IS QUANTUM MECHANICS NON-LOCAL?

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Abstract. In this paper I look at a number of issue surrounding the question of the title. I will begin with a report of a recent paper by Beckman *et al* where they examine the issue of a certain type of locality and differentiate it from the idea of causality (no "faster than light" signaling), and show that, surprisingly, the former is slighlty more restrictive than the latter. I then discuss H. Stapp's argument that one can demonstrate the non-locality of Quantum mechanics by showing that one obtains a contradiction with th predictions of quantum mechanics if one assumes that quantum mechanics is local, and that certain counterfactual lines of argument are valid. Finally I conclude with a statement of my position that quantum mechanics **is** local, presenting a simple field theoretic analysis of a Bell type experiment. Using this I argue that the quantum correlations present in such a Bell experiment are the result of a common cause, just as they are in the case of classical correlations.

Ever since Bell published his famous paper, in which he showed that no local classical model could duplicate the correlations seen in quantum systems, the argument around the question of the title has raged. Here I will present three different ways of examining the question, all tending toward answering the question in the negative, that quantum mechanics is local. It is also, I hope a glimpse into why answering the question is so much less straightforward than it would first seem.

1. Local vs causal operators

Beckman *et al* [1] have recently written a paper in which they examine quantum operations which transform a quantum state shared by two space-like separated observers in a space-time. Let us define two spatially and temporally limited regions A and B, such that both regions are entirely space-like separated from each other. Each of these regions contains operators with in the context say of a quantum field theory, which are localized within each of these regions. These sets of operators:

$$M_A = \{O_A\}, M_B = \{O_B\} \tag{1}$$

are such that any operator in M_A commutes with any operator in M_B. These are the local operators available in the two regions. They are, for example, the various field operators in a quantum field theory smeared by functions whose support lies in the regions A and B.

We will consider a set of operators which operate on a subspace of the total Hilbert space operated on by the sets $M_A \times M_B$. Define a general linear transformation on the

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initial density matrix ρ by

$$E(\rho) = \sum_\lambda p_\lambda U_\lambda \rho U_\lambda^\dagger \tag{2}$$

with

$$\sum_\lambda p_\lambda = 1 \tag{3}$$

in order to preserve the trace of ρ. If more than one of the coefficients p_λ is non-zero, this is a dissipative or entropy non-preserving interaction. We define this transformation to be a local transformation if each of the U_λ is the product of separate unitary transformation in region A and in B. I.e.,

$$U_\lambda = U_{A\lambda} U_{B\lambda}. \tag{4}$$

In contrast, define the the transformation as causal from B to A if

$$Tr\left(A\, E(W_B \rho W_B^\dagger)\right) = Tr(AE(\rho)) \tag{5}$$

with A being any Hermitian operators is a local operators in the region A, W_B, being an arbitrary unitary operator in region B, ρ being an arbitrary density matrix for the combined system.

In words, this requirement is that no local transformation made by B on any initial density matrix before the transformation E can make any difference to the expectation value of any local operator in region A (and thus not to the outcome of any measurement made in region A). B cannot do anything to signal A even in a probabilistic sense.

It is easy to show that any local transformation is causal, since the trace reduces to the sum of terms of the form

$$Tr(AU_{\lambda A} U_{\lambda B} W_B \rho W_B^\dagger U_{\lambda B}^\dagger) = Tr(AU_{\lambda A} W_B^\dagger U_{\lambda B}^\dagger U_{\lambda B} W_B \rho) \tag{6}$$

because both U_λ and W_B commute with A and $U_{\lambda A}$. But since both U_λ and W_B are unitary, this reduces to

$$Tr(AU_{\lambda A} U_{\lambda B} B \rho B^\dagger U_{\lambda B}^\dagger) = Tr(AU_{\lambda A} \rho U_{\lambda A}^\dagger), \tag{7}$$

which is independent of W_B, the requirement of causality from B to A.

The crucial result of their paper is to point out that one can have transformations E which are non-local (i.e., cannot be written in terms of any sum of local transformations), but are still causal. This is most easily seen by a specific example of such a trasformation.

The easiest way to characterize the transformation is to write it out in detail. Consider the universe of discourse for A and B to be that each has two spin 1/2 (or rather more generally, two two level systems). Define the Pauli spin operators on each to be (X, Y, Z) with subscripts indicating which particle and in which region they operate. Thus Z_{1A} means the 'z' Pauli operator operating on the first particle in the A region. These operators all have two eigenvalues with values of ± 1.

