks to be a physical one. Nevertheless we are confident that those mathematicians working in the area of quantum cohomology who have no prejudices against reading physical papers will be benefitted.

The lecture notes of Vitaly Tarasov "q-Hypergeometric Functions and Representation Theory" introduces the reader to another branch of the theory of integrable systems originated in the theory of form factors in massive integrable models of quantum field theory. This branch now developed into a part of representation theory of quantum affine algebras and of the corresponding vertex operators. Tarasov explains how to compute the matrix elements of the vertex operators in the terms of solutions to the quantized

Knizhnik - Zamolodchikov equation, and derives integral representations for these solutions.

The course of Gang Tian introduces the reader to the techniques of symplectic topology involved in the construction of Gromov - Witten invariants of compact symplectic manifolds. The main technical tool is the theory of virtual fundamental class on the moduli spaces of pseudoholomorphic curves. Tian applies this technique to the definition of quantum cohomology of symplectic manifolds and to constructions of certain nontrivial examples of symplectic manifolds.

We believe that the School was successful in reaching its aims, and we express our gratitude to the speakers for the high quality of their lectures and their availability for discussions during the School.

We also thank Prof. R.Conti and CIME Scientific Committee for the invitation to organize the School.

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Quantum Cohomology

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