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TIME AND PHYSICS – A NONCOMMUTATIVE REVOLUTION

Abstract. Basic ideas of noncommutative geometry are briefly presented. This mathematical theory, being global from the very beginning, can be used to model physics in which local concepts, such as those of time instant and space point, are meaningless. In spite of the lack of the standard time concept a "noncommutative dynamics" can be defined. Noncommutative generalizations of causality, probability and chance are discussed.

1. INTRODUCTION

Modern philosophy of time is inseparably connected with the progress in modern physics. Newtonian mechanics and the idea of absolute time, classical thermodynamics and the problem of the arrow of time, the study of non-linear dynamical systems and the problem of the origin of irreversibility, theories of relativity (both special and general) together with their revolutionary results concerning time measurements – are but main headlines of this magnificent story on time and physics. There are strong reasons to believe that the story is far from being completed. The problem of time is intricately involved in currently discussed interpretative issues of quantum mechanics (to mention only the measurement problem: a sudden collapse of the unitary evolution to a fixed measurement result), and it turns out to be one of the major stumbling blocks of almost all current attempts to unify quantum physics with general relativity. The strategy adopted in many recent models with respect to this problem is to eliminate it altogether by claiming that there is no time on the fundamental level of physics. There are more and more hints that this might indeed be the case. In the present study I shall not deal with these fascinating issues; instead I shall try to introduce the reader into a new field of mathematics, having broad spectrum of possible physical applications, which in the near future can cause a new revolution in our understanding of time.

If we look at the history of the problem, we can easily notice that the real progress was always connected with beautiful mathematics. Newtonian mechanics with its absolute time was based on beautiful mathematical structures (classical dynamical systems, symplectic manifolds...), thermodynamics, both linear and non-linear, was the result of beautiful mathematical structures (probability calculus, stochastic processes...), the theories of special and general relativity are implementation of beautiful mathematical structures (Minkowski and Lorentz geometries), whereas all unresolved problems pertaining to time (e. g., the collapse of the wave function in the act of measurement in quantum mechanics) arise on the basis of some partial models and computational tricks. The latter ones are often

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met in various attempts to construct quantum gravity or final unifying theories. In what follows, I shall briefly present a relatively new mathematical theory, namely noncommutative geometry, and discuss some of its applications to physics. It is certainly a beautiful mathematical theory, and it is radical indeed. Being global from the very beginning, it can be used to model an entirely new physics – physics with no local concepts such as time instant or space point. It can mark another breakthrough in our understanding of time and of the limitation of applicability of this concept.

Noncommutative geometry has its sources in quantum physics where the family of observables forms a noncommutative algebra, and in these branches of pure mathematics in which analysis, algebra and geometry interact with each other. It has emerged, however, as a more or less independent chapter of mathematics in the works of Alain Connes (which are summarized in his book published in 1994). The number of publications in this field, both in pure mathematics and in various physical applications (e. g., gravity theories, unified gauge theories, classical singularity problem, generalized Kaluza-Klein model), is rapidly growing (see the books by Landi 1997 and Madore 1999). In the present paper, after briefly presenting main ideas of noncommutative geometry, I shall explore its possibilities to deal with these problems which could contribute to our understanding of time and its origin.

The organization of the paper runs as follows. In section 2 I show how the main idea of noncommutative geometry emerges from previous investigations. The non-local character of noncommutative spaces (briefly presented in section 3) allows one (as it is demonstrated in section 4) to introduce a dynamics without the usual concept of time (a "timeless dynamics"). In section 5 I show that, by imposing some additional constraints on a given noncommutative algebra, we can gradually improve its temporal properties to finally obtain the usual time of classical physics. It turns out (in section 6) that the standard dynamics, the emergence of time, and the origin of probability have common roots in noncommutative geometry.

