3. THE MATHEMATICS OF SOCIAL PROCESSES

As I intend to translate the considerations of the last chapters into mathematical models, it seems appropriate to make some preliminary remarks about the specific concept of mathematical models I shall introduce. The main reason for these preambles is the fact that there are a lot of misunderstandings in social sciences when someone is speaking in terms of mathematics. That is not only due to the tradition of social theory, which is by and large an unmathematical one. It is also due to certain developments in the last decades, in particular to the emergence of mathematical models that have not much to do with the traditional tools of mathematical physics and strongly rely on their implementation into computer programs, i.e. they are validated by computer simulations. In 1.2. I already mentioned the Boolean networks (BNs) used and investigated by Kauffman, which can be seen as a paradigm for the new mathematical approaches to biological or social complexity.

Usually mathematical models of systems dynamics consist of ensembles of differential or difference equations like the famous Lotka-Volterra equations for the mathematical analysis of predator-prey systems, or the logistic equation described in 2.3. Sometimes the use of equations of this type is identified with the concept of mathematical model; therefore, some scholars postulate the existence of a "third symbol system", mathematics and natural language being the first two and the new computational models, which do not use classical equations being the third. That is rather misleading as it suggests that these new formal models are not mathematical ones. But if they are not a kind of mathematics – what else can they be?

Since the development of set theory and the foundations of mathematics based on mathematical logics and set theory by Hilbert and others, mathematics is generally defined as the science of all formal systems, regardless which formalisms are used to capture the particular structure of these systems. Therefore the mathematics that is commonly used in the empirical sciences, like the calculus or statistical procedures is only a part of the far greater realm of mathematics, though certainly a very important one.[31] The traditional mathematical methods have often shown their fruitfulness in the branches of empirical science, yet they were not very useful for dealing with the classical problems of theoretical sociology and the other disciplines of social complexity. Therefore it is necessary to give the concept of mathematical models in the social sciences more enlarged meanings. Some of the possibilities and results that are gained by this new approach are shown in the next subchapters.[32]

3.1 SOCIAL EVOLUTION AND THE GEOMETRY OF SOCIAL SYSTEMS

When I speak of "social geometry" I do not mean the geometry of the physical space, in which all social interactions occur. To be sure, geophysical factors often play an important role in the development of social systems, and physical barriers like mountains or oceans may be decisive for the particular paths of evolution taken by isolated societies. These are, as physicists would say, boundary conditions, i.e., they may influence the particular development of societies but have no impact on the *general* logics of sociocultural evolution, if there is such a thing. By speaking of the geometry of social systems I mean certain properties of their rules, which generate the dynamics and metadynamics of these systems (see above 1.3.), and which may be described in a mathematical manner by using some concepts of geometry. The concept of geometry is therefore used in a general mathematical sense, that is, as the description of structural features of rule systems that can be applied to physical systems as well as social ones.

In a very general sense it is possible to characterise the geometry of any set, or system, by defining a) the number of dimensions of this set and b) its topology and/or metrics. Mathematicians call a set with these properties a geometrical space which just means that certain geometrical properties can be defined according to this set. In the case of physical systems it is usually assumed that the geometrical properties of the

[31] These methods are of course also used in the social sciences (cf. e.g. Helbing 1995 and Gütschl 2001) and are far from worthless there.

[32] These new mathematical approaches and results are discussed in more detail in Klüver 2000. I give a short summary here because I cannot presuppose that readers interested in sociocultural evolution are acquainted with methods and results in recent mathematical social systems theory.

physical space define the geometry of these systems. In the case of social systems it is a bit more complicated.

Let us remember: in subchapter 1.3. I defined social systems by the rule governed interactions of social actors, which generate dynamics and metadynamics, respectively, evolution of the systems. The logical key concept in this definition is obviously that of *social rules* as I am speaking here only of *social* dynamics, I omit for the time being the additional importance of social roles for socio*cultural* evolution. As there is basically nothing else in social reality than social actors and their interactions and as all classical sociological concepts like structure, institutions or organisations must be understood in terms of rule governed interactions (cf. Giddens 1984), the main research question for social analysis must be the search for specific properties of ensembles of social rules, in particular such features that explain the dynamics and evolution observed in social reality. Mathematical analysis of social systems therefore means the search for mathematical features of rule ensembles that generate certain dynamical and evolutionary behaviour of the systems. It is the main thesis of this book that there are indeed such mathematical properties, i.e. parameters of different kinds, which together explain at least general features of social dynamics and evolution.

