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CONDITIONAL LOGIC

Prior to 1968 several writers had explored the conditions for the truth or assertability of conditionals, but this work did not result in an attempt to provide formal models for the semantical structure of conditionals. It had also been suggested that a proper logic for conditionals might be provided by combining modal operators with material conditionals in some way, but this suggestion never led to any widely accepted formal logic for conditionals.[1] Then Stalnaker [1968] provided both a formal semantics for conditionals and an axiomatic system of conditional logic. This important paper effectively inaugurated that branch of philosophical logic which we today call conditional logic. Nearly all the work on the logic of conditionals for the next ten years, and a great deal of work since then, has either followed Stalnaker's lead in investigating possible worlds semantics for conditionals or posed problems for such an approach. But in 1978, Peter Gärdenfors [1978] initiated a new line of inquiry focused on the use of conditionals to represent policies for belief revision. Thus, two main lines of development appeared, one an ontological approach concerned with truth or assertability conditions for conditionals and the other an epistemological approach focused on conditionals and change of belief.

With these two major lines of development, the material which has appeared on conditionals is prodigious. Consequently, we have had to focus upon certain aspects of conditional logic and to give other aspects less attention. We have followed the trend set in the literature and given the most attention to the analysis of so-called subjunctive conditionals as they are used in ordinary discourse and to triviality results for the Ramsey test. Accordingly, our discussion of conditionals and belief revision will be more heavily technical than our discussion of subjunctive conditionals. Other topics are discussed in less detail. Some of the important papers which it has not been possible to review are included in the accompanying bibliography, but the bibliography itself is far from complete.

1 ONTOLOGICAL CONDITIONALS

1.1 Introduction

Conditional logic is, in the first place, concerned with the investigation of the logical and semantical properties of a certain class of sentences occurring

[1] Another suggestion which has never been fully developed (but see Hunter [1980; 1982] is that an adequate theory of ordinary conditionals may be derived from relevance logic. We will say no more about this suggestion than it seems to us that conditional logic and relevance logic are concerned with very different problems, and it would be a tremendous coincidence if the correct logic for the conditionals of ordinary usage should turn out to resemble some version of relevance logic at all closely.

D.M. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, 2nd Edition, Volume 4, 1–98.

in a natural language. We will draw our examples from English, but much of what we have to say can be applied, with due caution, to other natural languages.

Paradigmatically, a conditional declarative sentence in English is one which contains the words 'if' and 'then'. Examples include

1. If it is raining, then we are taking a taxi.

and

2. If I were warm, then I would remove my jacket.

We could delete the occurrences of 'then' in (1) and (2) and we would still have perfectly acceptable sentences of English. In the case of (2), we can omit both 'if' and 'then' if we change the word order. Example (2) surely says the same thing as

3. Were I warm, I would remove my jacket.

Other conditionals in which neither 'if' nor 'then' occur include

4. When I find a good man, I will praise him.

and

5. You will need my number should you ever wish to call me.

Notice that all of these examples involve two component sentences or clauses, one expressing some sort of condition and another expressing some sort of claim which in some way depends upon the condition. The conditional or 'if' part of a conditional sentence is called the antecedent, and the main or 'then' part its consequent even when 'if' and 'then' do not actually occur. Notice that the antecedent precedes the consequent in (1)–(4), but the consequent comes first in (5). These examples should give the reader a fair idea of the types of sentences with which conditional logic is concerned.

While the verbs in (1) are in the indicative mood, those in (2) are in the subjunctive mood. Researchers often rephrase (2), forming a new conditional in which the verbs contained in antecedent and consequent are in the indicative mood. This practice implicitly assumes that (2) has the same content as

6. If it were the case that I am warm, then it would be the case that I remove my jacket.

Even without the rephrasing, it is sometimes said that 'I am warm' is the antecedent of both (2) and (6). Thus the mood of the verbs in the grammatical antecedent and consequent of (2) are taken logically to be a component of the conditional construction, while the logical antecedent and consequent are viewed as containing verbs in the indicative mood. Seen in this way, the conditional constructions in (1) and (2) look quite different and investigators have as a consequence made a distinction between indicative conditionals like (1) and subjunctive conditional like (2). This distinction is important because it appears that these two kinds of conditionals have different logical and semantical properties.

Much of the work done in conditional logic has focused on conditionals having antecedents and consequents which are false. Such conditionals are called counterfactuals. In actual practice, little distinction is made between counterfactuals and subjunctive conditionals which have true antecedents or consequents. Authors frequently refer to conditionals in the subjunctive mood as counterfactuals regardless of whether their antecedents or consequents are true or false. Another special kind of conditional is the so-called counterlegal conditional whose antecedent is incompatible with physical law. An example is

7. If the gravitational constant were to take on a slightly higher value in the immediate vicinity of the earth, then people would suffer bone fractures more frequently.

