

CHAPTER FOUR

VARIATIONS ON THE NON-STATEMENT VIEW OF SCIENCE¹

1. INTRODUCTION

In the Introduction I have briefly mentioned the basic differences between the statement and the non-statement approaches. The advocates of the statement approach depict scientific theories as axiomatised deductively closed sets of sentences within some appropriate syntactic system, and discuss the "empirical interpretations" of these theories in terms of some set of correspondence rules or bridge principles. The defenders of the non-statement approach, on the other hand, do not view the formal formulation of scientific theories in some appropriate language as the most useful characterisation of "theories". Rather, they depict these "theories" in terms of sets of mathematical structures that are the models of the theory in question.

In what follows I shall discuss a few of the main non-statement programmes offered by current philosophy of science. This is necessary since these programmes acknowledge the role of mathematical models in science's processes, which is of paramount importance to my model-theoretic view of science. Any kind of comparison is always limited and I do not claim aspects of the different accounts depicted, discussed, and compared here to be "exactly" the same. Rather I view these to be, at most "comparable" in a loose sense of the word, and offer these discussions more for clarification purposes than for making any naive claims of reducing any of these views to any other.

2. PATRICK SUPPES'S SET-THEORETIC APPROACH TO SCIENCE

Suppes offers one of the first viable alternatives to the "received (statement) view" of scientific theories and so brings about a radical turn in philosophy of science. Other than the structuralists who stress the use of formal semantics and meta science to appeal to the structural aspects of theories, Suppes finds the axioms of set-theory sufficient, and claims mathematics, rather than meta-mathematics to be the language of science.²

The class of structures (systems) that Suppes considers is described by giving one "generic" structure, with parameters, which can be specified to deliver all the systems in the class. Suppes (in Morgenbesser, 1967, p.60) acknowledges this when he points out that one of the simplest ways in which to provide an extrinsic

characterisation of a theory is to define the intended class of models of the theory; and then asking if the theory can be axiomatised, merely comes down to asking if a set of axioms can be stated such that the models of these axioms are precisely the models in the defined class. He (*ibid.*, pp.61, 62) remarks however that

... the problem of intrinsic axiomatisation of a scientific theory is more complicated and considerably more subtle Fortunately, it is precisely by explicit consideration of the class of models of the theory that the problem can be put into proper perspective and formulated in a fashion that makes possible consideration of its exact solution.

I agree, and, in model-theoretic terms, moreover, such consideration of the class of models of a given theory shows the continuous character of science (see Chapters 2 and 5).

Suppes addresses the philosophically problematic relations between empirical systems and theories (i.e. my "second set" of interpretational relations) in terms of a hierarchy of models that focuses on the complex nature of the experimental process.³ He (Suppes, 1954, p.243) already points out very early on in his work that progress in foundational studies of philosophy of science requires distinction between theory and experiment, since the reconstruction of the experimental stage of science is rather more problematic in comparison to the theoretical stage which may be axiomatised "quite easily" with the help of set-theoretic predicates. He (*ibid.*, p.246) wants to provide philosophy of science with "... a kind of algebra of experimentally realisable operations and relations" and emphasises that discussion of the empirical interpretations of the primitive notions for certain defined notions of some empirical theory imply interpretations of quantitative notions, which necessitates some systematic theory of measurement.

He is not interested in the classic notion of absolute objective truth, nor is he interested in the kind of framework offered by instrumentalists, rather he wants to speak more pragmatically about truth in terms of modern statistical decision theory. Thus, one of the most important issues in Suppes's philosophy of science is the emphasis he puts on the "experimental stage" of science.⁴

... the point of a theory of measurement is to lay bare the structure of a collection of empirical relations which may be used to measure the characteristics of empirical phenomena corresponding to the concept. Why a collection of relations? From an abstract standpoint a set of empirical data consists of a collection of relations between specified objects. For example, data on the relative weights of a set of physical objects are easily represented by an ordering relation on the set; additional data, and a fortiori an additional relation, are needed to yield a satisfactory quantitative measurement of the masses of objects" (Scott & Suppes, 1958, p.113).

