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PHILOSOPHICAL PERSPECTIVES ON QUANTIFICATION IN TENSE AND MODAL LOGIC

INTRODUCTION

The trouble with modal logic, according to its critics, is quantification into modal contexts—i.e. *de re* modality. For on the basis of such quantification, it is claimed, essentialism ensues, and perhaps a bloated universe of *possibilia* as well. The essentialism is avoidable, these critics will agree, but only by turning to a Platonic realm of individual concepts whose existence is no less dubious or problematic than mere *possibilia*. Moreover, basing one's semantics on individual concepts, it is claimed, would in effect render all identity statements containing only proper names either necessarily true or necessarily false—i.e. there would then be no contingent identity statements containing only proper names.

None of these claims is true quite as it stands, however; and in what follows we shall attempt to separate the chaff from the grain by examining the semantics of (first-order) quantified modal logic in the context of different philosophical theories. Beginning with the primary semantics of logical necessity and the philosophical context of logical atomism, for example, we will see that essentialism not only does not ensue but is actually rejected in that context by the validation of the modal thesis of anti-essentialism, and that in consequence all *de re* modalities are reducible to *de dicto* modalities.

Opposed to logical atomism, but on a par with it in its referential interpretation of quantifiers and proper names, is Kripke's semantics for what he properly calls metaphysical necessity. Unlike the primary semantics of logical necessity, in other words, Kripke's semantics for metaphysical necessity is in direct conflict with some of the basic assumptions of logical atomism; and in the form which that conflict takes, which we shall refer to here as the form of a *secondary* semantics for necessity, Kripke's semantics amounts to the initial step toward a proper formulation of Aristotelian essentialism. (A secondary semantics for necessity stands to the primary semantics in essentially the same way that non-standard models for second-order logic stand to standard models.) The problem with this initial step toward Aristotelian essentialism, however, is the problem of all secondary semantics; viz. that of its objective, as opposed to its merely formal, significance—a problem which applies all the more so to Kripke's deepening of his formal semantics by the introduction of an accessibility relation between possible worlds. This, in fact, is the real problem of essentialism.

There are no individual concepts, it will be noted in what follows, in either logical atomism or Kripke's implicit philosophical semantics, and yet in both contexts proper names are rigid designators; that is, in both there can be no contingent identity statements containing only proper names. One need not, accordingly, turn to a Platonic realm of individual concepts in order to achieve this result. Indeed, quite the opposite is the case. That is, it has in fact been for the defence of contingent identity, and not its rejection, that philosophical logicians have turned to a Platonic realm of individual concepts, since, on this view, it is only through the mere coincidence of the denotations of the individual concepts expressed by proper names that an identity statement containing those names can be contingent. Moreover, unless such a Platonic realm is taken as the intensional counterpart of logical atomism (a marriage of dubious coherence), it will not validate the modal thesis of anti-essentialism. That is, one can in fact base a Platonic or logical essentialism—which is not the same thing at all as Aristotelian essentialism—upon such a realm. However, under suitable assumptions, essentialism can also be avoided in such a realm; or rather it can in the weaker sense in which, given these assumptions, all *de re* modalities are reducible to *de dicto* modalities.

Besides the Platonic view of intensionality, on the other hand, there is also a socio-biologically based conceptualist view according to which concepts are not independently existing Platonic forms but cognitive capacities or related structures of the human mind whose realisation in thought is what informs a mental act with a predicable or referential nature. This view, it will be seen, provides an account in which there can be contingent identity statements, but not such as to depend on the coincidence of individual concepts in the platonic sense. Such a conceptualist view will also provide a philosophical foundation for quantified tense logic and paradigmatic analyses thereby of metaphysical modalities in terms of time and causation. The problem of the objective significance of the secondary semantics for the analysed modalities, in other words, is completely resolved on the basis of the nature of time, local or cosmic. The related problem of a possible ontological commitment to *possibilia*, moreover, is in that case only the problem of how conceptualism can account for direct references to past or future objects.

1 THE PRIMARY SEMANTICS OF LOGICAL NECESSITY

We begin by describing what we take to be the primary semantics of logical necessity. Our terminology will proceed as a natural extension of the syntax and semantics of standard first-order logic with identity. Initially, we shall assume that the only singular terms are individual variables. As primitive *logical constants* we take $\rightarrow$, $\neg$, $\forall$, $=$, and $\Box$ for the material conditional sign, the negation sign, the universal quantifier, the identity sign and the

necessity sign, respectively. (The conjunction, disjunction, biconditional, existential quantifier and possibility signs— $\wedge, \vee, \leftrightarrow, \exists$ and $\Diamond$, respectively—are understood to be defined in the usual way as metalinguistic abbreviatory devices.) The only non-logical or *descriptive constants* at this point are predicates of arbitrary (finite) degree. We call a set of such predicates a *language* and understand the well-formed formulas (*wffs*) of a language to be defined in the usual way.

