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DIAGRAMMATIC LOGIC

The many diagrammatic systems in use include Euler circles, Venn diagrams, state diagrams, control-flow diagrams, line graphs, circuit diagrams, category-theory diagrams, Hasse diagrams, and geometry diagrams. A *diagrammatic logic* seeks to describe the syntax, semantics, proof theory, etc., of some such diagrammatic system.

The diagrams of a diagrammatic system have a (typically two-dimensional) syntactic structure that can be described using concepts such as labeling, connectedness, inclusion, direction, etc. They also have a meaning that can be described using techniques from model theory or algebra. Thus, a diagrammatic logic differs from an ordinary logic only in the type of well-formed representations it describes (though these may well have properties not common to more familiar logics).

Diagrams can have unusual properties that distinguish them from expressions of many languages, properties that might motivate the formulation and analysis of a diagrammatic logic. The structure of a diagram might have a close correspondence with what they represent. Its meaning might be invariant under certain topological transformations. It might be unusually easy to understand. A diagrammatic logic *need* illuminate none of these matters (though some of them may be connected to the system's logical properties and hence addressed by the logic). In particular, philosophical and psychological questions about the nature of the diagrammatic system that is the target of a logic could be left to philosophy and psychology.

To reveal the typical characteristics of diagrammatic logics more directly, several examples will be presented. These include Venn diagrams, a variation due to Peirce that will be called *Peirce-Venn diagrams*, and a historically important system developed by Peirce called *existential graphs*. Other diagrammatic logics that have been developed include logics of state transition diagrams,[1] blocks world diagrams,[2] circuit diagrams,[3] conceptual graphs,[4] and geometry diagrams.[5] Relevant collections include Allwein and Barwise [1996] and Glasgow, Narayanan, and Chandrasekaran [1995].

1 FOUNDATIONS

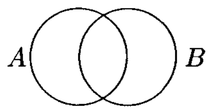
Venn diagrams and Peirce-Venn diagrams (covered in the next two sections) are constructed from circles or, more generally, closed curves, that overlap in

[1]Harel [1988].
[2]Barwise and Etchemendy [1995].
[3]Johnson, Barwise and Allwein [1996].
[4]Sowa [1984].
[5]Luengo [1995].

D.M. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, 2nd Edition, Volume 4, 395–422.

all combinations. Some simple syntactic and semantic concepts are common to both of these systems and so are handled jointly in this section.

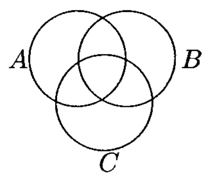
The circles of Venn diagrams represent sets, and the overlapping combinations of the circles represent combinations of the sets. For example, in the case of two circles the four combinations of circles represent the intersection, the two differences, and the complement of the union.



In particular, this diagram consists of four *minimal regions*[6] which can be described by four corresponding combinations of the two labels:

Term	Corresponds to minimal region
AB	within both
$A\overline{B}$	within A, not B
$\overline{A}B$	within B, not A
$\overline{AB}$	within neither

A term such as $A\overline{B}$ is said to *correspond* to the minimal region of the diagram within left one circle but outside of the right circle.[7] Likewise, $\overline{AB}$ corresponds to the minimal region outside of both circles, AB corresponds to the minimal region within both circles, and $B\overline{A}$ corresponds to the minimal region within the right but not the left circle. A three-circle diagram such as



has eight corresponding terms:

$$ABC \quad \overline{A}BC \quad AB\overline{C} \quad \overline{A}B\overline{C} \quad A\overline{B}C \quad \overline{AB}C \quad A\overline{BC} \quad \overline{ABC}$$

The term $A\overline{B}C$ corresponds to the minimal region within both A and C but outside of B, etc. More generally, with an n-circle diagram labeled by n

[6] Minimal regions are described in Shin [1994], p. 51.

[7] Correspondence is described in Hammer [1994], pp. 77–78.

letters, there should be a minimal region and a corresponding term for each of the 2^n combinations of circles. One way to think of this is that there should be a term for each row of an n-variable truth table, the variables of which are the letters labeling the circles, with truth indicating that the region falls within the circle and falsity indicating that it falls outside of the circle.

For the purposes of logic, minimal regions are entirely described by which of the circles they fall within (and hence also which they fall outside of). So any subset of the n circles should describe a minimal region: that minimal region falling within all the circles in the subset and outside of the rest of the circles of the diagram.

Given n circles, the following are the conditions desired for a Venn-type diagram:

1. For each of the 2^n terms, there is a minimal region corresponding to it.

2. There is no more than one region corresponding to any term.

The first condition ensures that every Boolean combination of the n sets is represented in the diagram. The second prevents any redundancy by ensuring that each combination is represented only once.