Let us first define the concept of social geometry in the double sense mentioned above. The *topology* of a set is its mathematical structure, by which the spatial relations of the set's elements are described. In particular the topological structure of a set determines if any two elements are connected, directly or by a chain of coupling elements, or if there are "holes" in the structure of the set, i.e., some elements are not connected at all. For example, two geometrical figures in the Euclidean plane are, in this sense, topologically equivalent if they have no holes or the same number of holes. Therefore the topology of a set defines measures of connectedness with regard to the elements of the set.

A special kind of topology defines a *metric* of a set, that is, a structure, which measures distances between the set's elements. A metric is formally defined by a relation $d(x,y)$ between two elements x and y with $d(x,y) \geq 0$ and which fulfils the conditions

(1) $d(x,x) = 0$

(2) $d(x,y) = d(y,x)$ and

(3) $d(x,y) + d(y,z) \leq d(x,z)$ (triangle inequality).

Intuitively these concepts are clear when one is imagining the usual physical space in which we live. As we have "only" rules of interaction with regard to social "spaces" we have to translate these mathematical terms into sociological ones. That is rather easily done by use of social structural analysis (cf. Freeman 1989):

Two actors x and y can be defined as (directly) connected if they may interact with each other. In this case we define $d(x,y) = 1$. Two actors x and y are connected indirectly with each other if they can interact via a chain of interacting actors. In this case $d(x,y) = n+1$ if n is the smallest number of interacting actors, i.e. the size of the smallest chain of interacting actors (cf. the "Small World" experiments by Milgram 1967); if there is no such finite chain, $d(x,y) = \infty$. It is easy to demonstrate that these rough definitions fulfil the strict criteria of the mathematical concepts but this is not important here. As the possibilities to interact directly or via a chain of interacting actors is a question of social rules, we can obviously define some properties of social rules by geometrical concepts. For example, a worker in a factory can interact directly with his/her foreman or forewoman and therefore the distance between the two is $d = 1$. As the worker is usually not allowed to interact directly with the chairman of the firm he/she needs some interacting persons, whose number depends on the size of the firm. This number $n > 1$ defines the social distance between the worker and the chairman; the proportion between n (the actual distance defined by the rules of the firm) and m, understood as the greatest possible number of intermediating persons, defines a measure of hierarchy of the organisation (which is certainly greater in an army than in one of the Internet firms).

These topological or metrical definitions are, though not in these terms, well known to structural social theorists. We shall see in the next subchapters that certain topological features of rule systems also have consequences for the dynamics of social systems. More unusual is the use of the concept of dimensions in regard to social systems. In the mathematical theory of vector spaces, the number of dimensions is usually defined as the number of *linear independent unit vectors* one needs to describe any element of the vector space completely. The Euclidean plane has two dimensions because each point can be described by exactly two co-ordinates, which in turn are linear multiples of the two unit vectors x and y; for the same reason the physical space of our perception has three dimensions, and the strange Hilbert spaces of quantum mechanics have millions of dimensions. Well known in mathematical systems analysis are the state spaces or phase spaces whose numbers of dimensions depend on the number of (independent) variables one needs to completely describe any event of the system, and by this its trajectories.

When referring to the space of our perceptions we may understand the concept of dimensions in a more simple way, which is of course nothing more than the abstract definition just quoted. We perceive any object or event in a three-dimensional way because we always need three independent spatial indications: the event A is "before/behind me", "left/right of me" and "above/below me". By adding the indication "after/before now" we get a fourth dimension of description, which gives us the four-dimensional space-time.

By translating this rather general concept of dimensions into our sociological language of rules, actors and social roles, we get a rather "natural" definition of the number of dimensions of a social system: it is the number of distinctions necessary to describe any social event, that is mainly any other social actor and his/her actions, completely in regard to the particular society. Such descriptions may of course be very different but as we are interested in the *general* features of any given society we may safely assume that the most important distinctions are "familiar versus strange", "above versus below" and "active versus passive". In other words, if an actor meets another one and has to classify him, it is in principle enough to judge if the other belongs to one own group or not, if the other is socially above oneself or below (or equal) and if the other's role means that one is the active part in the interaction or the passive one, or if both are occupants of active or passive roles respectively. For example, a worker's interactions with his foreman are characterised by the fact that both belong to the same organisation, familiar, that the worker is socially below the foreman and that both roles are active ones. A doctor on the other hand interacts with his patients usually based on

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