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PARACONSISTENT LOGIC

Indeed, even at this stage, I predict a time when there will be mathematical investigations of calculi containing contradictions, and people will actually be proud of having emancipated themselves from 'consistency'. Ludwig Wittgenstein, 1930.[1]

1 INTRODUCTION

Paraconsistent logics are those which permit inference from inconsistent information in a non-trivial fashion. Their articulation and investigation is a relatively recent phenomenon, even by the standards of modern logic. (For example, there was no article on them in the first edition of the *Handbook*.) The area has grown so rapidly, though, that a comprehensive survey is already impossible. The aim of this article is to spell out the basic ideas and some applications. Paraconsist logic has interest for philosophers, mathematicians and computer scientists. As befits the *Handbook*, I will concentrate on those aspects of the subject that are likely to be of more interest to philosopher-logicians. The subject also raises many important philosophical issues. However, here I shall tread over these very lightly—except in the last section, where I shall tread over them lightly.

I will start in part 2 by explaining the nature of, and motivation for, the subject. Part 3 gives a brief history of it. The next three parts explain the standard systems of paraconsistent logic; part 4 explains the basic ideas, and how, in particular, negation is treated; parts 5 and 6 discuss how this basic apparatus is extended to handle conditionals and quantifiers, respectively. In part 7 we look at how a paraconsistent logic may handle various other sorts of machinery, including modal operators and probability. The next two parts discuss the applications of paraconsistent logic to some important theories; part 8 concerns set theory and semantics; part 9, arithmetic. The final part of the essay, 10, provides a brief discussion of some central philosophical aspects of paraconsistency.

In writing an essay of this nature, there is a decision to be made as to how much detail to include concerning proofs. It is certainly necessary to include many proofs, since an understanding of them is essential for anything other than a relatively modest grasp of the subject. On the other hand, to prove everything in full would not only make the essay extremely long, but distract from more important issues. I hope that I have struck a happy *via media*.

[1] Wittgenstein [1975], p. 332.

D. Gabbay and F. Guenthner (eds.),
Handbook of Philosophical Logic, Volume 6, 287–393.

Where proofs are given, the basic definitions and constructions are spelled out, and the harder parts of the proof worked. Routine details are usually left to the reader to check, even where this leaves a considerable amount of work to be done. In many places, particularly where the material is a dead end for the purposes of this essay, and is easily available elsewhere, I have not given proofs at all, but simply references. Those for whom a modest grasp of the subject is sufficient may, I think, skip all proofs entirely.

Paraconsistent logic is strongly connected with many other branches of logic. I have tried, in this essay, not to duplicate material to be found in other chapters of this *Handbook*, and especially, the chapter on Relevant Logic. At several points I therefore defer to these. There is no section of this essay entitled 'Further Reading'. I have preferred to indicate in the text where further reading appropriate to any particular topic may be found.[2]

2 DEFINITION AND MOTIVATION

2.1 Definition

The major motivation behind paraconsistent logic has always been the thought that in certain circumstances we may be in a situation where our information or theory is inconsistent, and yet where we are required to draw inferences in a sensible fashion. Let $\vdash$ be any relationship of logical consequence. Call it *explosive* if it satisfies the condition that for all α and β, $\{\alpha, \neg\alpha\} \vdash \beta$, *ex contradictione quodlibet* (ECQ). (In future I will omit set braces in this context.) Both classical and intuitionist logics are explosive. Clearly, if $\vdash$ is explosive it is not a sensible inference relation in an inconsistent context, for applying it gives rise to *triviality*: everything. Thus, a minimal condition for a suitable inference relation in this context is that it not be explosive. Such inference relationships (and the logics that have them) have come to be called *paraconsistent.*[3]

Paraconsistency, so defined, is something of a minimal condition for a logic to be used as envisaged; and there are logics that are paraconsistent but not really appropriate for the use. For example, Johansson's minimal logic is paraconsistent, but satisfies $\alpha, \neg\alpha \vdash \neg\beta$. One might therefore attempt a stronger constraint on the definition of 'paraconsistent', such as: for no syntactically definable class of sentences (e.g., negated sentences), Σ, do

[2]The most useful general reference is Priest *et al.* [1989] (though this is already a little dated). That book also contains a bibliography of paraconsistency up to about the mid-1980s.