My interest in noncommutative geometry has also a philosophical motivation. Progress in science is intrinsically linked with evolution of concepts, and many concepts of great philosophical importance, such as the concepts of causality and chance, undergo a radical change when passing from commutative geometry to noncommutative geometry. This change consists in far-reaching generalizations. It turns out that no standard concept of time and of event is necessary to make one able to meaningfully speak of causality and chance. This is certainly an important philosophical lesson: our seemingly most universal concepts have their limited domain of applicability. Since this lesson is based on strict mathematical results and not on any of their particular applications, it will remain valid even if it turns out that these particular applications fail to model the fundamental level of physics. This question is discussed in section 7.

2. SOURCES OF NONCOMMUTATIVE GEOMETRY

Noncommutative geometry has two sources: quantum mechanics and problems that arise at a rather fuzzy borderline between differential geometry, algebra and functional analysis (see Manin 1991, pp. 3-8). It often happens in science that germs of future achievements are present in well-known theories or models long before somebody realizes that they are powerful enough to initiate a major breakthrough in a certain field of research. For a long time it was very well known that noncommutativity plays an important role in quantum physics. If two observables do not commute, the corresponding physical properties cannot be measured simultaneously with any desired precision. For instance, this is valid as far as the position and momentum observables are concerned. The famous Heisenberg uncertainty relations are simple consequences of the noncommutativity of some observables. Noncommutative geometry can be regarded as a reconstruction, in geometrical terms, of a "logical scheme" underlying the aforementioned properties. In this sense, the Planck constant can be regarded as a "deformation parameter" from commutativity of classical physics to noncommutativity of quantum theory.

Both in physics and in mathematics there is an ever-growing need for more and more general spaces. In recent years physicists and mathematicians have often been confronted with spaces which seem highly pathological; for instance, some spaces studied by them are reduced, from the topological point of view, to a single point (technically, they are called non-Hausdorff spaces). In such a case, one could follow two strategies: either to regard such spaces as "meaningless" and banish them from science, or to look for more powerful mathematical methods to cope with such spaces. Noncommutative geometry is precisely the result of the latter approach. In modern differential geometry the notion of a "well behaved space" has assumed the form of the concept of *differential manifold* (or *manifold*, for simplicity). Examples of differential manifolds are: n-dimensional Euclidean space, Minkowski's spacetime of special relativity and spacetimes of general relativity (if singularities are not treated as parts of them) and many other spaces considered in physics. The usual way of defining manifolds is in terms of coordinate systems, but it can be shown that the entire information about a given differential manifold is contained in the family of all smooth functions on this manifold. In fact, one can equivalently define a manifold as the pair (M, C) where M is a non-empty set and C is the family of all smooth functions on M. This family forms what mathematicians call an algebra. This means that functions belonging to C can be added and multiplied with each other and multiplied by scalars (numbers). Working with coordinates is easier for practical (i.e., computational) purposes, but the method of the smooth function algebras turns out to be better adapted for further generalizations. The main idea is to replace a given functional algebra C by an (associative) algebra A. The algebra C is commutative because functions are multiplied in the commutative way, but a general algebra need not be commutative, and this is how noncommutative geometry is born.

3. NONLOCAL SPACES

Noncommutative geometry is indeed a powerful generalization of the standard geometry. Many "pathological spaces" can be regarded as noncommutative spaces. For example, those spaces which, from the topological point of view, are reduced to a single point, turn out to be workable, noncommutative spaces when treated as pairs (M, A) with A a suitable noncommutative algebra. This is due to a striking property of noncommutative spaces — they are non-local entities: the concepts of point and its neighborhood are, in principle, meaningless in them.[1]

Functions on a manifold "feel" points. This fact is used in the definition of function multiplication. Two functions, f and g, are multiplied by multiplying their values at every point x, i.e., $(f \cdot g)(x) = f(x) \cdot g(x)$. (Of course, this is the same as $g(x) \cdot f(x)$ and, consequently, the multiplication of functions is commutative). A given point x can be identified with the set of all these functions that vanish at x. This set is called a *maximal ideal* of the given algebra C. In other words, points in the manifold (M, C) can be identified with maximal ideals of the algebra C. In the family of all algebras to have maximal ideals is an exception rather than a rule, and noncommutative algebras, in general, have no maximal ideals. This is why noncommutative spaces are non-local. They do not consist of points, and only global concepts refer to them.