The transformation E be composed of the following four transformations. To begin, I take $E_1(\rho)$ to be the identity, $E_1(\rho) = \rho$. The other three transformations are

$$E_2(\rho) = \frac{1}{2}\left[\rho + Z_{1A}\rho Z_{1A}\right] \tag{8}$$

$$E_3(\rho) = \frac{1}{2}\left[\rho + Z_{1B}\rho Z_{1B}\right] \tag{9}$$

$$U_5 = 1 + \frac{1}{4}(1 + Z_{1A})(1 + Z_{1B})(-1 + U_{2B}) \tag{10}$$

$$E_4(\rho) = U_{2B}(\rho)U_{2B}^{\dagger} \tag{11}$$

$$E(\rho) = E_4(E_3(E_2(E_1(\rho)))). \tag{12}$$

E_2 decohers the system over the eigenvalues of Z_{1A} while E_3 does the same over Z_{1B}. E_4 rotates the second spin at B by the transformation U_{2B} if and only if both of the first spins at A and B have Z eigenvalue of $+1$. The E_2 and E_3 are clearly local, while E_5 is clearly non-local (the easiest way to demonstrate this is to show that it is not causal). Furthermore, this conjunction of the three is still non-local, as Beckman *et al* show. However, this transformation is causal from B to A—i.e. any transformation at B is invisible at A.

To see this, define the projection operator

$$P_{\mu 1A} = \frac{1}{2}(1 + \mu Z_{1A}); \quad \mu = \pm 1 \tag{13}$$

and similarly for other projectors at the first particle at B. Then

$$\begin{aligned} E_4(E_3(E_2(E_1(\rho)))) = \frac{1}{4}(&P_{+1A}P_{+1B}U_{2B}E_1(\rho)U_{2B}^{\dagger}P_{+1A}P_{+1B} + \\ &P_{-1A}P_{+1B}E_1(\rho)P_{-1A}P_{+1B} + P_{+1A}P_{-1B}E_1(\rho)P_{+1A}P_{-1B} + \\ &P_{-1A}P_{-1B}E_1(\rho)P_{-1A}P_{-1B}). \end{aligned} \tag{14}$$

Then, in the first term of the trace, $Tr(AE(W_B\rho W_B^{\dagger}))$, we have

$$\begin{aligned} &\frac{1}{4}Tr(AP_{+1A}P_{+1B}U_{2B}E_1(W_B\rho W_B)U_{2B}^{\dagger}P_{+1A}P_{+1B}) = \\ &Tr(AP_{+1A}P_{+1B}E_1(W_B\rho W_B)P_{+1A}P_{+1B}) \end{aligned} \tag{15}$$

because U_{2B} commutes with A, P_{1A}, and P_{1B}. Thus as far as this trace is concerned, E_4 acts as the identity transformation, and

$$Tr(AE(W_B\rho W_B^{\dagger})) = Tr(AE_3(E_2(E_1(W_B\rho W_B^{\dagger})))). \tag{16}$$

But, the transformation $E_3E_2E_1$ is a local transformation and thus is causal, making the RHS independent of W_B. I.e., this transformation is a causal transformation from B to A.

However, this transformation as it stands is not causal from A to B. For example, starting from an intial state in which particles 1A, 1B, and 2A are all in the Z eigenstate with value $+1$ and 2B is in some arbitraty state, the expectation values for an operator at

B on the second particle will in general be different than if particle 1A is rotated to the -1 eigenstate of Z_{1A}, because of the absense of the rotation U_{2B} on the state of the second B particle during the application of the operator E to this state. I.e., expectation values of B do depend on the operations carried out at A.

What is now needed is to make all expectation values of operators on particle 2B be independent of U_{2B}. The way to do this is to apply another transformation, E_1 which makes the reduced density matrix at particle 2B be the identity. In that case any expectation value at B will be independent of U_{2B}. They did this by making the transformation E_1 be an incoherent projection into a Bell state basis between the particles 2A and 2B.

Define the local operator

$$E_1(\rho) = \frac{1}{4}(\rho + X_{2A}X_{2B}\rho X_{2B}X_{2A} + Y_{2A}Y_{2B}\rho Y_{2B}Y_{2A} + Z_{2A}Z_{2B}\rho Z_{2B}Z_{2A}). \tag{17}$$

The operator

$$\tilde{E}(\rho) = E_4(E_3(E_2(E_1(\rho)))) \tag{18}$$

is now both non-local and causal. The causality from B to A follows from a similar argument as the one just given with the trivial E_1. Causality from A to B follows from the insensitivity of any operators at B to the rotation U_{2B}. After the operation of E_1, the density matrix over the second two particles will have a terms of the form

$$\lambda(|++\rangle + |--\rangle)(\langle ++| + \langle --|),$$

one of the projectors onto a Bell state, where the two eigenstates correspond to the Z operators on the two particles 2A and 2B. λ will in general be a density matrix over the first particles at A and B, and will depend on the intial density matrix and the unitary transformation which is applied at A to test causality (W_A). But taking the trace over the second particle at A leaves us with the identity matix for the particle 2B, which will be insensitive to the rotation U_{2B}. I.e., any expectation value at B will be the same as if U_{2B} had been the identity. But with the identity for U_{2B}, the transformation E_4 is trivial and the total operation is local and causal. I.e., the changes in expectation values at B are the same as they would have been had the U_{2B} been the identity, and thus is causal from A to B. This operator is thus a causal operator. The only non-trivial part in their paper is then to show that it is still non-local, which I will skip. I refer to their original paper for the proof.

