

COMBINATIONS OF TENSE AND MODALITY

1 INTERACTIONS WITH TIME

Physics should have helped us to realise that a temporal theory of a phenomenon X is, in general, more than a simple combination of two components: the statics of X and the ordered set of temporal instants. The case in which all functions from times to world-states are allowed is uninteresting; there are too many such functions, and the theory has not begun until we have begun to restrict them. And often the principles that emerge from the interaction of time with the phenomena seem new and surprising. The most dramatic example of this, perhaps, is the interaction of space with time in relativistic space-time.

The general moral, then, is that we shouldn't expect the theory of time + X to be obtained by mechanically combining the theory of time and the theory of X .¹

Probability is a case that is closer to our topic. Much ink has been spilled over the evolution of probabilities: take, for instance, the mathematical theory of Markov processes (Howard [1971a; 1971b] make a good text), or the more philosophical question of rational belief change (see, for example, Chapter 11 of Jeffrey [1990] and Harper [1975].) Again, there is more to these combinations than can be obtained by separate reflection on probability measure and the time axis.

probability shares many features with modalities and, despite the fact that (classical) probabilities are numbers, perhaps in some sense probability *is* a modality. It is certainly the classic case of the use of possible worlds in interpreting a calculus. (Sample points in a state space are merely possible worlds under another name.) But the literature on probability is enormous, and almost none of it is presented from the logician's perspective. So, aside from the references I have given, I will exclude it from this survey. However, it seems that the techniques we will be using can also help to illuminate problems having to do with probability; this is illustrated by papers such as D. Lewis [1981] and Van Fraassen [1971]. For lack of space, these are not discussed in the present essay.

¹For a treatment that follows this procedure, see [Woolhouse, 1973]; [Werner, 1974] may also fit into this category, but I have not been able to obtain a copy of it. The tense logic of Woolhouse's paper is fairly crude: e.g. moments of time appear both in models and in the object language. The paper seems mainly to be of historical interest.

2 INTRODUCTION TO HISTORICAL NECESSITY

Modern modal logic began with necessity (or with things definable with respect to necessity), and the earliest literature, like C. I. Lewis [1918], confuses this with validity. Even in later work that is formally scrupulous about distinguishing these things, it is sometimes difficult to tell what concepts are really metalinguistic. Carnap, for instance [1956, p. 10], begins his account of necessity by directing our attention to *L*-truth; a sentence of a semantical system (or language) is *L*-true when its truth follows from the semantical rules of the language, without auxiliary assumptions. This, of course, is a metalinguistic notion. But later, when he introduces necessity into the object language [Carnap, 1956, p. 174], he stipulates that $\Box\varphi$ is true if and only if φ is *L*-true.

Carnap thinks of the languages with which he is working as fully determinate; in particular, their semantical rules are fixed. This has the consequence that whatever is *L*-true in a language is eternally *L*-true in that language. (See [Schlipp, 1963, p. 921], for one passage in which Carnap is explicit on the point: he says 'analytic sentences cannot change their truth-value'.) Combining this consequence with Carnap's explication of necessity, we see that²

$$(1) \quad \Box\varphi \rightarrow HG\Box\varphi$$

will be valid in languages containing both necessity and tense operators: necessary truths will be eternally true. The combination of necessity with tense would then be trivialised.

But there are difficulties with Carnap's picture of necessity; indeed, it seems to be drastically misconceived.³ For one thing, many things appear to be necessary, even though the sentences that express them can't be derived from semantical rules. In Kripke [1982], for instance, published 26 years after *Meaning and Necessity*, Saul Kripke argues that it is necessary that Hesperus is Phosphorous, though 'Hesperus' and 'Phosphorous' are by no means synonymous. Also at work in Kripke's conception of necessity, and that of many other contemporaries, is the distinction between φ expressing a necessary truth, and φ necessarily expressing a truth. In a well-known defence of the analytic-synthetic distinction, Grice and Strawson [1956] write as follows:

²I use the tense logical notation of the first Chapter in this volume.