[3]The word was coined by Miró Quesada at the Third Latin American Symposium on Mathematical Logic, in 1976. Note that a paraconsistent logic need not itself have an inconsistent set of logical truths: most do not. But there are some that do, e.g., any logic produced by adding the connexivist principle $\neg(\alpha \rightarrow \neg\alpha)$ to a relevant logic at least as strong as B. See Mortensen [1984].

we have $\alpha, \neg\alpha \vdash \sigma$, for all $\sigma \in \Sigma$. This seems too strong, however. In many logics, $\alpha, \neg\alpha \vdash \beta$, for every logical truth, β. If the logic is decidable, then there is a clear sense in which the set of logical truths is syntactically characterisable. Yet such logics would still be acceptable for many paraconsistent purposes. Hence, this definition would seem to be too strong.[4]

In his [1974], da Costa suggests another couple of natural constraints on a paraconsistent logic, of a rather different nature. One is to the effect that the logic should not contain $\neg(\alpha \wedge \neg\alpha)$ as a logical truth. The rationale for this is not spelled out. However, I take it that the idea is that if one has information that contains α and $\neg\alpha$ one does not want to have a logical truth that contradicts this. Why not though? Since one is not ruling out inconsistency *a priori*, there would seem to be nothing *a priori* against this (though maybe for particular applications one would not want the situation to arise). As a general condition, then, it seems too strong. And certainly a number of the logics that we will consider have $\neg(\alpha \wedge \neg\alpha)$ as a logical truth.

Another of the constraints that da Costa suggests is to the effect that the logic should contain as much of classical—or at least intuitionist—logic, as does not interfere with its paraconsistent nature. The condition is somewhat vague, though its intent is clear enough; and again, it is too strong. It assumes that a paraconsistent logician must have no objection to other aspects of classical or intuitionist logic, and this is clearly not true. For example, a relevant logician might well object to paradoxes of implication, such as $\alpha \to (\beta \to \alpha)$.[5]

As an aside, let me clarify the relationship between relevant logics and paraconsistent logics. The motivating concern of relevant logic is somewhat different from that of paraconsistency, namely to avoid paradoxes of the conditional. Thus, one may take a relevant (propositional) logic to be one such that if $\alpha \to \beta$ is a logical truth then α and β share a propositional parameter. The interests of relevant and paraconsistent logics clearly converge at many points. Relevant logics and paraconsistent logics are not coextensive, however. There are many paraconsistent logics that are not relevant, as we shall see. The relationship the other way is more complex, since there are different ways of using a relevant logic to define a consequence relation. A natural way is to say that $\alpha \vdash \beta$ iff $\alpha \to \beta$ is a logical truth. Such a consequence relation is clearly paraconsistent. Another is to define logical consequence as deducibility, defined in the standard way, using some set of axioms and rules for the relevant logic. Such a consequence relation may, but need not, be relevant. For example, Ackermann's original formulation of E contained the rule γ: if $\vdash \alpha$ and $\vdash \neg\alpha \vee \beta$ then $\vdash \beta$. This gives explo-

[4]Further attempts to tighten up the definition of paraconsistency along these lines can be found in Batens [1980] (in the definition of 'A-destructive', p. 201, clause (i) should read $\nvdash_L A$), and Urbas [1990].

[5]Indeed, it is just this principle that ruins minimal logic for serious paraconsistent purposes. For α and $\alpha \to \bot$ (i.e., $\neg\alpha$) give $\bot$, and the principle then gives $\beta \to \bot$.

sion by an argument often called the 'Lewis Independent Argument', that we will meet in a moment.

Anyway, and to return from the digression: the definition of paraconsistency given here is weaker than sufficient to *guarantee* sensible application in inconsistent contexts; but an elegant stronger definition is not at hand, and since the one in question has become standard, I will use it to define the contents of this essay.

