Preface

What's Here

This is a book about intensional logic. It also provides a thorough look at higher-type classical logic, including tableaus and a completeness proof for them. It also provides a formal examination of the Gödel ontological argument. These are not disparate topics. Higher-type classical logic is intensional logic with the intensional features removed, so this is a good place to start. Ontological arguments, Gödel's in particular, are natural examples of intensional logic at work, so this is a good place to finish.

The term *formal logic* covers a broad range of inventions. At one end are small, special-purpose systems; at the other are rich, expressive ones. Higher-type modal logic—intensional logic—is one of the rich ones. Originating with Carnap and Montague, it has been applied to provide a semantics for natural language, to model intensional notions, and to treat long-standing philosophical problems. Recently it has also supplied a semantic foundation for some complex database systems. But besides being rich and expressive, it is also tremendously complex, and requires patience and sympathy on the part of its students.

There are two quite different reasons to be interested in a logic. There is its formal machinery for its own sake, and there is using the formal machinery to address problems from the outside world. The mechanism of higher-type modal logic is complex and requires serious mathematics to develop properly. Models are not simple to define, and tableau systems are quite elaborate. A completeness argument, to connect the two, is difficult. But, the machinery is of considerable interest, if this is the sort of thing you have a considerable interest in. If you are such a reader, applications concerning the existence of God can simply be skipped. On the other hand, if philosophical applications are what you are after, the

Gödel ontological argument is a prime example. If this is the kind of reader you are, much of the mathematical background can be taken on faith, so to speak. It is a rare reader who will be interested equally in both the formal and the applied aspects of intensional logic. In a sense, then, this book has no audience—there are separate audiences for different parts of it. (But I encourage these audiences to do some 'crossing over.')

If you are interested in ontological arguments for their own sakes, start with Part III, and pick up material from earlier chapters as it is needed. If you are interested in the mathematical details of the formal system, its semantics and its proof theory, Parts I and II will be of interest—you can skimp on reading Part III. Part I is entirely devoted to classical logic, and Part II to modal. Here is a more detailed summary.

Part I presents higher-type classical logic. It begins with a discussion of syntax matters, Chapter 1. I present types in Schütte's style, rather than following Church. Types can be somewhat daunting and I've tried to make things go as smoothly as I can.

Chapter 2 examines semantics in considerable detail. What are sometimes called "true" higher-order models are presented first. After this, Henkin's generalization is given, and finally a non-extensional version of Henkin models is defined. Henkin himself mentioned such models, but knowledge of them does not seem to be widespread. They are natural, and should become more familiar to the logic community—the philosophical logic community in particular.

Classical higher-order tableaus are formulated in Chapter 3. These are not original here—versions can be found in several places. A number of worked out examples of tableau proofs are given, and more are in exercises. The system is best understood if used. I do not attempt a consideration of automation—the system is designed entirely for human application. There is even some discussion of why.

Soundness and completeness are proved in Chapter 4. Tableaus are complete with respect to non-extensional Henkin models. The completeness argument is not original; it is, however, intricate, and detailed proofs are scarce in the literature.

After the hard work has been done, equality and extensionality are easy to add using axioms, and this is done in Chapters 5 and 6. And this concludes Part I. Except for the explicit formulation of non-extensional models, the material in Part I is not original—see [Tak67, Pra68, Tol75, And86, Sha91, Lei94, Koh95, Man96], for example.

Part II is devoted to the complications that modality brings. Chapter 7 adds the usual box and diamond to the syntax, and possible worlds to the semantics. It is now that choices must be made, since quantified modal logic is not a thing, but a multitude.

First, at ground level quantifiers could be actualist or possibilist—they can range over what actually exists at a world, or over what might exist. This corresponds to the varying domain, constant domain split familiar to many from first-order modal discussions. However, either an actualist or a possibilist approach can simulate the other. I opt for a possibilist approach, with an explicit existence predicate, because it is technically simpler.

Next, we must go up the ladder of higher types. Doing so extensionally, as in classical logic, means we take subsets of the ground-level domain, subsets of these, and so on. Going up intensionally, as Montague did, means we introduce functions from possible worlds to sets of ground-level objects, functions from possible worlds to sets of such things, and so on. What is presented here mixes the two notions—both extensional and intensional objects are present. I refer you to [Fit00b, Fit00a] for applications of these ideas to database theory—intensional and extensional objects make natural sense even in such a context.

Classical tableau rules are adapted in Chapter 8, using *prefixes*, to produce modal systems. While the modal tableau rules are rather straightforward, they are new to the literature, and should be of interest. Since things are already quite complex, no completeness proof is given. If it were given, it would be a direct extension of the classical proof of Part I.

