

# FREE LOGICS

## I: Introduction

### 1 WHAT ARE FREE LOGICS?

Some theorems of **CQC**=, such as those of the form

$$(1) \quad \exists x(x = \tau)$$

and

$$(2) \quad \varphi[\tau/x] \rightarrow \exists x\varphi,$$

are often accused of introducing into that theory—and thus into the very core of ‘our logic’—undesired ‘existential commitments’. However, the mere derivability of these sequences of symbols can hardly accomplish such a major feat by itself, and even when the theory is supplied with the usual ‘referential’ semantics, metaphysics is still far from being determined one way or another. 1 and 2 certainly require—by way of this semantics—that every singular term of the language receive an interpretation in the domain of quantification, but so what? The formal instrument does not specify the metaphysical counterpart of the relation between a symbol and its interpretation, nor does it tell you which things can or cannot belong to a domain of quantification. The formal instrument is neutral with respect to all these questions, and thus *by itself* cannot introduce any metaphysical commitments, existential or otherwise.

Things get more complicated when one takes into account the ideology most commonly associated with **CQC**= and its referential semantics. Then it becomes very ‘natural’ to think of a singular term as *denoting* its interpretation, hence to read the semantical requirement evoked by 1 and 2 as the requirement that every singular term denote. Even more importantly, if one agrees with Quine that ‘to be is to be a value of a bound variable’<sup>1</sup>—that is, if one assigns ‘existential import’ to quantifiers—the domain of quantification becomes the set of all and only those objects which exist in a given (possible) situation, and the above requirement is drastically strengthened, to the demand that every singular term denote *an existing object*. Now the ontological commitments are certainly apparent, and someone is bound to react to them in the name of logic’s ‘purity’.

---

<sup>1</sup>See for example [Quine, 1939]. In what follows, we will sometimes refer to this statement as *Quine’s dictum*.

Free logics<sup>2</sup> result from this reaction. However, since what they are a reaction to is a very delicate combination of many factors—a certain philosophical understanding of a certain formal interpretation of a certain formal system—it is difficult to say exactly what they are and how far they extend. To say—as is often said—that they are ‘logics free of existence assumptions with respect to their singular terms’ is too vague to be of much help, and also somewhat inaccurate from a historical point of view. For *every* formal system and *every* formal semantics can be free in this sense, given a suitable ideology, but this much tolerance was certainly not in the minds of the people who created free logics.<sup>3</sup> They wanted to *reform* classical logic, and *substitute* for it a *better* instrument, they thought that both the usual formal systems and the usual formal semantics were faulty in important ways, and it is only fair to define free logics so as to make sense of the precise task that they set for themselves.

On the other hand, it would not do to identify free logics with a certain class of theorems. For one thing, there is no *one* such class (as the expression ‘free logics’ should make clear),<sup>4</sup> and there is even some debate as to whether free logics result from restricting or rather extending classical logic.<sup>5</sup> But more importantly, we suggested above that all these modifications—whether restrictions or extensions—would make no sense (and in particular would not be legitimately referred to as free logics) if not in the context of certain interpretations of the formal systems, and of a certain understanding of these interpretations.

And finally, it would be totally unsatisfactory to define free logics in terms of a given semantics, or even a given *class* of semantics. For not only is a formal semantics (as well as a formal system) not enough to characterise the present enterprise in the absence of some ‘intuitive reading’ of it, but also the choice of a semantics is probably the most important question in this area, and we have to be careful not to prejudice such a fundamental issue by a biased *definition*.

Keeping all these reservations in mind will inevitably result in a less than straightforward characterisation of our subject, but the complications we will have to go through will prove instructive. For in this subject more than in others, logic, philosophy of logic and philosophy in general (especially metaphysics) are intertwined in a very delicate way, and it does not hurt if this delicate relation is emphasised right from the beginning.