Thus, what they showed was that locality (as defined by them) of transformations is a slightly stronger requirement than causality. However, I also note that the use of entropy non-preseving local transformations was crucial to the above argument. If the transformation E is a unitary transformation

$$E(\rho) = U\rho U^{\dagger}, \tag{19}$$

then it would seem that locality and causality are equivalent. Thus, the existence of decohering transformations seems to be crucial in the breakdown of the equivalence between locality and causality. While such decohering transformations can be implimented by the use of local unitary transformations and the use of (correlated) ancilla (extra physical systems at A and B), this would be philosophically unsatisfactory as one would naturally be drawn to simply increasing the size of what one called the system to include the ancilla. This would restore the equivalence between causality and locality. I.e., one would need require a theory in which the decohering interactions were fundamental to the theory, and did not arise out of the neglect of auxilliary degrees of freedom. A non-local, but causal theory would thus look quite different from any of our current theories of quantum fields.

2. Stapp's argument for non-locality

Let me now examine a different tactic to try to separate locality from causality in quantum theories. This is an attempt by Stapp [2] to define locality not in a technical quantum operator sense as above, but logically, by the use of counterfactual reasoning.

Henry Stapp [2] has tried to argue for many years that quantum mechanics can be shown to be non-local without any additional assumptions. In one of his latest attempts, he uses the Hardy state (see below) to derive a conclusion from one sequence of possible measurements, plus some locality assumptions in the context of counterfactual reasoning, to derive a contradiction with other conclusions of quantum mechanics for the same state.

The state I will describe is a generalization of a state given by Hardy [3]. It is a state for two 2-level systems, located I will assume on the left and the right, and designated by L and R. The two levels will be labeled by the values 1 and 0. The operators corresponding to the Pauli projection operators in the direction n, namely $(1+\mathbf{n}\cdot\sigma)/2$ will be denoted by L_n or R_n. They have eigenvalues 1 or 0.

Consider the state defined in the L_z and R_z basis given by

$$\Psi = N\left(\cos(\theta)\mid 11\rangle + \sin(\theta)\mid 10\rangle + \frac{1+\sin(\theta)^2}{\cos(\theta)}\right)\mid 01\rangle - \sin(\theta)\mid 00\rangle, \qquad (20)$$

where $\mid 10\rangle$ is the eigenstate of value $+1$ for L_z and 0 for R_z, and $N = \frac{\cos(\theta)}{\sqrt{(2(1+\sin(\theta)^2))}}$. Now, this same state can be rewritten in the following three different ways.

$$\begin{aligned}\Psi &= N(\mid 1\rangle(\cos(\theta)\mid 1\rangle + \sin(\theta)\mid 0\rangle) + \mid 0\rangle(\frac{1+\sin(\theta)^2}{\cos(\theta)}\mid 1\rangle - \sin(\theta)\mid 0\rangle))\\ &= N((\mid 1\rangle + \mid 0\rangle)(\cos(\theta)\mid 1\rangle + \sin(\theta)\mid 0\rangle) + 2\tan(\theta)\mid 0\rangle(\sin(\theta)\mid 1\rangle - \cos(\theta)\mid 0\rangle))\\ &= N((\mid 1\rangle + \mid 0\rangle)\frac{1}{\cos(\theta)}\mid +\rangle + (\mid 1\rangle - \mid 0\rangle)(-\frac{\sin^2(\theta)}{\cos(\theta)}(\mid 1\rangle + \sin(\theta)\mid 0\rangle))).\end{aligned} \qquad (21)$$

The first line says that if one measures the operator L_z and find the value 1, and measure the $R_\theta = \cos(2\theta)R_z + \sin(2\theta)R_x$, you will with certainly also measure value $1i$ (the quantity in brakets multiplying the state $\mid 1\rangle$ for the left particle is the eigenstate with value $+1$ for the operator R_θ). The second line says that if you measure R_θ and find value 1 and L_x you will also find value 1 with certainty (since $(\mid 1\rangle + \mid 0\rangle)/\sqrt{2}$ is the $+1$ eigenstate of L_x. Finally the third line says that if you measure L_x and find value 1 and you measure R_z you will find value 1 with certainty. However, if we look at the original expression for the state Ψ, if we measure L_z and find value 1, we will with probability $\cos(\theta)^2$ find value 1 and with probability $\sin(\theta)^2$ find value 0 for R_z.

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