A *model* $\mathfrak{A}$ indexed by a language L , or for brevity, an *L-model*, is a structure of the form $\langle D, R \rangle$, where D , the *universe* of the model, is a non-empty set and R is a function with L as domain and such that for each positive integer n and each n -place predicate F^n in L , $R(F^n) \subseteq D^n$, i.e. $R(F^n)$ is a set of n -tuples of members of D . An *assignment in D* is a function A with the set of individual variables as domain and such that $A(x) \in D$, for each variable x . Where $d \in D$, we understand $A(d/x)$ to be that assignment in D which is exactly like A except for its assigning d to x . The *satisfaction* of a wff φ of L in $\mathfrak{A}$ by an assignment A in D , in symbols $\mathfrak{A}, A \models \varphi$, is recursively defined as follows:

1. $\mathfrak{A}, A \models (x = y)$ iff $A(x) = A(y)$;
2. $\mathfrak{A}, A \models P^n(x_1, \dots, x_n)$ iff $\langle A(x_1), \dots, A(x_n) \rangle \in R(P^n)$;
3. $\mathfrak{A}, A \models \neg\varphi$ iff $\mathfrak{A}, A \not\models \varphi$;
4. $\mathfrak{A}, A \models (\varphi \rightarrow \psi)$ iff either $\mathfrak{A}, A \not\models \varphi$ or $\mathfrak{A}, A \models \psi$;
5. $\mathfrak{A}, A \models \forall x\varphi$ iff for all $d \in D$, $\mathfrak{A}, A(d/x) \models \varphi$; and
6. $\mathfrak{A}, A \models \Box\varphi$ iff for all R' , if $\langle D, R' \rangle$ is an L -model, then $\langle D, R' \rangle, A \models \varphi$.

The *truth* of a wff in a model (indexed by a language suitable to that wff) is as usual the satisfaction of the wff by every assignment in the universe of the model. *Logical truth* is then truth in every model (indexed by any appropriate language). One or another version of this primary semantics for logical necessity, it should be noted, occurs in [Carnap, 1946]; [Kanger, 1957]; [Beth, 1960] and [Montague, 1960].

2 LOGICAL ATOMISM AND QUANTIFIED MODAL LOGIC

These definitions, as already indicated, are extensions of essentially the same semantical concepts as defined for the modal free wffs of standard first-order predicate logic with identity. The clause for the necessity operator has a particularly natural motivation within the framework of logical atomism. In such a framework, a model $\langle D, R \rangle$ for a language L represents a *possible world* of a *logical space* based upon (1) D as the universe of objects of that space and (2) L as the predicates characterising the *atomic states of affairs*

of that space. So based, in other words, a logical space consists of the totality of atomic states of affairs all the constituents of which are in D and the characterising predicates of which are in L . A possible world of such a logical space then amounts in effect to a partitioning of the atomic states of affairs of that space into two cells: those that obtain in the world in question and those that do not.

Every model, it is clear, determines both a unique logical space (since it specifies both a domain and a language) and a possible world of that space. In this regard, the clause for the necessity operator in the above definition of satisfaction is the natural extension of the standard definition and interprets that operator as ranging over *all* the possible worlds (models) of the logical space to which the given one belongs.

Now it may be objected that logical atomism is an inappropriate framework upon which to base a system of quantified modal logic; for if any framework is a paradigm of anti-essentialism, it is logical atomism. The objection is void, however, since in fact the above semantics provides the clearest validation of the modal thesis of anti-essentialism. Quantified modal logic, in other words, does not in itself commit one to any non-trivial form of essentialism (cf. [Parsons, 1969]).

The general idea of the modal thesis of anti-essentialism is that if a predicate expression or open wff φ *can be true of* some individuals in a given universe (satisfying a given identity-difference condition with respect to the variables free in φ), then φ *can be true of* any individuals in that universe (satisfying the same identity-difference conditions). In other words, no conditions are essential to some individuals that are not essential to all, which is as it should be if necessity means logical necessity.

