

Contents

Preface to the English Edition	VII
From the Preface to the Russian Edition	XI
Introduction	1
0.1 Statement of Model Problems	3
0.2 Difference Potentials	7
0.3 Solution of Model Problems	25

**Part I. Justification of Algorithms of the Method
of Difference Potentials for Calculating
Numerical Solutions of Interior
Boundary-Value Problems
for the Laplace Equation**

1. Preliminaries	37
1.1 Local Splines	37
1.2 Finite Fourier Series	40
1.3 Calculation of the Solution of a Difference Analog of the Dirichlet Problem for the Poisson Equation in a Square Region	41
1.4 Hölder Spaces	44
1.5 Schauder and Thomee Estimates	48
1.6 On the Proximity of Solutions of the Differential and Difference Dirichlet Problems for the Poisson Equation in a Square Domain	49
2. Differential and Difference Potentials	53
2.1 Fundamental Solution and Green Functions and Operators ..	53
2.2 Potentials and Their Integral and Operator Representations ..	56
2.3 Definition and Calculation of a Difference Potential	66
2.4 Approximation of a Differential Potential by a Difference Potential	72

3. Reduction of Boundary-Value Problems for the Laplace Equation to Boundary Equations of Calderón–Seeley Type	81
3.1 Boundary Projection and a Boundary Condition of Calderón–Seeley Type	81
3.2 Passage from Boundary-Value Problems in a Domain to Equations on Its Boundary	84
4. Numerical Solution of Boundary-Value Problems	87
4.1 Intermediate Discretization	87
4.2 Final Discretization	97
4.3 Conjugate Gradient Method	117
4.4 Reduction of the Discrete Problem to a Form Convenient for the Solution by the Conjugate Gradient Method	123
4.5 An Algorithm for the Numerical Solution of the Discrete Problem by the Conjugate Gradient Method	129
4.6 A Computational Example	135

Part II. General Constructions of Surface Potentials and Boundary Equations on the Basis of the Concept of a Clear Trace

1. Generalized Potentials and Boundary Equations with Projections for Differential Operators	141
1.1 Clear Trace and General Constructions of Differential Potentials and Boundary Equations with Projections	141
1.2 Conditionality of Boundary Equations with Projections	151
1.3 Comments on the Literature	154
2. General Constructions of Potentials and Boundary Equations for Difference Operators	159
2.1 General Constructions	159
2.2 Examples	167
2.3 Cauchy-Type Potentials for General Linear Systems of Difference Equations on Abstract Grids	172
2.4 Cauchy-Type Potentials and Uniquely Solvable Difference Boundary-Value Problems	179
2.5 Reznik’s Algorithm for Calculating the Difference Potential	198
2.6 Comments on the Literature	201

3. Lazarev's Results on the Algebraic Structure of the Set of Surface Potentials of a Linear Operator	207
3.1 Preliminaries	207
3.2 Potentials with Density from the Space of Clear Traces and Boundary Equations with Projections of an Abstract Operator	209

**Part III. A General Scheme of the Method of Difference
Potentials for the Numerical Solution
of Differential and Difference Boundary-Value
Problems of Mathematical Physics**

1. A General Scheme of the Method of Difference Potentials for Differential Problems	217
1.1 Nonclassical Auxiliary Problems	217
1.2 Admissible Arbitrariness in the Choice of the Construction of a Clear Trace	221
1.3 Scheme for Approximating Differential Potentials by Difference Potentials	225
1.4 The Reznik Theorems on the Approximation of the Surface Potentials of Elliptic Operators by Difference Potentials	231
1.5 Intermediate Discretization of Boundary Equations with Projections	234
1.6 Constructive Discretization Scheme for Boundary Equations with Projections	240
1.7 Fragments of Other Methods of Constructive Discretization	249
1.8 Methods of Deriving an Algebraic System of Simple Structure	252
1.9 On the Operator adjoint to the Green Operator of the Difference Auxiliary Problem	257
2. Illustrations of Constructions of the Method of Difference Potentials	273
2.1 Examples of Interior Problems	273
2.2 Examples of Exterior Problems	279
2.3 An Example of Constructing the Difference Potential for Solving Numerically Boundary-Value Problems in a Domain with a Cut	282
2.4 An Example of Boundary Equations with Projections for the Stokes System	287