In conclusion, I propose the following definition. A free logic is a formal system of quantification theory, with or without identity, which allows for some singular terms in some circumstances to be thought of as denoting no

<sup>2</sup>This expression was first used by Karel Lambert in 1960.

<sup>3</sup>See for example [Leonard, 1956] and [Lambert, 1967].

<sup>4</sup>Thus ‘free logics’ is the correct expression to refer to the whole subject, but ‘free logic’ is also very common.

<sup>5</sup>In this regard, see van Fraassen’s position sketched in Section 11.

existing object, and in which quantifiers are invariably thought of as having existential import.

A few comments and clarifications are in order. First of all, a terminological matter. The expression ‘thought of’, which occurs twice in the definition, must be regarded as inclusive of both the formal interpretation of the system and the intuitive (or philosophical) reading of this interpretation. When the formal semantics is missing (as was the case in free logics for several years), this ‘thinking of’ reduces entirely to its intuitive component.

Secondly, the definition requires that there be in the language of a free logic expressions construed as singular terms. A language containing no individual constants or descriptions and allowing individual variables to occur only bound in well-formed formulas (and there are languages of this sort for **CQC**, for example some of Quine’s) would hardly satisfy the present requirement.

Thirdly, the definition does not exclude the possibility that every singular term denotes in every circumstance, only that every singular term denotes *an existing object* in every circumstance. There are philosophers (Meinongians for example) who think that there are non-existing objects, and that singular terms may well denote them: the definition is neutral with respect to such views. However, to avoid awkwardness, usually I will refer to singular terms not denoting an existent simply as *non-denoting*.

Fourthly, the definition is concerned not with whether there *actually* are non-denoting singular terms, but only with whether there *may be*. A free logic is after all a *logic*; hence all that it can reasonably care for is logical possibility. When a logic acknowledges the possibility of non-denoting singular terms, we will say that it *allows for* non-denoting singular terms.

Fifthly, not every logic allowing for non-denoting singular terms is a free logic by our definition. In particular, all attempts at saving the formal system (and the formal semantics) of classical logic by some substitutional or Meinongian reading of the quantifiers are ruled out. On the other hand, it is perfectly possible to *add* to a free logic substitutional or Meinongian quantifiers, thus extending its expressive power.

Finally, even though referential semantics played a major role in the discussion above, the definition does not mention this semantics. The reason is that the existential import of quantifiers, and even the distinction between denoting and non-denoting singular terms, can be effectively mimicked in some non-referential semantics (for example, in Leblanc’s truth-value semantics),<sup>6</sup> even if the best way to understand what is going on in these semantics is still to compare them with their referential analogues. Thus the three factors to whose combination a free logic is a reaction come to play different roles in its definition: a free logic is the result of a modification of

---

<sup>6</sup>On this and other alternatives to the standard referential approach, see the chapter by Leblanc in Volume 2 of the present 2nd edition of this *Handbook*.

the *formal system* of **CQC** (or **CQC=**), motivated by a certain *intuitive reading* of it, which is best understood (at least so far) in the context of the usual referential *interpretation* of that system.

## 2 WHY FREE LOGICS?

The most general answer to this question has already been suggested in the discussion preceding my definition of a free logic. Though vigorously attacked from some quarters, the neopositivistic suspicion towards metaphysics is still highly influential in contemporary logic. Whether they regard metaphysics as sheer 'nonsense' or as a set of 'synthetic' statements to be neatly distinguished from the 'analytic' ones constituting their discipline, many logicians like logic to be metaphysically 'pure', or not to carry any metaphysical 'baggage'—as the many debates in the area of quantified modal logic show sometimes quite dramatically. To apply such a general motivation to the present case, it is enough to regard even the simplest existential statements as metaphysical in nature.

