J. MICHAEL DUNN AND GREG RESTALL

RELEVANCE LOGIC

1 INTRODUCTION

1.1 Delimiting the topic

The title of this piece is not 'A Survey of Relevance Logic'. Such a project was impossible in the mid 1980s when the first version of this article was published, due to the development of the field and even the space limitations of the *Handbook*. The situation is if anything, more difficult now. For example Anderson and Belnap and Dunn's two volume [1975; 1992] work *Entailment: The Logic of Relevance and Necessity*, runs to over 1200 pages, and is their summary of just some of the work done by them and their co-workers up to about the late 1980s. Further, the comprehensive bibliography (prepared by R. G. Wolf) contains over 3000 entries in work on relevance logic and related fields.

So, we need some way of delimiting our topic. To be honest the fact that *we* are writing this is already a kind of delimitation. It is natural that you shall find emphasised here the work that we happen to know best. But still rationality demands a less subjective rationale, and so we will proceed as follows.

Anderson [1963] set forth some open problems for his and Belnap's system **E** that have given shape to much of the subsequent research in relevance logic (even much of the earlier work can be seen as related to these open problems, e.g. by giving rise to them). Anderson picks three of these problems as major: (1) the admissibility of Ackermann's rule γ (the reader should not worry that he is expected to already know what this means), (2) the decision problems, (3) the providing of a semantics. Anderson also lists additional problems which he calls 'minor' because they have no 'philosophical bite'. We will organise our remarks on relevance logic around three major problems of Anderson. The reader should be told in advance that each of these problems are closed (but of course 'closed' does not mean 'finished'—closing one problem invariably opens another related problem). This gives then three of our sections. It is obvious that to these we must add an introduction setting forth at least some of the motivations of relevance logic and some syntactical specifications. To the end we will add a section which situates work in relevance logic in the wider context of study of other logical systems, since in the recent years it has become clear that relevance logics fit well among a wider class of 'resource-conscious' or 'substructural' logics [Schroeder-Heister and Došen, 1993; Restall, 2000] [and cite the S–H article in this volume]. We thus have the following table of contents:

D. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, Volume 6, 1–128.

1. Introduction
2. The Admissibility of γ
3. Semantics
4. The Decision Problem
5. Looking About

We should add a word about the delimitation of our topic. There are by now a host of formal systems that can be said with some justification to be 'relevance logics'. Some of these antedate the Anderson–Belnap approach, some are more recent. Some have been studied somewhat extensively, whereas others have been discussed for only a few pages in some journal. It would be impossible to describe all of these, let alone to assess in each and every case how they compare with the Anderson–Belnap approach. It is clear that the Anderson–Belnap-style logics have been the most intensively studied. So we will concentrate on the research program of Anderson, Belnap and their co-workers, and shall mention other approaches only insofar as they bear on this program. By way of minor recompense we mention that Anderson and Belnap [1975] have been good about discussing related approaches, especially the older ones.

Finally, we should say that our paradigm of a relevance logic throughout this essay will be the Anderson–Belnap system **R** or relevant implication (first devised by Belnap—see [Belnap, 1967a; Belnap, 1967b] for its history) and not so much the Anderson–Belnap favourite, their system **E** of entailment. There will be more about each of these systems below (they are explicitly formulated in Section 1.3), but let us simply say here that each of these is concerned to formalise a species of implication (or the conditional—see Section 1.2) in which the antecedent suffices *relevantly* for the consequent. The system **E** differs from the system **R** primarily by adding necessity to this relationship, and in this **E** is a modal logic as well as a relevance logic. This by itself gives good reason to consider **R** and not **E** as the paradigm of a relevance logic.[1]

1.2 Implication and the Conditional

Before turning to matters of logical substance, let us first introduce a framework for grammar and nomenclature that is helpful in understanding the ways that writers on relevance logic often express themselves. We draw

[1]It should be entered in the record that there are some workers in relevance logic who consider both **R** and **E** too strong for at least some purposes (see [Routley, 1977], [Routley *et al.*, 1982], and more recently, [Brady, 1996]).

heavily on the 'Grammatical Propaedeutic' appendix of [Anderson and Belnap, 1975] and to a lesser extent on [Meyer, 1966], both of which are very much recommended to the reader for their wise heresy from logical tradition.

Thus logical tradition (think of [Quine, 1953]) makes much of the grammatical distinction between 'if, then' (a connective), and 'implies' or its rough synonym 'entails' (transitive verbs). This tradition opposes

1. If today is Tuesday, then this is Belgium

to the pair of sentences

2. 'Today is Tuesday' implies 'This is Belgium',

3. That today is Tuesday implies that this is Belgium.

And the tradition insists that (1) be called a *conditional*, and that (2) and (3) be called *implications*.

Sometimes much philosophical weight is made to rest on this distinction. It is said that since 'implies' is a verb demanding nouns to flank it, that implication must then be a relation between the objects stood for by those nouns, whereas it is said that 'if, then' is instead a connective combining that implication (unlike 'if, then') is really a metalinguistic notion, either overtly as in (2) where the nouns are names of sentences, or else covertly as in (3) where the nouns are naming propositions (the 'ghosts' of linguistic entities). This last is then felt to be especially bad because it involves ontological commitment to propositions or some equally disreputable entities. The first is at least free of such questionable ontological commitments, but does raise real complications about 'nested implications', which would seem to take us into a meta-metalanguage, etc.