For logical purposes, these two conditions are really the only desiderata of a (formal or informal) syntax of the circles of a system of Venn-type diagrams. All that is relevant is that there is exactly one minimal region for each term representing each combination of circles.[8]

A *region* of a diagram consists of one or more minimal regions. Hence, a region can be entirely represented as a set of one or more of the terms corresponding to the minimal regions of a diagram.[9] In the case of a two-circle diagram with labels A and B, the set $\{A\overline{B}, \overline{A}\overline{B}\}$ represents the region outside of the circle labeled by B.

Since a region consists of any one or more minimal regions, there are as many regions as there are sets of minimal region, minus the empty set. So there are $2^{(2^n)} - 1$ regions.

If two regions of two diagrams are represented by the same set of terms, they are said to be *counterparts*.[10] Because regions that are counterparts have to be assigned the same set by any model, for convenience below they are sometimes spoken of as though they were the same region. This makes some discussions and proofs easier to read.

[8] Formal models of the syntax of overlapping circles have been provided for which these two conditions are satisfied for any finite number of circles, though the concept of circle must be extended to include non-convex closed curves. An example of such a model is presented in More [1959].

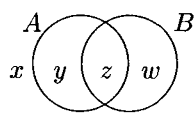
[9] See Shin [1994], p. 51.

[10] The counterpart relation is defined in Shin [1994], pp. 53–57.

A *model* has a domain of discourse which can be an arbirary set, and assigns subsets of the domain to the circles of the diagrams in question, assigning the same subset to circles labeled by the same letter. For example, a model might assign $\{x, y\}$ to the domain, assign $\{x\}$ to one circle of a diagram and $\{x, y\}$ to the other circle.

A model can also be understood as assigning subsets of the domain to minimal regions. A minimal region such as $\overline{A}BC\overline{D}E$ would be assigned $\overline{A} \cap B \cap C \cap \overline{D} \cap E$ (where $\overline{A}$ is the domain minus the set assigned to the circle labeled by A, B is the set assigned to the circle labeled B, etc.).[11] Likewise, a region can be understood as being assigned the union of the sets assigned to the minimal regions composing it.

Just as a model determines the sets assigned to minimal regions, conversely, an assignment to minimal regions can be used to specify a model. For example, suppose the four minimal regions of the following diagram are assigned sets x, y, z, and w, as shown:



This specifies the model:

$$\begin{cases} A & = & y \cup z \\ B & = & z \cup w \\ \text{domain} & = & x \cup y \cup z \cup w \end{cases}$$

The two systems, Venn diagrams and Peirce-Venn diagrams, discussed in the next two sections build on the basic diagrams described here by adding additional syntactic devices that can be used to mark various regions and thereby make assertions about the sets they represent.

2 VENN DIAGRAMS

This section presents the logical theory of Venn diagrams. Venn diagrams were introduced by John Venn in 1880 for the purpose of clearly representing categorical sentences and syllogistic reasoning.[12] Venn's system is a modification of a previous, incompleted system of Leonhard Euler's developed in 1761.[13]

[11]This definition of model is given in Hammer and Danner [1996]. A similar concept is defined in Shin [1994], pp. 64–68.

[12]See Venn [1880] and Venn [1894].

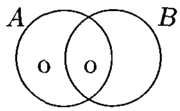
[13] Euler [1846]. For an analysis of Euler's system see Hammer and Shin [1996].

The particular version of Venn diagrams presented here is based on modifications made by Peirce in 1903[14] and Shin in 1994.[15] Peirce provided syntactic rules of inference for manipulating his variation on Venn diagrams while Shin formulated a coherent fragment of Peirce's system and reconstructed and analyzed it in modern form.

Venn diagrams are based on the syntax and semantics developed in the previous section. In addition, the system allows any region of a diagram to be marked as either representing an empty set or a non-empty set (more briefly: to be marked as empty or non-empty).

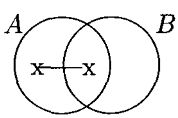
To assert that a region (rather, the set it represents) is empty is simply to assert that each of the minimal regions that make it up is empty. A minimal region is marked as empty by adding the symbol 'o' to it. This is Peirce's notation replacing Venn's shading of the minimal region.

For example, the following diagram asserts that A is empty (that both $\overline{A}B$ and AB are empty):



It is redundant to mark a minimal region with more than one 'o'. If the region is empty it's empty. Therefore well-formed diagrams will be required to have at most one 'o' in each minimal region.

To assert that a region is non-empty (rather, the set it represents) is not the same as asserting that each of the minimal regions composing it is empty. Rather, it is to assert that at least one of them is non-empty. With Venn diagrams, this is done by adding a chain of 'x's connected by lines to the region, with one 'x' falling in each of its minimal regions. For example, the following diagram asserts that A is non-empty (that either $\overline{A}B$ or AB is non-empty):



The region consisting of all the minimal regions with 'x's of the chain is said to *have* the chain. In particular, larger regions will not be said to have a chain falling in some proper subregion of it. For example in the

[14] Peirce [1958], pp. 294–319.
[15] Shin [1994].

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