Preface

This book grew out of my attempt in August 1998 to compare Carleson's and Fefferman's proofs of the pointwise convergence of Fourier series with Lacey and Thiele's proof of the boundedness of the bilinear Hilbert transform. I started with Carleson's paper and soon realized that my summer vacation would not suffice to understand Carleson's proof.

Bit by bit I began to understand it. I was impressed by the breathtaking proof and started to give a detailed exposition that could be understandable by someone who, like me, was not a specialist in harmonic analysis. I've been working on this project for almost two years and lectured on it at the University of Seville from February to June 2000. Thus, this book is meant for graduate students who want to understand one of the great achievements of the twentieth century.

This is the first exposition of Carleson's theorem about the convergence of Fourier series in book form. It differs from the previous lecture notes, one by Mozzochi [38], and the other by Jørsboe and Mejlbro [26], in that our exposition points out the motivation of every step in the proof. Since its publication in 1966, the theorem has acquired a reputation of being an isolated result, very technical, and not profitable to study. There have also been many attempts to obtain the results by simpler methods. To this day it is the proof that gives the finest results about the maximal operator of Fourier series.

The Carleson analysis of the function, one of the fundamental steps of the proof, has an interesting musical interpretation. A sound wave consists of a periodic variation of pressure occurring around the equilibrium pressure prevailing at a particular time and place. The sound signal f is the variation of the pressure as a function of time. The Carleson analysis gives the score of a musical composition given the sound signal f. The Carleson analysis can be carried out at different levels. Obviously the above assertion is true only if we consider an adequate level.

Carleson's proof has something that reminds me of living organisms. The proof is based on many choices that seem arbitrary. This happens also in living organisms. An example is the **error** in the design of the eyes of the vertebrates. The photoreceptors are situated in the retina, but their outputs emerge on the **wrong** side: inside the eyes. Therefore the axons must finally be packed in the optic nerve that exit the eyes by the so called **blind spot**. But so many fibers (125 million light-sensitive cells) will not pass by a small spot. Hence evolution has solved the problem packing another layer of neurons inside the eyes that have rich interconections with the photoreceptors and with each other. These neurons process the information before it is send to the brain, hence the number of axons that must leave the eye is sustantially reduced (one million axons in each optic nerve). The incoming light must traverse these neurons to reach the photoreceptors, hence evolution has the added problem of making them transparent.

We have tried to arrange the proof so that these things do not happen, so that these arbitrary selections do not shade the idea of the proof. We have had the advantage of the text processor TEX, which has allowed us to rewrite without much pain. (We hope that no signs of these rewritings remain).

By the way, the eyes and the ears process the information in totally different ways. The proof of Carleson follows more the ear than the eyes. But what these neurons are doing in the inside of the eyes is just to solve the problem: How must I compress the information to send images using the least possible number of bits? A problem for which the wavelets are being used today.

I would like this book to be a commentary to the Carleson paper. Therefore we give the Carleson-Hunt theorem following more Carleson's than Hunt's paper.

The chapter on the maximal operator of Fourier series S^*f, gives the first exposition of the consequences of the Carleson-Hunt theorem. Some of the results appear here for the first time.

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