# Preface

There is a long history of the application of monotone methods and comparison arguments in deterministic dynamical systems (see, e.g., Smith [102] and the literature quoted there). Monotonicity methods are now fully integrated within the framework of deterministic dynamical systems theory. The situation is quite different for random systems and stochastic differential equations. Monotonicity arguments have mainly been used for one-dimensional random or stochastic differential equations, relying on well-known comparison theorems for solutions of one-dimensional ordinary random (see, e.g., Ladde/Lakshmikantham [75]) or stochastic (see, e.g., Ikeda/Watanabe [57]) equations. In particular, these theorems and also the analysis of some explicitly solvable models make it possible to give a complete description of random attractors and bifurcation scenarios for several rather complicated situations (see, e.g., Arnold [3, Chap.9]).

Let us also mention that products of positive random matrices have been the subject of numerous studies (comprising, in particular, a random version of Perron-Frobenius theory) with applications notably in economics and biology (for a survey, see Arnold/Demetrius/Gundlach [8]). Kellerer [65] found that independent identically distributed iterations of monotone random mappings on $\mathbb{R}_+$ are a model ideally suited for extending discrete Markov chain theory to uncountable state spaces.

Our main goal in this book is to present the basic ideas and methods for order-preserving (or monotone) random dynamical systems that have been developed over the past few years. We focus on the qualitative behaviour of these systems and our main objects are equilibria and attractors.

There is a deep analogy between the theory of random dynamical systems and the classical theory of dynamical systems. This analogy makes it possible to develop qualitative theory for stochastic systems relying on ideas of classical dynamical systems. In this book we try to expose this analogy in a clear and transparent way. We hope it makes the book accessible not only to experts in stochastic analysis but also to people working in the field of deterministic dynamical systems. It provides a bridge from classical theory to stochastic dynamics and it can be also used as an introductory textbook on random dynamical systems at the graduate level.

Our main application is to the so-called cooperative random and stochastic ordinary differential equations. These systems arise naturally from mathematical models in the field of ecology, epidemiology, economics and biochemistry (see, e.g., the literature quoted in Smith [102]). Deterministic cooperative differential equations have been studied by many authors (see, e.g., Smith [102] and the references therein). The books by Krasnoselskii [68, 69] and the series of papers by Hirsch [52, 53, 54] (see also the references in Smith [102]) lay the groundwork for the qualitative theory of deterministic cooperative systems. Monotone methods and comparison arguments are of prime importance in the study of these systems.

The results presented in this book rely on ideas and methods developed in collaboration with Ludwig Arnold (see Arnold/Chueshov [5], [6] and [7]). The author is extremely grateful to him for very stimulating and fruitful discussions on the subject. Warmest thanks are also due to Gunter Ochs, James Robinson and Björn Schmalfuss for their comments and suggestions, all of which improved the book.

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