³For an early appreciation of the philosophical importance of making necessity time-dependent (the point I myself am leading up to), see [Lehrer and Taylor, 1965]. The puzzles they raise in this paper are genuine and well presented. But the solution they suggest is very implausible, and the considerations that motivate it seem to confuse semantic and pragmatic phenomena. This is a good example of a case in which philosophical reflections could have been aided by an appeal to the technical apparatus of model theory (in this case, to the model theory of tense logic).

Any form of words at one time held to express something true may, no doubt, at another time come to be held to express something false. but it is not only philosophers who would distinguish between the case where this happens as the result of a change of opinion solely as to matters of fact, and the case where this happens at least partly as a result of shift in the sense of the words (p. 157).

This distinction, at least in theory, makes it possible that a sentence φ should necessarily (perhaps, because of semantical rules) express a truth, even though the truth that it expresses is contingent. This idea is developed most clearly in [Kaplan, 1978].

On this view of necessity, it attaches not primarily to sentences, but to propositions. A sentence will express a proposition, which may or may not be necessary. This can be explicated using possible worlds: propositions take on truth values in these worlds, and a proposition is necessary if and only if it is true in all possible worlds.⁴

This conception can be made temporal without trivialising the results. Probably the simplest way of managing this is to begin with nonempty sets T of times and W of worlds;⁵ T is linearly ordered by a relation $<$. I will call this the $T \times W$ approach.

Recall that a tensed formula, say $F\varphi$, is true at $\langle w, t \rangle$, where $w \in W$ and $t \in T$, if and only if φ is true at $\langle w, t' \rangle$, for some t' such that $t < t'$.⁶ We now want to ask under what conditions $\Box\varphi$ is true at $\langle w, t \rangle$. (In putting it this way we are suppressing propositions; this is legitimate, as long as we treat propositional attitudes as unanalysed, and assume that sentences express the same proposition everywhere.)

If we appeal to intuitions about languages like English, it seems that we should treat formulas like $\Box\varphi$ as nontrivially tensed. This is shown most clearly by sentences involving the adjective 'possible', such as 'In 1932 it was possible for Great Britain to avoid war with Germany; but in 1937 it was impossible'. This suggests that when $\Box\varphi$ is evaluated at $\langle w, t \rangle$ we

⁴To simplify matters, I confine the discussion to the absolute necessity of S5. But perhaps I should mention in passing that in explicating the relative breeds of necessity, such as that of S4, it is easy to confuse modal relations with temporal relations, relative necessity with evanescent necessity. And, of course, tense logic was inspired in part by work on relative necessity. But the two notions are separate; an S5 breed of necessity, for instance, can be evanescent. And when tense and modality are combined, it is very important to attend to the distinction.

⁵I dislike this way of arranging things for philosophical reasons. it doesn't strike me as a logical truth that all worlds have the same temporal orderings: some may have an earliest time, for instance, and others not. Also, the notion of different worlds sharing the same time is philosophically problematic; it is hard to reconcile with a plausible theory of time, when the possible worlds differ widely. Finally, I like to think of possible worlds as overlapping, so that at the same moment may have alternative futures. This requires a more complicated representation. However, the $T \times W$ arrangement will do for now.

⁶See the discussion of the interpretation of tense in Chapter 1 of this volume.

are considering what is *then necessary*; what is true in all worlds at that particular time, t .

The rule then is that $\Box\varphi$ is true at $\langle w, t \rangle$ if and only if φ is true at $\langle w', t \rangle$ for all $w' \in W$. If we like, we can make this relational. Let $\{\approx_t: t \in T\}$ be a family of equivalence relations on W , and let $\Box\varphi$ be true at $\langle t, w \rangle$ if and only if φ is true at $\langle t, w' \rangle$ for all $w' \in W$ such that $w \approx_t w'$.