However, this motivation by itself does not go very far towards motivating anything close to free logics. As we will see in the next section, classical logic has its own ways of dealing with these matters, and certainly many classical logicians would not accept without a fight the claim—presupposed by the alleged 'justification' of free logics suggested above—that classical logic is in any sense existentially committed or metaphysically 'impure'. To get closer to the justification we are looking for, we need to weaken that claim as follows. Classical logic (if filtered through the usual interpretation, and the usual reading of this interpretation) does not allow for non-denoting singular terms. To be sure, this logic can be *used* in such a way as to avoid any philosophical commitments or any problems resulting from the limitation in question, but this requires the adoption of convoluted and *ad hoc* procedures of translation from natural language into the formal language and back (in a word, of a number of epicycles). Free logics, on the other hand, represent a much more straightforward and direct approach to the same problems: they make the translations easier, they allow expressions of natural language to be taken more often at face value, and they require fewer *ad hoc* assumptions.

This justification is certainly better than the first one, but still, it does not entirely fulfil its purpose. For it does not take into account the fact that the classical logician can shape his philosophy of language so as to make it fit his logic perfectly (and make his logic the most 'natural' thing in the world): Russell's position—to be mentioned briefly in the next section—is in this respect typical. And this makes it clear once and for all that the adoption of some specific view in the philosophy of language is an essential step towards the justification of free (and perhaps all) logics.

There is a whole spectrum of such views that would do the job nicely, ranging from an extremely ‘metaphysical’ one to an extremely ‘pragmatic’ one. For the sake of illustration, let me briefly discuss these two extremes.

The ‘metaphysical’ extreme states simply that in natural language *there* are non-denoting singular terms. A singular term is an expression that *purports* to denote a single object, and many a singular term fails to achieve this purpose. Nonetheless, they are still singular terms: ‘Pegasus’ is as much a singular term as ‘Caesar’ or ‘3’, and ‘the winged horse’ or ‘the round square’ are as much singular terms as ‘the President of the USA in 2000’. Hence no formal system can give a faithful representation of the structure of natural language (and so be reasonably applied to it) if it does not allow for non-denoting singular terms.

The ‘pragmatic’ extreme, on the other hand, regards the *real existence* of non-denoting singular terms in natural language as totally irrelevant. Whether there are or there aren’t any, there are contexts in which some people use expressions *as* singular terms without assuming that they denote anything, or maybe even in the process of wondering whether they denote or not. For example, an attempt by a person to prove that God exists—or that ‘God’ denotes—might be conceived as a case in point. Whether these people are right or not, a logic allowing for non-denoting singular terms would also allow for a more direct and faithful representation (and evaluation) of their reasoning in those contexts. So this logic would be an instrument of wider and simpler applicability than classical logic, and would not prejudice important issues which it is inappropriate for logic to decide.

Of course, the classical logician can be expected to have responses to these motivations. It is certainly not news that in philosophy, or anywhere else, you can’t get something valuable for nothing. In the present case, this suggests that you need a position in between the two above extremes to transform the fear of metaphysical commitment so well entrenched in most contemporary logicians into a defence of free logics.

### 3 CLASSICAL LOGIC AND NON-DENOTING SINGULAR TERMS

As suggested earlier, the classical logician is not *forced* to modify his formal instrument by the mere presence in natural language of expressions like ‘Pegasus’ or ‘the round square’. He has at his disposal several techniques for dealing with alleged non-denoting singular terms within his own framework. Since all these techniques are treated extensively in other parts of the *Handbook*.<sup>7</sup> I will limit myself here to little more than listing them.

In the first section, I pointed out that the problem free logicians see in classical logic (and try to solve with their logics) is the following: classical

---

<sup>7</sup>In particular, in Hodges’ chapter in Volume 1 and Salmon’s chapter “Reference and Information Contents: Names and Descriptions” in a later volume.

Handbook of Philosophical Logic

Gabbay, D.; Guenther, F. (Eds.)

2002, XIII, 360 p. 1 illus., Hardcover

ISBN: 978-1-4020-0235-9