

Contents

I Curves

1 Geometric fundamentals

1.1	Affine spaces	3
1.2	Affine combinations	4
1.3	Affine maps	5
1.4	Parametric curves and surfaces	6
1.5	Problems	7

2 Bézier representation

2.1	Bernstein polynomials	9
2.2	Bézier Representation	11
2.3	The de Casteljau algorithm	13
2.4	Derivatives	15
2.5	Singular parametrization	17
2.6	A tetrahedral algorithm	17
2.7	Integration	19
2.8	Conversion to Bézier representation	20
2.9	Conversion to monomial form	22
2.10	Problems	22

3 Bézier techniques

3.1	Symmetric polynomials	25
3.2	The main theorem	27
3.3	Subdivision	27
3.4	Convergence under subdivision	29
3.5	Curve generation by subdivision	30
3.6	Curve generation by forward differences	32
3.7	Intersections	32
3.8	The variation diminishing property	34
3.9	The symmetric polynomial of the derivative	35
3.10	Simple C^r joints	36

3.11	Degree elevation	37
3.12	Convergence under degree elevation	39
3.13	Problems	40
4	Interpolation and approximation	
4.1	Interpolation	43
4.2	Lagrange form	44
4.3	Newton form	47
4.4	Hermite interpolation	48
4.5	Piecewise cubic Hermite interpolation	49
4.6	Approximation	52
4.7	Least squares fitting	53
4.8	Improving the parameter	55
4.9	Problems	56
5	B-spline representation	
5.1	Splines	59
5.2	B-splines	60
5.3	A recursive definition of B-splines	61
5.4	The de Boor algorithm	63
5.5	The main theorem in its general form	65
5.6	Derivatives and smoothness	67
5.7	B-spline properties	68
5.8	Conversion to B-spline form	69
5.9	The complete de Boor algorithm	70
5.10	Conversions between Bézier and B-spline representations	72
5.11	B-splines as divided differences	73
5.12	Problems	74
6	B-spline techniques	
6.1	Knot insertion	77
6.2	The Oslo algorithm	79
6.3	Convergence under knot insertion	80
6.4	A degree elevation algorithm	81
6.5	A degree elevation formula	82
6.6	Convergence under degree elevation	83
6.7	Interpolation	84
6.8	Cubic spline interpolation	86
6.9	Problems	88

7 Smooth curves

7.1	Contact of order r	91
7.2	Arc length parametrization	93
7.3	Gamma-splines	94
7.4	Gamma B-splines	95
7.5	Nu-splines	96
7.6	The Frenet frame	97
7.7	Frenet frame continuity	98
7.8	Osculants and symmetric polynomials	100
7.9	Geometric meaning of the main theorem	102
7.10	Splines with arbitrary connection matrices	103
7.11	Knot insertion	105
7.12	Basis splines	105
7.13	Problems	106

8 Uniform subdivision

8.1	Uniform B-splines	109
8.2	Uniform subdivision	110
8.3	Repeated subdivision	112
8.4	The subdivision matrix	114
8.5	Derivatives	115
8.6	Stationary subdivision	115
8.7	Convergence theorems	116
8.8	Computing the difference scheme	117
8.9	The four-point scheme	119
8.10	Analyzing the four-point scheme	120
8.11	Problems	120

II Surfaces

9 Tensor product surfaces

9.1	Tensor products	125
9.2	Tensor product Bézier surfaces	127
9.3	Tensor product polar forms	130
9.4	Conversion to and from monomial form	131
9.5	The de Casteljau algorithm	132
9.6	Derivatives	133
9.7	Simple C^r joints	135
9.8	Piecewise bicubic C^1 interpolation	135

