

# Contents

<b>Preface</b> .....	v
<b>1. General Technical Results</b> .....	1
1.1 Introduction .....	1
1.1.1 Function Spaces .....	1
1.1.2 Regularity of Domains .....	10
1.1.3 Poincaré Inequality .....	12
1.1.4 Covering of Domains .....	18
1.2 Useful Techniques .....	25
1.2.1 Reverse Hölder's Inequality .....	25
1.2.2 Gehring's Result .....	36
1.2.3 Hole-Filling Technique of Widman .....	38
1.2.4 Inhomogeneous Hole-Filling .....	40
1.3 Green Function .....	44
1.3.1 Statement of Results .....	44
1.3.2 Proof of Theorem 1.26 .....	45
1.3.3 Estimates on $\log G$ .....	46
1.3.4 Estimates on Positive and Negative Powers of $G$ .....	49
1.3.5 Harnack's Inequality .....	52
1.3.6 Proof of Theorem 1.27 .....	57
<b>2. General Regularity Results</b> .....	63
2.1 Introduction .....	63
2.2 Obtaining $W^{1,p}$ Regularity .....	63
2.2.1 Linear Equations .....	63
2.2.2 Nonlinear Problems .....	66
2.3 Obtaining $C^\delta$ Regularity .....	70
2.3.1 $L^\infty$ Bounds for Linear Problems .....	70
2.3.2 $C^\delta$ Regularity for Dirichlet Problems .....	73
2.3.3 $C^\delta$ Regularity for Linear Mixed Boundary Value Problems .....	82
2.3.4 $C^\delta$ Regularity in the Case $n = 2$ .....	85
2.4 Maximum Principle .....	87
2.4.1 Assumptions .....	87

2.4.2	Proof of Theorem 2.16	88
2.5	More Regularity	89
2.5.1	From $C^\delta$ and $W^{1,p_0}$ , $p_0 > 2$ , to $H_{\text{loc}}^2$	89
2.5.2	Using the Linear Theory of Regularity	96
2.5.3	Full Regularity for a General Quasilinear Scalar Equation	98
<b>3.</b>	<b>Nonlinear Elliptic Systems Arising from Stochastic Games</b>	<b>113</b>
3.1	Stochastic Games Background	113
3.1.1	Statement of the Problem and Results	113
3.1.2	Bellman Equations	115
3.1.3	Verification Property	116
3.2	Introduction to the Analytic Part	118
3.3	Estimates in Sobolev spaces and in $C^\delta$	120
3.3.1	Assumptions and Statement of Results	120
3.3.2	Preliminaries	122
3.3.3	Proof of Theorem 3.7	125
3.4	Estimates in $L^\infty$	127
3.4.1	Assumptions	127
3.4.2	Statement of Results	128
3.5	Existence of Solutions	129
3.5.1	Setting of the Problem and Assumptions	129
3.5.2	Proof of Existence	130
3.5.3	Existence of a Weak Solution	132
3.6	Hamiltonians Arising from Games	133
3.6.1	Notation	133
3.6.2	Verification of the Assumptions for Hölder Regularity	135
3.6.3	Verification of the Assumptions for the $L^\infty$ Bound	136
3.7	The Case of Two Players with Different Coupling Terms in the Payoffs	143
3.7.1	Description of the Model and Statement of Results	144
3.7.2	$L^\infty$ Bounds	145
3.7.3	$H_0^1$ Bound	150
<b>4.</b>	<b>Nonlinear Elliptic Systems Arising from Ergodic Control</b>	<b>153</b>
4.1	Introduction	153
4.2	Assumptions and Statement of Results	154
4.2.1	Assumptions on the Hamiltonians	154
4.2.2	Statement of Results	156
4.3	Proof of Theorem 4.4	156
4.3.1	First Estimates	156
4.3.2	Estimates on $u_\epsilon^\nu - \bar{u}_\epsilon^\nu$	158
4.3.3	End of Proof of Theorem 4.4	161
4.4	Verification of the Assumptions	162
4.4.1	Notation	162

4.4.2	The Scalar Case .....	163
4.4.3	The General Case .....	167
4.5	A Variant of Theorem 4.4 .....	169
4.5.1	Statement of Results .....	169
4.5.2	Proof of Theorem 4.13 .....	170
4.6	Ergodic Problems in $R^n$ .....	175
4.6.1	Presentation of the Problem .....	175
4.6.2	Existence Theorem for an Approximate Solution .....	176
4.6.3	Proof of Theorem 4.17 .....	189
4.6.4	Growth at Infinity .....	191
4.6.5	Uniqueness .....	192
<b>5.</b>	<b>Harmonic Mappings</b> .....	197
5.1	Introduction .....	197
5.2	Extremals .....	198
5.3	Regularity .....	200
5.4	Hardy Spaces .....	201
5.4.1	Basic Properties .....	201
5.4.2	Main Regularity Result in the Hardy Space .....	204
5.5	Proof of Theorem 5.13 .....	208
5.5.1	Continuity when $n = 2$ .....	208
5.5.2	Proof of (5.35) and (5.36) .....	216
5.5.3	Proof of (5.37) .....	218
5.5.4	Atomic decomposition .....	221
<b>6.</b>	<b>Nonlinear Elliptic Systems Arising from the Theory of Semiconductors</b> .....	229
6.1	Physical Background .....	229
6.2	Stationary Case Without Impact Ionization .....	230
6.2.1	Mathematical Setting .....	230
6.2.2	Proof of Theorem 6.1 .....	233
6.2.3	A Uniqueness Result .....	240
6.2.4	Local Regularity .....	245
6.3	Stationary Case with Impact Ionization .....	246
6.3.1	Setting of the Model .....	246
6.3.2	Proof of Theorem 6.5 .....	248
6.4	Impact Ionization Without Recombination .....	257
6.4.1	Statement of the Problem .....	257
6.4.2	Proof of Theorem 6.7 .....	259
<b>7.</b>	<b>Stationary Navier–Stokes Equations</b> .....	265
7.1	Introduction .....	265
7.2	Regularity of “Maximum-Like Solutions” .....	266
7.2.1	Setting of the Problem .....	266