The resulting theory generates some validities arising from the assumption that the worlds share a common temporal ordering. Formulas (2) and (3) are two such validities, corresponding to the principle that one world has a first moment if and only if all worlds do.

$$(2) \quad P[\varphi \vee \neg\varphi] \leftrightarrow \Box P[\varphi \vee \neg\varphi]$$

$$(3) \quad H[\varphi \wedge \neg\varphi] \leftrightarrow \Box H[\varphi \wedge \neg\varphi]$$

In case $\approx_t$ is the universal relation for every t (or the relations $\approx_t$ are simply omitted from the satisfaction conditions) there are other validities, such as (4) and (5).

$$(4) \quad P\Box\varphi \rightarrow \Box P\varphi$$

$$(5) \quad F\Box\varphi \rightarrow \Box F\varphi$$

As far as I know, the general problem of axiomatising these logics has not been solved. But I'm not sure that it is worth doing, except as an exercise. The completeness proofs should not be difficult, using Gabbay's techniques (described in Section 4, below). and these logics do not seem particularly interesting from a philosophical point of view.

But a more interesting case is near to hand. The tendency we have noted to bring Carnap's metalinguistic notion of necessity down to earth has made room for the reintroduction of one of the most important notions of necessity: practical necessity, or historical necessity.⁷ This is the sort of necessity that figures in Aristotle's discussion of the Sea Battle (*De Int.* 18^b25–19^b4), and that arises when free will is debated. It also seems to be an important background notion in practical reasoning. Jonathan Edwards, in his usual lucid way, gives a very clear statement of the matter.

Philosophical necessity is really nothing else than the full and fixed connection between the things signified by the subject and predicate of a proposition, which affirms something to be true [This connection] may be fixed and made certain, because the existence of that thing is already come to pass; and either

⁷I am not sure if there are personal, or relational varieties of inevitability; it seems a bit peculiar to my ear to speak of an accident John caused as inevitable for Mary, but not inevitable for John. If there are such sorts of inevitability I mean to exclude them, and to speak only of impersonal inevitability. Thus 'inevitable' does not belong to the same modal family as 'able', since the latter is personal.

now is, or has been; and so has as it were made sure of existence. And therefore, the proposition which affirms present and past existence of it, may be this means be made certain, and necessarily and unalterably true; the past event has fixed and decided that matter, as to its existence; and has made it impossible but that existence should be truly predicted of it. Thus the existence of whatever is already come to pass, is not become necessity; 'tis become impossible it should be otherwise than true, that such a thing has been. [Edwards, 1957, pp. 152–3]

Historical necessity can be fitted into the $T \times W$ framework; it is merely a matter of adjusting the relations $\approx_t$ so that if $w \approx_t w'$, then w and w' share the same past up to and including t . So for $t' < t$, atomic formulas must be treated the same way in w and w' . Furthermore, we have to stipulate that historical possibilities diminish monotonically with the passage of time: if $t < t'$, then $\{w' : w \approx_{t'} w'\} \subseteq \{w' : w \approx_t w'\}$. This interaction between time and relative necessity creates distinctive validities, such as (6) and (7).

(6) $\varphi \leftrightarrow \Box\varphi$, if φ contains no occurrences of F .

(7) $P\Box\varphi \rightarrow \Box P\varphi$

Formula (8), on the other hand, is clearly invalid.

(8) $\Box Pp \rightarrow P\Box p$

These correspond to rather natural intuitions relating the flow of time to the loss of possibilities.

There is another way of representing historical necessity, which perhaps will seem less straightforward to logicians steeped in possible worlds. Time can be treated as non-linear (branching only towards the future), and worlds represented as branches on the resulting ordered structure. This corresponds very closely to the $T \times W$ account: (6) and (7) remain valid, and (8) invalid. But the validities are not the same. This matter will be taken up below, in Section 4.

So much for necessity; I will deal more briefly with 'ought' and conditionals.

As Aristotle points out, we don't deliberate about just anything; in particular, we deliberate only about what is in our power to determine. [Ne 1112^a 19f.] But the past, and the instantaneous present, are not in our power: deliberation is confined to future alternatives.

This suggests that deontic logic, insofar as it investigates practical oughts, should identify its possibilities with the ones of historical necessity. Unfortunately, this conception played little or no role in the early interpretation of deontic logic; those who developed the deontic applications of possible worlds semantics seemed to think of deontic possibilities ahistorically, as

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