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Notation

Set notation

x, y, z, \dots	points of \mathbb{R}^d
X, Y, Z, \dots	subsets of \mathbb{R}^d
$\mathcal{X}, \mathcal{Y}, \dots$	classes of subsets of \mathbb{R}^d
$\mathfrak{X}, \mathfrak{Y}, \dots$	family of classes of subsets of \mathbb{R}^d
X^c	complement of X in \mathbb{R}^d
$X \cup Y$	union of X and Y
$X \cap Y$	intersection of X and Y
$X \setminus Y = X \cap Y^c$	set difference of X and Y
$\check{X} = \{-x : x \in X\}$	reflection of X
X_x	X shifted by \vec{ox} (o = origin of \mathbb{R}^d)
$X \oplus Y = \cup_{y \in Y} X_y$	Minkowski sum of X and Y
$X \ominus Y = \cap_{y \in Y} X_y$	Minkowski difference of X and Y
$X^Y = (X \ominus \check{Y}) \oplus Y$	morphological opening of X w.r.t. Y
$X^Y = (X \oplus \check{Y}) \ominus Y$	morphological closure of X w.r.t. Y
$X \downarrow S$	projection of X onto subspace S
$\overset{\circ}{X}$	topological interior of X
\overline{X}	topological closure of X
∂X	boundary of X
$ X $	volume of X (case X infinite)
$\#X$	cardinality of X (case X finite)
$B(x, r)$	ball with center x and radius r
$B_d = B(o, 1)$	standard unit ball in \mathbb{R}^d
$\omega_d = B_d $	volume of the unit ball in \mathbb{R}^d
$S_d = \partial B_d$	standard unit sphere in \mathbb{R}^d
$H(\alpha, p)$	hyperplane with direction α and location p
Y^X	set of all mappings from X to Y

Classes of subsets

\mathcal{F}_d	closed subsets of \mathbb{R}^d
\mathcal{G}_d	open subsets of \mathbb{R}^d
\mathcal{K}_d	compact subsets of \mathbb{R}^d
\mathcal{K}_d^*	nonempty compact subsets of \mathbb{R}^d
\mathcal{C}_d	convex subsets of \mathbb{R}^d
\mathcal{R}_d	polyconvex subsets of \mathbb{R}^d

Analysis

1_X	indicator function of set X
$d(X, Y)$	Hausdorff distance between $X, Y \in \mathcal{K}_d^*$
$W_i(X)$	i^{th} Minkowski functional of $X \in \mathcal{R}_d$
$\chi(X) = W_d(X)$	Euler-Poincaré characteristic of $X \in \mathcal{R}_d$
$L^2(X, p)$	set of scalar functions defined on X and square integrable for p
$\langle f, g \rangle$	scalar product between $f, g \in L^2(X, p)$
$\ f\ _2$	norm of $f \in L^2(X, p)$
$\ \mu\ $	total variation of measure μ

Probability and statistics

$A \vee B$	union of the events A and B
$A \wedge B$	intersection of the events A and B
$\mathcal{D}(X)$	distribution of X
$X \sim p$	the distribution of X is p
$X \stackrel{df}{\sim} F$	the distribution function of X is F
$X \stackrel{\mathcal{D}}{\equiv} Y$	X and Y have the same distribution
T_x	hitting functional of $X \in \mathcal{F}_d$
$P(x, A)$	transition kernel
$P^{(n)}(x, A)$	n^{th} iterate of the transition kernel P
$s^2(X Y)$	dispersion variance of X in Y

Distributions

$Unif$	Uniform distribution on $]0, 1[$
$Unif(D)$	Uniform distribution on D
$Gauss$	Standard Gaussian distribution
$Gauss(m, \sigma^2)$	Gaussian distribution with mean value m and variance σ^2
Exp	Exponential distribution with scale factor 1
$Exp(b)$	Exponential distribution with scale factor b
$Gamma(\alpha, b)$	Gamma distribution with parameter α and scale factor b
$Beta(\alpha, \beta)$	Beta distribution with parameters α and β
$Ber(p)$	Bernoulli distribution with parameter p
$Bin(n, p)$	Binomial distribution with index n and parameter p
$Poisson(\theta)$	Poisson distribution with mean θ
$Geom(p)$	Geometric distribution with parameter p
$Nbd(\nu, p)$	Negative binomial distribution with index ν and parameter p
$Sichel(\theta, \alpha)$	Sichel distribution with parameters α and θ
$InvG(\alpha, \beta)$	Inverse Gaussian distribution with parameters α and β
$Bigauss(\rho)$	standard bigaussian distribution with correlation ρ
$Multb(n; p_1, \dots, p_n)$	Multinomial distribution with index n and parameters p_1, \dots, p_n

Miscellaneous

$a \equiv b \bmod m$	a and b are congruent modulo m
$\left(\frac{a}{b}\right)$	Legendre coefficient of a and b
(X, \preceq)	poset
$M_x = \{y \in X : x \preceq y\}$	upper bounds of x in (X, \preceq)
$M^x = \{y \in X : y \preceq x\}$	lower bounds of x in (X, \preceq)

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