7.2.2	Some Regularity Properties of “Maximum-Like Solutions”	267
7.2.3	The Navier–Stokes Inequality	273
7.2.4	Hole-Filling	275
7.2.5	Full Regularity	279
7.3	Maximum Solutions and the NS Inequality	280
7.3.1	Notation and Setup	280
7.3.2	Proof of Theorem 7.8	281
7.4	Existence of a Regular Solution for $n \leq 5$	283
7.4.1	Green Function Associated with Incompressible Flows	283
7.4.2	Approximation	288
7.4.3	Proof of Existence of a Maximum Solution for $n \leq 5$	289
7.5	Periodic Case: Existence of a Regular Solution for $n < 10$	291
7.5.1	Approximation	291
7.5.2	A Specific Green Function	292
7.5.3	Main Results	295
<b>8.</b>	<b>Strongly Coupled Elliptic Systems</b>	<b>299</b>
8.1	Introduction	299
8.2	$H^2_{\text{loc}}$ and Meyers’s Regularity Results	300
8.3	Hölder Regularity	305
8.3.1	Preliminaries	305
8.3.2	Representation Using Spherical Functions	308
8.3.3	Statement of the Main Result	311
8.3.4	Additional Remarks	317
8.3.5	Hölder’s Continuity up to the Boundary	319
8.4	$C^{1+\alpha}$ Regularity	329
8.4.1	Auxiliary Inequalities	329
8.4.2	Main Result	334
8.5	Almost Everywhere Regularity	338
8.5.1	Regularity on Neighborhoods of Lebesgue Points	338
8.5.2	Proof of Theorem 8.22	339
8.6	Regularity in the Uhlenbeck Case	343
8.6.1	Setting of the Problem	343
8.6.2	Proof of Theorem 8.24	344
8.7	Counterexamples	348
8.8	Regularity for Mixed Boundary Value Systems	352
8.8.1	Stating the Problem	352
8.8.2	Proof of Theorem 8.25	354
8.8.3	Proof of Lemma 8.28	359
8.8.4	Further Regularity	364
8.8.5	Domain with a Corner. Mixed Boundary Conditions	369
8.8.6	Domain with a Corner. Dirichlet Boundary Conditions	371

<b>9. Dual Approach to Nonlinear Elliptic Systems</b>	375
9.1 Introduction	375
9.2 Preliminaries	377
9.2.1 Notation	377
9.2.2 Properties of the Operators $\epsilon(u)$ and $Du$	378
9.3 Elasticity Models	379
9.3.1 Primal and Dual Problems	379
9.3.2 A Hybrid Model	380
9.4 $H_{\text{loc}}^1$ Theory for the Nonsymmetric Case	381
9.4.1 Presentation of the Problem	381
9.4.2 $H_{\text{loc}}^1$ Regularity	382
9.5 $H_{\text{loc}}^1$ Theory for the Symmetric Case	391
9.5.1 Presentation of the Problem	391
9.5.2 $H_{\text{loc}}^1$ Regularity	391
9.5.3 Reducing the Symmetric Case to the Nonsymmetric Case	396
9.6 $L_{\text{loc}}^\infty$ Theory for the Nonsymmetric Uhlenbeck Case	398
9.6.1 Setting of the Problem and Statement of Results	398
9.6.2 Proof of Theorem 9.8	399
9.7 $W_{\text{loc}}^{1,p}$ Theory for the Nonsymmetric Case	401
9.7.1 Assumptions and Results	401
9.7.2 Proof of Theorem 9.9	402
9.8 $C_{\text{loc}}^{1+\delta}$ Regularity for the Nonsymmetric Case	405
9.8.1 Setting of the Problem and Statement of Results	405
9.8.2 Preliminary Results	406
9.8.3 Proof of Theorem 9.10	410
9.9 $C^\delta$ Regularity on Neighborhoods of Lebesgue Points for the Nonsymmetric Case	413
9.9.1 Setting of the Problem and Statement of Results	413
9.9.2 Proof of Theorem 9.11	414
9.9.3 Additional Results in the Uhlenbeck Case	418
<b>10. Nonlinear Elliptic Systems Arising from plasticity Theory</b>	421
10.1 Introduction	421
10.2 Description of Models	422
10.2.1 Spaces $U(\Omega)$ , $\Sigma(\Omega)$	422
10.2.2 Hencky model	423
10.2.3 Norton–Hoff Model	424
10.2.4 Passing to the Limit	426
10.3 Estimates on the Displacement	427
10.3.1 The $f_j$ Derive from a Potential	427
10.3.2 Strict Interior Condition	428
10.3.3 Constituent Law for the Hencky model	429
10.4 $H_{\text{loc}}^1$ Regularity	430

10.4.1 Preliminaries .....	430
10.4.2 Uniform Estimates and Main Regularity Result .....	432
<b>References</b> .....	435
<b>Index</b> .....	441

Regularity Results for Nonlinear Elliptic Systems and  
Applications

Bensoussan, A.; Frehse, J.

2002, XII, 443 p., Hardcover

ISBN: 978-3-540-67756-7