Thus, as far as the co-ordinating principles or bridge principles of the statement approach are concerned, Suppes stresses (in Morgenbesser, 1967, p.62) that the practice of testing scientific theories is a much more complicated issue than is implied by the usual comment about these issues.⁵ (See Chapter 3.) I agree with this, but I do not choose to turn to statistical methodology to examine these relations that Suppes (in Morgenbesser (1967), and also in Suppes (1969), Suppes (1989), and Suppes (1993)) insists on. For the purposes of a realist philosophy of science, it is sufficient — and a more philosophically challenging prospect, I might add — to analyse the various

semantic model-theoretic relations involved.

I am, of course, not denying the use of statistical methodology to clarify and determine as precisely as possible the chances of a theory's models having connections to some systems in reality. I merely prefer a model-theoretic analysis of theories, combined with a(n adapted) Van Fraassenian notion of empirical adequacy and non-monotonic preferential analyses of empirical over-determination instead of the mathematical tools of statistics being employed to answer inherently *philosophical* questions about possible relations between science and reality. Also, I agree with Wójcicki (in Humphreys, 1994, p.130) that Suppes's set-theoretical position may reduce philosophy of science to no more than "selected problems of metamathematics". Wójcicki also remarks:

Needless to say, as long as an empirical theory is not provided with any factual interpretation, it remains merely a certain formal system. But ... one may wonder whether the *differentia specifica* allowing us to tell an empirical theory from a piece of pure mathematics does not consist in the fact that the former has some intended empirical applications.

This was essentially Adams's (1959) argument, as Suppes also points out. He started the idea of an empirical theory consisting of two classes of structures, the one the class of all the theory's realisations and the other the class of all *intended applications* which is a class of empirical structures (physical systems) of which the theory is expected to be true. Adams also pointed out that not every intended application necessarily has to be a realisation of the theory. Sneed (see Section 4.3) modified these notions in the sense that he requires that no component of an intended application be T-theoretical, while Adams saw the intended applications of structures of the same set-theoretic type as the realisations of the theory itself, which is close to how model-theoretic realism also portrays it.

Suppes and Dana Scott in their article *Foundational aspects of theories of measurement* (1958) ground the foundational analysis of measurement in general model theory. Suppes (in Morgenbesser, 1967, p.58) points out that the essential characteristic of a theory of measurement is that it can study (in a precise way) the transformation or development of "qualitative observations" into the "quantitative assertions" characteristic of the more theoretical stages of the scientific process. He approaches this problem in terms of representation theorems, mainly because he views the models of the theory and the models of the data (see below) to be of different logical types. He (ibid., p.58) writes:

Given an axiomatised theory of measurement of some empirical quantity such as mass, distance, or force, the mathematical task is to prove a representation theorem for *models* of the theory which establishes, roughly speaking, that any empirical model is isomorphic to some numerical model of the theory. The existence of this isomorphism between models justifies the application of numbers to things. ... What we can do is to show that the structure of a set of phenomena under certain empirical operations is the same as the structure of some set of numbers under arithmetical operations and relations.⁶

Although I would discuss the above in terms of empirical reducts and empirical models and although model-theoretic realism does not need representation theorems, this is essentially (as far as these views are comparable) my view of the "verification"

of the models of scientific theories too. In the model theoretic context it is however not necessary to use a separate language — from the one talking about the content of a theory's interpretative models — to talk about the empirical models of theories⁷ — although of course it can be done, and then Suppes's use of representation theorems will become applicable too.⁸

Suppes (1954, p.245) sets out the various stages of formulating a set-theoretic predicate for (or axiomatising) a particular branch of empirical science as follows:

- In the beginning some kind of statement of what other theories are assumed (e.g. in axiomatising rigid body mechanics, one would assume the standard branches of mathematics and particle mechanics) is needed.
- Then the “primitive” notions of the theory are listed, and their set-theoretic nature (in particle mechanics, notions like “set of particles”, the “interval of elapsed time”, the “position function”, the “mass function”, and so on) is indicated.
- The set-theoretic definition can then be completed by listing the axioms which have to be satisfied, because one will then be able to examine the deductive consequences of the definition.