The restriction to identity-difference conditions mentioned (parenthetically) above can be dropped, it should be noted, if nested quantifiers are interpreted exclusively and not (as we have done) inclusively where, e.g. it is allowed that the value of y in $\forall x \exists y \varphi(x, y)$ can be the same as the value of x . (Cf. [Hintikka, 1956] for a development of the exclusive interpretation.) Indeed, as Hintikka has shown, when nested quantifiers are interpreted exclusively, identity and difference wffs are superfluous—which is especially apropos of logical atomism where an identity wff does not represent an atomic state of affairs. (Cf. Wittgenstein's *Tractatus Logico-Philosophicus* 5.532–5.53 and [Cocchiarella, 1975a, Section V].)

Retaining the inclusive interpretation and identity as primitive, however, an *identity-difference condition* for distinct individual variables $x_1, \dots, x_n$ is a conjunction of one each but not both of the wffs $(x_i = x_j)$ or $(x_i \neq x_j)$, for all i, j such that $1 \leq i < j \leq n$. It is clear of course that such a conjunction specifies a complete identity-difference condition for the variables $x_1, \dots, x_n$. Since there are only a finite number of non-equivalent such conditions for $x_1, \dots, x_n$, moreover, we understand $ID_j(x_1, \dots, x_n)$, relative to an assumed ordering of such non-equivalent conjunctions, to be the j th

conjunction in the ordering. *The modal thesis of anti-essentialism* may now be stated as the thesis that every wff of the form

$$\begin{aligned} & \exists x_1 \dots \exists x_n (ID_j(x_1, \dots, x_n) \wedge \Diamond \varphi) \\ & \rightarrow \forall x_1 \dots \forall x_n (ID_j(x_1, \dots, x_n) \rightarrow \Diamond \varphi) \end{aligned}$$

is to be logically true, where $x_1, \dots, x_n$ are all the distinct individual variables occurring free in φ . (Where $n = 0$, the above wff is understood to be just $(\Diamond \varphi \rightarrow \Diamond \varphi)$; and where $n = 1$, it is understood to be just $\exists x \Diamond \varphi \rightarrow \forall x \Diamond \varphi$.) The validation of the thesis in our present semantics is easily seen to be a consequence of the following lemma (whose proof is by a simple induction on the wffs of L).

LEMMA If L is a language, $\mathfrak{A}, \mathfrak{B}$ are L-models, and h is an isomorphism of $\mathfrak{A}$ with $\mathfrak{B}$, then for all wffs φ of L and all assignments A in the universe of $\mathfrak{A}$, $\mathfrak{A}, A \models \varphi$ iff $\mathfrak{B}, A/h \models \varphi$.

One of the nice consequences of the modal thesis of anti-essentialism in the present semantics, it should be noted, is the reduction of all *de re* wffs to *de dicto* wffs. (A *de re* wff is one in which some individual variable has a free occurrence in a subwff of the form $\Box \psi$. A *de dicto* wff is a wff that is not *de re*.) Naturally, such a consequence is a further sign that all is well with our association of the present semantics with logical atomism.

THEOREM (De Re Elimination Theorem) For each *de re* wff φ , there is a *de dicto* wff ψ such that $(\varphi \leftrightarrow \psi)$ is logically true.¹

These niceties aside, however, another result of the present semantics is its essential incompleteness with respect to any language containing at least one relational predicate. (It is not only complete but even decidable when restricted to monadic wffs—of which more anon.) The incompleteness is easily seen to follow from the following lemma and the well-known fact that the modal free non-logical truths of a language containing at least one relational predicate is not recursively enumerable (cf. [Cocchiarella, 1975b]). (It is also for the statement of the infinity condition of this lemma that a relational predicate is needed.)

LEMMA If ψ is a sentence which is satisfiable, but only in an infinite model, and φ is a modal and identity-free sentence, then $(\psi \rightarrow \neg \Box \varphi)$ is logically true iff φ is not logically true.

¹A proof of this theorem can be found in [McKay, 1975]. Briefly, where $x_1, \dots, x_n$ are all the distinct individual variables occurring free in φ and $ID_1(x_1, \dots, x_n), \dots, ID_k(x_1, \dots, x_n)$ are all the non-equivalent identity-difference conditions for $x_1, \dots, x_n$, then the equivalence in question can be shown if ψ is obtained from φ by replacing each subwff $\Box \chi$ of φ by:

$$\begin{aligned} & [ID_1(x_1, \dots, x_n) \wedge \Box \forall x_1 \dots \forall x_n (ID_1(x_1, \dots, x_n) \rightarrow \chi)] \vee \dots \\ & \vee [ID_k(x_1, \dots, x_n) \wedge \Box \forall x_1 \dots \forall x_n (ID_k(x_1, \dots, x_n) \rightarrow \chi)]. \end{aligned}$$

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