2.2 Inconsistency and Dialetheism

Numerous examples of inconsistent information/theories from which one might want to draw inferences in a controlled way have been offered by paraconsistent logicians. For example:

1. information in a computer data base;
2. various scientific theories;
3. constitutions and other legal documents;
4. descriptions of fictional (and other non-existent) objects;
5. descriptions of counterfactual situations.

The first of these is fairly obvious. As an example of the second, consider, e.g., Bohr's theory of the atom, which required bound electrons both to radiate energy (by Maxwell's equations) and not to (since they do not spiral inwards towards the nucleus). As an example of the third, just consider a constitution that gives persons of kind A the right to do something, x, and forbids persons of kind B from doing x. Suppose, then, that a person in both categories turns up. (We may assume that it had never occurred to the legislators that there might be such a person.) In the fourth case, the information (in, say, a novel or a myth) characterises an object, and turns out—deliberately or otherwise—to be inconsistent. To illustrate the fifth, suppose, for example, that we need to compute the truth of the conditional: if you were to square the circle, I would give you all my money. Applying the Ramsey-test, we see what follows from the antecedent (which is logically impossible), together with appropriate background assumptions. (And I would *not* give you all my money!)[6]

There is no suggestion here that in every case one must remain content with the inconsistent information in question. One might well like to remove

[6]Many of these examples are discussed further in Priest *et al.* [1989], ch. 18. The Bohr case is discussed in Brown [1993]. Another kind of example that is sometimes cited is the information provided by witnesses at a trial. I find this less persuasive. It seems to me that the relevant information here is all of the form: witness x says so and so. (That a witness is lying, or making an honest mistake, is always a possibility to be taken into account.) And any collection of statements of this form is quite consistent.

some of the inconsistent information in the data base; reject or revise the scientific theory; change the law to eliminate the inconsistency. But this is not possible in all of the cases given, e.g., for counterfactual conditionals with impossible antecedents. And even where it is, this not only may take time; it is often not clear how to do so satisfactorily. (The matter is certainly not algorithmic.) While we figure out how to do it, we may still be in a situation where inference is necessary, perhaps for practical ends, e.g., so that we can act on the information in the data base; or manipulate some piece of scientific technology; or make decisions of law (on other than an obviously inconsistent case). Moreover, since there is no decision procedure for consistency, there is no guarantee that any revision will achieve consistency. We cannot, therefore, be sure that we have succeeded. (This is particularly important in the case of the data base, where the deductions go on "behind our back", and the need to revise may never become apparent.)

In cases of this kind, then, even though we may not, ideally, be satisfied with the inconsistent information, it may be desirable—indeed, practically necessary—to use a paraconsistent logic. Moreover, we know that many scientific theories are false; they may still be important because they make correct predications in most, or even all, cases; they may be good *approximations* to the truth, and so on. These points remain in force, even if the theories in question contain contradictions, and so are (thought to be) false for logical reasons. Of course, this is not so if the theories are trivial; but that's the whole point of using a paraconsistent logic.

One can thus subscribe to the use of paraconsistent logics for some purposes without believing that inconsistent information or theories may be *true*. The view that some *are* true has come to be called *dialetheism*, a dialetheia being a true contradiction.[7] If the truth about some subject is dialetheic then, clearly, a paraconsistent logic needs to be employed in reasoning about that subject. (I take it to be uncontentious that the set of truths is not trivial. Why this is so, especially once one has accepted dialetheism is, however, a substantial question.)

Examples of situations that may give rise to dialetheias, and that have been proposed, are of several kinds, including:

1. certain kinds of moral and legal dilemmas;
2. borderline cases of vague predicates;
3. states of change.

Thus, one may suppose, in the legal example mentioned before, that a person who is A and B both has and has not the right to do x; or that in

[7]The term was coined by Priest and Routley in 1981. See Priest *et al.* [1989], p. xx. Note that some writers prefer 'dialethism'.

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