The response of relevance logicians to this distinction has been largely one of 'What, me worry?' Sometime sympathetic outsiders have tried to apologise for what might be quickly labelled a 'use–mention confusion' on the part of relevance logicians [Scott, 1971]. But 'hard-core' relevance logicians often seem to luxuriate in this 'confusion'. As Anderson and Belnap [1975, p. 473] say of their 'Grammatical Propaedeutic': "the principle aim of this piece is to convince the reader that it is philosophically respectable to 'confuse' implication or entailment with the conditional, and indeed philosophically suspect to harp on the dangers of such a 'confusion'. (The suspicion is that such harpists are plucking a metaphysical tune on merely grammatical strings.)"

The gist of the Anderson–Belnap position is that there is a generic conditional-implication notion, which can be carried into English by a variety of grammatical constructions. Implication itself can be viewed as a connective requiring prenominalisation: 'that __ implies that __', and as such it nests. It is an incidental feature of English that it favours sentences with main subjects and verbs, and 'implies' conforms to this reference by

the trick of disguising sentences as nouns by prenominalisation. But such grammatical prejudices need not be taken as enshrining ontological presuppositions.

Let us use the label 'Correspondence Thesis' for the claim that Anderson and Belnap come close to making (but do not actually make), namely, that *in general* there is nothing other than a purely grammatical distinction between sentences of the forms

4. If A, then B, and

5. That A implies that B.

Now undoubtedly the Correspondence Thesis overstates matters. Thus, to bring in just one consideration, [Castañeda, 1975, pp. 66 ff.] distinguishes 'if A then B' from 'A only if B' by virtue of an essentially pragmatic distinction (frozen into grammar) of 'thematic' emphases, which cuts across the logical distinction of antecedent and consequent. Putting things quickly, 'if' introduces a sufficient condition for something happening, something being done, etc. whereas 'only if' introduces a necessary condition. Thus 'if' (by itself or prefixed with 'only') always introduces the state of affairs thought of as a condition for something else, then something else being thus the focus of attention. Since 'that A implies that B' is devoid of such thematic indicators, it is not equivalent at *every* level of analysis to either 'if A then B' or 'A only if B'.

It is worth remarking that since the formal logician's $A \to B$ is equally devoid of thematic indicators, 'that A implies that B' would seem to make a better reading of it than either 'if A then B' or 'A only if B'. And yet it is almost universally rejected by writers of elementary logic texts as even an acceptable reading.

And, of course, another consideration against the Correspondence Thesis is produced by notorious examples like Austin's

6. There are biscuits on the sideboard if you want some,

which sounds very odd indeed when phrased as an implication. Indeed, (6) poses perplexities of one kind or another for any theory of the conditional, and so should perhaps best be ignored as posing any special threat tot he Anderson and Belnap account of conditionals. Perhaps it was Austin-type examples that led Anderson and Belnap [1975, pp. 491–492] to say "we think every use of 'implies' or 'entails' as a connective can be replaced by a suitable 'if-then'; however, the converse may not be true". They go on to say "But with reference to the uses in which we are primarily interested, we feel free to move back and forth between 'if-then' and 'entails' in a free-wheeling manner".

Associated with the Correspondence Thesis is the idea that just as there can be contingent conditionals (e.g. (1)), so then the corresponding implications (e.g. (3)) must also be contingent. This goes against certain Quinean

tendencies to 'regiment' the English word 'implies' so that it stands only for *logical* implication. Although there is no objection to thus giving a technical usage to an ordinary English word (even requiring in this technical usage that 'implication' be a metalinguistic relation between sentences), the point is that relevance logicians by and large believe we are using 'implies' in the ordinary non-technical sense, in which a sentence like (3) might be true without there being any logical (or even necessary) implication from 'Today is Tuesday' to 'This is Belgium'.

Relevance logicians are not themselves free of similar regimenting tendencies. Thus we tend to differentiate 'entails' from 'implies' on precisely the ground that 'entails', unlike 'implies', stands only for *necessary* implication [Meyer, 1966]. Some writings of Anderson and Belnap even suggest a more restricted usage for just *logical* implication, but we do not take this seriously. There does not seem to be any more linguistic evidence for thus restricting 'entails' than there would be for 'implies', though there may be at least more excuse given the apparently more technical history of 'entails' (in its logical sense—cf. The OED).

This has been an explanation of, if not an apology for, the ways in which relevance logicians often express themselves. but it should be stressed that the reader need not accept all, or any, of this background in order to make sense of the basic aims of the relevance logic enterprise. Thus, e.g. the reader may feel that, despite protestations to the contrary, Anderson, Belnap and Co. are hopelessly confused about the relationships among 'entails', 'implies', and 'if-then', but still think that their system **R** provides a good formalisation of the properties of 'if-then' (or at least 'if-then relevantly'), and that they system **E** does the same for some strict variant produced by the modifier 'necessarily'.

One of the reasons the recent logical tradition has been motivated to insist on the fierce distinction between implications and conditionals has to do with the awkwardness of reading the so-called 'material conditional' $A \to B$ as corresponding to any kind of implication (cf. [Quine, 1953]).

The material conditional $A \to B$ can of course be defined as $\neg A \vee B$, and it certainly does seem odd, modifying an example that comes by oral tradition from Anderson, to say that:

7. Picking a guinea pig up by its tail implies that its eyes will fall out.

just on the grounds that its antecedent is false (since guinea pigs have no tails). But then it seems equally false to say that:

8. If one picks up a guinea pig by its tail, then its eyes will fall out.

And also both of the following appear to be equally false:

9. Scaring a pregnant guinea pig implies that all of her babies will be born tailless.

http://www.springer.com/978-1-4020-0583-1

Handbook of Philosophical Logic
Gabbay, D.; Guenthner, F. (Eds.)
2002, XIII, 406 p. 1 illus., Hardcover
ISBN: 978-1-4020-0583-1