9.9	Surfaces of arbitrary topology	136
9.10	Singular parametrization	138
9.11	Bicubic C^1 splines of arbitrary topology	139
9.12	Problems	140
10 Bézier representation of triangular patches		
10.1	Bernstein polynomials	141
10.2	Bézier simplices	143
10.3	Linear precision	145
10.4	The de Casteljau algorithm	146
10.5	Derivatives	147
10.6	Convexity	149
10.7	Limitations of the convexity property	150
10.8	Problems	152
11 Bézier techniques for triangular patches		
11.1	Symmetric polynomials	155
11.2	The main theorem	157
11.3	Subdivision and reparametrization	158
11.4	Convergence under subdivision	160
11.5	Surface generation	160
11.6	The symmetric polynomial of the derivative	162
11.7	Simple C^r joints	162
11.8	Degree elevation	164
11.9	Convergence under degree elevation	165
11.10	Conversion to tensor product Bézier representation	166
11.11	Conversion to triangular Bézier representation	167
11.12	Problems	168
12 Interpolation		
12.1	Triangular Hermite interpolation	171
12.2	The Clough-Tocher interpolant	172
12.3	The Powell-Sabin interpolant	173
12.4	Surfaces of arbitrary topology	174
12.5	Singular parametrization	175
12.6	Quintic C^1 splines of arbitrary topology	176
12.7	Problems	178
13 Constructing smooth surfaces		
13.1	The general C^1 joint	179

13.2	Joining two triangular cubic patches	181
13.3	A triangular G^1 interpolant	183
13.4	The vertex enclosure problem	184
13.5	The parity phenomenon	185
13.6	Problems	186
14	G^k-constructions	
14.1	The general C^k joint	189
14.2	G^k joints by cross curves	190
14.3	G^k joints by the chain rule	192
14.4	G^k surfaces of arbitrary topology	193
14.5	Smooth n -sided patches	198
14.6	Multi-sided patches in the plane	201
14.7	Problems	203
15	Stationary subdivision for regular nets	
15.1	Tensor product schemes	205
15.2	General stationary subdivision and masks	207
15.3	Convergence theorems	209
15.4	Increasing averages	211
15.5	Computing the difference schemes	212
15.6	Computing the averaging schemes	214
15.7	Subdivision for triangular nets	215
15.8	Box splines over triangular grids	218
15.9	Subdivision for hexagonal nets	219
15.10	Half-box splines over triangular grids	221
15.11	Problems	222
16	Stationary subdivision for arbitrary nets	
16.1	The midpoint scheme	225
16.2	The limiting surface	227
16.3	The standard parametrization	229
16.4	The subdivision matrix	230
16.5	Continuity of subdivision surfaces	231
16.6	The characteristic map	232
16.7	Higher order smoothness	232
16.8	Triangular and hexagonal nets	234
16.9	Problems	235

III Multivariate Splines

17 Box splines

17.1	Definition of box splines	239
17.2	Box splines as shadows	240
17.3	Properties of box splines	242
17.4	Derivatives of box splines	243
17.5	Box spline surfaces	244
17.6	Subdivision for box spline surfaces	247
17.7	Convergence under subdivision	249
17.8	Half-box splines	251
17.9	Half-box spline surfaces	253
17.10	Problems	256

18 Simplex splines

18.1	Shadows of simplices	259
18.2	Properties of simplex splines	260
18.3	Normalized simplex splines	262
18.4	Knot insertion	263
18.5	A recurrence relation	265
18.6	Derivatives	267
18.7	Problems	268

19 Multivariate splines

19.1	Generalizing de Casteljau's algorithm	271
19.2	B-polynomials and B-patches	273
19.3	Linear precision	274
19.4	Derivatives of a B-patch	275
19.5	Multivariate B-splines	277
19.6	Linear combinations of B-splines	279
19.7	A recurrence relation	280
19.8	Derivatives of a spline	282
19.9	The main theorem	283
19.10	Problems	284

References	287
-------------------	-----

Index	297
--------------	-----

Bézier and B-Spline Techniques

Prautzsch, H.; Boehm, W.; Paluszny, M.

2002, XIV, 304 p., Hardcover

ISBN: 978-3-540-43761-1