Also recognized are counteridenticals like

8. If I were the pope, I would support the use of the pill in India.

and countertemporals like

9. If it were 3.00 a.m., it would be dark outside.

Analysis of these special conditionals may involve special difficulties, but we can say very little about these special problems in a paper of this length.

Two other interesting conditional constructions are the even-if construction used in

10. It would rain even if the shaman did not do his dance.

and the might construction used in

11. If you don't take the umbrella, you might get wet.

We might paraphrase (10) using the word 'still' to get

12. It would still rain if the shaman did not do his dance.

even-if and might conditionals have somewhat different properties from those of other conditionals. It is believed by many, though, that these two kinds of conditionals can be analyzed in terms of subjunctive conditionals once we have an acceptable analysis of these. The strategy in this paper will be to concentrate on the many proposals for subjunctive conditionals, returning later (briefly) to the topics of indicative, even-if and might conditionals.

We will use two different symbols to represent indicative and subjunctive conditionals. For indicative conditionals we will use the double arrow $\Rightarrow$, and for the subjunctive conditional we will use the corner $>$. (Where context makes our intention clear, we will sometimes use symbols and formulas autonomously to refer to themselves.) With these devices we may represent (1) as

13. It is raining $\Rightarrow$ I am taking a taxi.

and represent (2) as

14. I am warm $>$ I remove my jacket.

Frequently we will have no particular antecedent or consequent in mind as we discuss one or the other of these two kinds of conditionals and as we examine forms which arguments involving these conditionals may take. In these cases we will use standard notation for classical first-order logic augmented by our symbols for indicative and subjunctive conditionals to represent the forms of sentences and arguments under discussion. We assume, as have nearly all investigators, that conditional have truth values and may therefore appear as arguments for truth-functional operators.

Students in introductory symbolic logic courses are normally taught to treat English conditionals as material conditionals. By material conditionals we mean certain truth-functional compounds of simpler sentences. A material condition $\phi \rightarrow \psi$ is true just in case ϕ is false or ψ is true. There can be little doubt that neither material implication nor any other truth function can be used by itself to provide an adequate representation of the logical and semantical properties of English conditionals or, presumably, the conditionals of any other language.

Consider the following two examples.

15. If I were seven feet tall, then I would be over two meters tall.

16. If I were seven feet tall, then I would be less than two yards tall.

In fact one of the authors is more than two yards tall but less than two meters tall, so for him the common antecedent and the two consequents of (15) and (16) are all false. Yet surely (15) is true while (16) is false. When both the antecedent and the consequent of an English subjunctive conditional are false, the conditional may be either true or false. Now consider two more examples.

17. If I were eight feet tall, I would be less than seven feet tall.

18. If I were seven feet tall, I would be over six feet tall.

Here we have two conditionals each of which has a false antecedent and a true consequent. but the first of these conditionals is false and the second is true. The moral of these examples is that when the antecedent of an English subjunctive conditional is false, the truth value of the conditional is not determined by the truth values of the antecedent and the consequent of the conditional alone. Some other factors must be involved in determining the truth values of such conditionals.

But what about English conditionals with true antecedents? It is generally accepted that any conditional with a true antecedent and a false consequent is false, but the situation is more controversial where the conditionals with true antecedents and true consequents are concerned. Some researchers have maintained that all such conditionals are true while others have claimed that such conditionals are sometimes false. Later we will consider some of the issues involved in this controversy. For now we simply recognize that there are some very good reasons for rejecting the view that all English conditionals can be represented adequately by material implication or by any other truth function.

1.2 Cotenability theories of conditionals

Chisholm [1946], Goodman [1955], Sellars [1958], Rescher [1964] and others have proposed accounts of conditionals which share some important features. Borrowing a term from Goodman, we can call these proposals *cotenability* theories of conditionals. The basic idea which these proposals share is that the conditional $\phi > \psi$ is true in case ϕ, together with some set of laws and true statements, entails ψ.

A crucial problem for such an analysis is that of determining the appropriate set of true statements to involve in the truth condition for a particular conditional. If the antecedent of the conditional is false, then of course its negation is true. But any proposition together with its negation will entail anything. The set of true statements upon which the truth of the conditional is to depend must at least be logically compatible with the antecedent of the conditional or the conditional will turn out to be trivially true on such an account. But logical compatibility is not enough either. We can have a true proposition χ such that ϕ and χ are logically compatible but such that $\chi > \neg\phi$ is also true. Then we should not wish to include χ in the set of propositions upon which the evaluation of $\phi > \psi$ depends. Goodman said of such a χ that it is not cotenable with ϕ. So Goodman's ultimate position is that $\phi > \psi$ is true just in case ψ is entailed by ϕ together with the set of all physical laws and the set of all true propositions cotenable with ϕ, i.e. with the set of all true propositions such that no member of that set counterfactually implies the negation of ϕ and the negation of no member

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