Using modal semantics and tableaus, in Chapter 9 I discuss the relationships between rigidity, *de re* and *de dicto* usages, and what I call Gödel's *stability* conditions, which arise in his proof of the existence of God. I also relate all this to definite descriptions. While this is not deep material, much of it does not seem to have been noted before, and many should find it of some significance.

Finally, Part III is devoted to ontological proofs. Chapter 10 gives a brief history and analysis of arguments of Anselm, Descartes, and Leibniz. This is followed by a longer, still informal, presentation of the Gödel argument itself. Formal methods are applied in Chapter 11, where Gödel's proof is examined in great detail. While Gödel's argument is formally correct, some fundamental flaws are pointed out. One, noted by Sobel, is that it is too strong—the modal system collapses. This could be seen as showing that free will is incompatible with Gödel's assumptions. Some ways out of this are explored. Another flaw is equally serious: Gödel assumes as an axiom something directly equivalent to a key conclusion of his argument. The problematic axiom is related to a principle Leibniz proposed as a way of dealing with a hole he found in an ontological proof of Descartes. Descartes, Leibniz, and Gödel (and also Anselm) all have proofs that stick at the same point: showing that the existence of God is possible.

If the Gödel argument is what you are interested in, start with Part III, and pick up earlier material as needed. Many of the uses of the formalism are relatively intuitive. Indeed, in Gödel's notes on his ontological argument, formal machinery is never discussed, yet it is possible to get a sense of what it is about anyway.

How Did This Get Written?

Having just completed work on a book about first-order modal logic, [FM98], a look at higher-order modal logic suggested itself. I thought I would use Gödel's ontological argument as a paradigm, because it is one of the few examples I have run across that makes essential use of higher-order modal constructs. Gödel's argument for the existence of God is not particularly well-known, but there is a growing body of literature on it. This literature sometimes gives formalizations of Gödel's rather sketchy ideas—generally along natural deduction or axiomatic lines. My idea was, I would design a tableau system within which the argument could be formalized, and this might lead to a nice paper illustrating the use of tableau methods. First, give tableau rules, then give Gödel's proof.

One cannot really develop semantic tableaus without a semantics behind it. The semantics of higher-order modal logic turned out to be of considerable intricacy, far beyond what could even be sketched in a paper. Clearly, an extended discussion of the semantics for higher-order modal logic was needed before the tableau rules could be motivated.

I soon realized that in presenting higher-order modal logic, I was trying to explicate ideas coming from two quite different sources. On the one hand, there are essentially *modal* problems, some of which already arise at the first-order level and have little to do with higher-order constructs. On the other hand, a number of higher-order modal complexities also manifest themselves in a *classical* setting, and can be discussed more clearly without modalities complicating things. So I decided that before modal operators were introduced, I would give a thorough presentation of a semantics and tableau system for higher-order classical logic. There are already treatments of tableau, or Gentzen, systems for higher-order classical logic in the literature, but I felt it would be useful to give things in full here. Detailed completeness proofs are hard to find, for instance.

Higher-order classical logic already has its hidden pitfalls. It is common knowledge, so to speak, that "true" higher-order classical models cannot correspond to any proof procedure. Henkin models are what is needed. But a "natural" formulation of tableaus is not complete with respect to Henkin models either. This is something known to experts—

it was not known to me when I started this book. A broader notion of Henkin model (also due to Henkin) is needed, a *non-extensional* version. Such models should be better known since they are actually quite plausible things, and address problems that, while not common in mathematics, do arise in linguistic applications of logic.

In the 1960's, cut-elimination theorems were proved for higher-order classical logic, using semantic methods that relied on non-extensional models. In effect, these cut-elimination proofs concealed a completeness argument within them, but the general notion of non-extensional model was not formulated abstractly—only the specific structure constructed by the completeness argument was considered. In short, a completeness theorem was never stated, only a consequence, albeit a very important one. So I found myself required to formulate a general notion of classical non-extensional Henkin model, then prove completeness for a suitable classical tableau system. After this I could move on to discuss modality.

What sort of modal features did I want? Formalizations of the Gödel argument by others had generally used some version of an intensional logic, with origins in work of Carnap, [Car56], developed and applied by Montague, [Mon60, Mon68, Mon70], and formally elaborated in [Gal75]. After several preliminary attempts I decided this logic was not quite what I wanted. In it, semantically speaking, all objects are intensional. I decided I needed a logic containing *both* intensional and extensional objects. Of course, one could bring extensional objects into the Montague setting by identifying them with objects that are rigid, in an appropriate sense, but it seemed much more natural to have extensional objects from the start. Thus the modal logic given in the second part of this book is somewhat different from what has been previously considered.

Once I had formulated the modal logic I wanted, tableau rules were easy, and I could finally formalize the Gödel argument. What began as a short paper had turned into a book. My after-the-fact justification is that there are few treatments of higher-order logic at all, and fewer still of higher-order modal logic. It is a rare flower in a remote field. But it is a pretty flower.

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