This is the radical change of perspective, which compels us to go outside the range of validity of the usual set theory. For the expression "belonging to a set" to have a meaning, there should be the possibility of identifying elements of the considered collection by means of at most a denumerable family of properties. In noncommutative geometry such a possibility does not exist, if we decide to use only measurable maps between spaces (Connes 1994, p. 74); but to use nonmeasurable maps would be as bad as going beyond the set theory.

4. TIMELESS DYNAMICS

To explore conceptual horizons that are open by noncommutative geometry let us make the bold assumption that the fundamental level of physics is modeled by a noncommutative geometry. Let us notice that in such a model there could be no space and no time in their usual meaning since space consists of points and time consists of instants. In fact, such a noncommutative model has been proposed (Heller et al. 1997; Heller and Sasin 1999, 2000) but in the present study we shall refer to it only for illustrative purposes. We treat the hypothesis that the fundamental level is noncommutative in a purely heuristic way. It is supposed to help us to realize the degree of generalization in passing from commutative geometry to noncommutative geometry.

The fundamental level of physics "is situated" beyond the so-called *Planck threshold*, which is characterized by the *Planck length* $l_{Pl} = 10^{-33}$ cm, and the *Planck time* $t_{Pl} = 10^{-44}$ s. This threshold can be found "in two directions": firstly, if we go backwards in time to the close vicinity of the Big Bang, when the "age of the Universe" was of the order of 10^{-44} s; secondly, if we go (now, i. e., at the present cosmic epoch) deeper and deeper into the strata of the Universe until we reach distances of the order of 10^{-33} cm. Since, however, on the strength of our hypothesis, there is no space and time (in their usual meaning) beyond the Planck threshold, both these directions (back in time and deeper in space) turn out to be the same direction!

What could physics look like in the absence of space and time? When we think about physics, we first of all think about dynamics, i. e., about physics of motion. With no space and no time there can be no motion in the usual sense, but there can be an authentic, albeit generalized, dynamics. To see this let us recall the following facts.

In dynamical equations (for instance, in the Newtonian equations of motion) time appears as a parameter which measures the change. However, instead of using equations we can equivalently describe the dynamics with the help of vector fields. In the usual setting such vector fields, called *integral vector fields*, consist of tangent vectors to the trajectories of a given dynamical system. Let us also recall that a vector is essentially a derivative of a function; for instance, the velocity vector is the derivative of the distance function with respect to time. The concept of a vector is a local concept, but the concept of a vector field has a global aspect that can be generalized to the noncommutative setting.

Let (M, C) be a manifold. Then, formally speaking, a vector (tangent to M) is a linear mapping

$$\partial: C \to C$$

satisfying the well-known Leibniz rule (which says how to differentiate the product of two functions belonging to C). In noncommutative geometry, there is a non-local counterpart of the above concept that can be thought of as a generalization (and a 'delocalization') of the vector field concept; it is called *derivation* of an algebra C, and is defined to be a linear mapping

$$D: A \to A$$

from a not necessarily commutative algebra A to itself, satisfying the Leibniz rule. This concept can be used to define a generalized noncommutative dynamics. In this way, the intuitive concept of motion, as a change of place in time, is replaced by an abstract idea of mapping from an algebra A into itself which satisfies properties analogous to those which are satisfied in the commutative case (linearity and the Leibniz rule).

There are various noncommutative algebras, and by choosing the 'correct one' we can more adequately model temporal properties of noncommutative dynamical systems.

5. EMERGENCE OF TIME

The concept of state was used in physics for a long time. Although the state of a given physical system can be represented as a point in a space (called the *phase*

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