3. General Scheme of the Method of Difference Potentials for Solving Numerically the Difference Analogs of Differential Boundary-Value Problems	291
3.1 Statement of Difference Problems	291
3.2 Abstract Equations with Projections	293
3.3 Reduction of Difference Problems to Equations for the Density of the Difference Potential and the Scheme for Calculating Solutions to These Equations	302
3.4 Methods for Obtaining Boundary Conditions with Projections that are Convenient for Iterations	305
3.5 Difference Single Layer Potential as an Example of Potentials of Special Form. Resonance in the Complementary Domain. Relation to the Capacity Matrix Method	315
3.6 Remark on the Combined Use of the Finite Element Method, the Fedorenko Multigrid Method, and the Method of Difference Potentials	321

**Part IV. Examples of MDP Algorithms for Solving
Numerically Boundary-Value Problems
of Mathematical Physics**

1. The Tricomi Problem	329
1.1 Difference Analogs of the Tricomi Problem	330
1.2 Algorithms of the Method of Difference Potentials	334
1.3 Computational Results	339
2. Constructions of the Method of Difference Potentials for the Computation of Stressed States of Elastic Compressible Materials	341
2.1 Difference Potential	341
2.2 Remarks on Algorithms of the Method of Difference Potentials	343
3. Problems of Internal Flows of Viscous Incompressible Fluids	345
3.1 An Algorithm for Solving the Two-Dimensional Stokes Problem Numerically in the Natural Variables (Torgashov Algorithm)	346
4. An Example of the MDP Algorithm for Computing the Stationary Acoustic Wave Field outside a Solid of Revolution	371
4.1 Difference Spherical Harmonics	371

4.2	Constructions of the Difference Potential for Exterior Problems	376
4.3	An Algorithm for Solving Exterior Problems for Solids of Revolution	380
4.4	Numerical Examples	389

Part V. Artificial Boundary Conditions for Stationary Problems

1.	An Efficient Algorithm for Constructing Artificial Boundary Conditions for a Model Problem	395
2.	On the Results of the Application of the Method of Difference Potentials to the Construction of Artificial Boundary Conditions for External Flow Computations	403
2.1	Introduction	403
2.2	Formulation of the Problem	404
2.3	Two-Dimensional Flows Around Airfoils	411
2.4	Three-Dimensional Flows Past a Wing	419
2.5	Three-Dimensional Flow with Jet Exhaust	429

Part VI. General Constructions of Difference Nonreflecting Artificial Boundary Conditions for Time-Dependent Problems

1.	Nonreflecting Difference Conditions on the Moving and Shape Varying Boundary of the Computational Domain	445
1.1	Introduction	445
1.2	Formulation of the Problem	446
1.3	Construction of NRABCs	450
1.4	Possibility of Speeding up Computations by Taking Account of the Properties of Specific Problems	457
1.5	Bibliographical Comments	460
2.	Spectral Approach to the Construction of Nonreflecting Boundary Conditions	461
2.1	Finite-Difference Nonreflecting Boundary Conditions	462
2.2	Algorithm for NRABC Approximation	469
2.3	Choice of a Particular Basis	476
2.4	Numerical Experiments	477

2.5 Potential Generalizations	481
-------------------------------------	-----

**Part VII. Nonreflecting Artificial Boundary Conditions
for Replacing the Rejected Equations
with Lacunas**

1. Problem of Constructing NRABCs and the Corresponding Auxiliary Cauchy Problem	489
1.1 Definition of Nonreflecting Artificial Boundary Conditions (NRABCs)	489
1.2 Auxiliary Difference Cauchy Problem for Constructing NRABCs	492
2. Algorithm for Solving the Cauchy Problem with the Help of Lacunas	495
2.1 Lacunas	495
2.2 Economical Algorithm for Computing the Solution of the Difference Cauchy Problem	499
2.3 Taking Account of the Special Properties of the Auxiliary Difference Cauchy Problem Used to Compute the Nonreflecting Artificial Boundary Conditions	505
2.4 Turchaninov's Phenomenon	506
2.5 Numerical Experiments	507
2.6 On Problems in a Moving Computational Domain	510

Part VIII. Problems of Active Shielding and Imitation

1. Active Shielding Control	515
1.1 Difference Stationary Problem of Active Shielding	515
1.2 Brief Bibliographic Review	518
2. Difference Imitation Problems	519
2.1 Difference Schemes	519
2.2 Statement and General Solution of the Imitation Problem ...	520
Appendix	523
References	525
Index	537



<http://www.springer.com/978-3-540-42633-2>

Method of Difference Potentials and Its Applications

Ryaben'kii, V.S.

2002, XVIII, 538 p., Hardcover

ISBN: 978-3-540-42633-2