Obviously one of the main tasks here is to rationally reconstruct within set theory the standard theorems of the branch being studied. One will also then be able to ask some of the questions of modern mathematics that have obvious implications for the structure of empirical theories, such as questions concerning the formulation of representation theorems which may be linked for instance to studies directed towards the problem of reduction between theories⁹. Then finally, one will be in a position to give an empirical interpretation of the axiomatised theory, which will have to take the complexity of the entire experimental enterprise into account.

Suppes thus articulates a more complex stratified view of the relations between models of theories and systems in reality, than I do in this text. However, I too stress the elaborate sophistication of the manoeuvres needed to find the possible links between particular systems in reality and certain models of a theory being examined at a given time, although he (Suppes, 1989, p.25) wants to show that the study of the relations between (empirical) theories and their data demands a study in terms of a hierarchy of models of different logical type¹⁰. He (Suppes, 1960, p.297) stresses the clarifying role of the set-theoretical notion of model in experimental design and the analysis of data:

The maddeningly diverse and complex experience which constitutes an experiment is not the entity which is directly compared with a model of a theory. Drastic assumptions of all sorts are made in reducing the experimental experience ... to a simple entity ready for comparison with a model of the theory.

Suppes sees the empirical relation between an interpretation (of a given theory or class of systems) and a system in reality as a highly articulated, composite relation, with an articulation which depends upon the experimental or observational situation in question. Suppes (1989, p.25) explains that

Theoretical notions are used in the theory which have no direct observable analogue in the experimental data. In addition, it is common for models of a theory to contain continuous functions or infinite sequences although the confirming data are highly discrete and finitistic in character. ... Corresponding to possible realisations of the theory, I introduce possible realisations of the data. As should be apparent, from a logical standpoint possible realisations of data are defined in just the same way as possible realisations of the theory being tested by the experiment from which the data come. The precise definition of models of the data for any given experiment requires that there be a theory of the data in the sense of the experimental procedure, as well as in the ordinary sense of the empirical theory of the phenomena being studied.

He (ibid.) gives two reasons why a possible realisation of a theory cannot be a possible realisation of its data: no actual experiment can include an infinite number of discrete trials, and the parameter of the experiment is not directly observable and is not part of the recorded data. In other words, models of the experiment and models of the theory are of a different logical type. Note that although in a model-theoretic approach none of this is denied, this still does not necessarily imply that different languages are needed to talk about the content of the interpretative models and empirical models.

I thus agree with his hierarchical view, but for my (semantic realist) purposes I collapse this complex relationship to a much simpler relation, indicated by "empirical adequacy" (in my terms). This simple relation results in fitting the empirical data — however elaborately extracted from the physical system¹¹, and subsequently formulated conceptually, i.e. mathematically — into the relevant interpretative model of the theory in question (i.e. the relevant conceptualisation of the empirical data in question is shown to be isomorphic to a substructure (i.e. the empirical reduct) of the interpretative model in question). A simple example: Observations over time deliver 113 different spatial positions $(x_1, y_1), (x_2, y_2), \dots, (x_{113}, y_{113})$ for the planet Neptune in the x-y-plane (of the planets in our solar system) of a coordinate system centred on the sun. These are the data. All 113 points *lie on the* (near-) ellipse with its uncountably many points which is the interpretative model of the orbit of Neptune (which in its turn is part of the interpretative model of the solar system, in which the — Newtonian or Einsteinian — theory of our solar system is true). What is suppressed or collapsed here is the process of distilling the data $(x_1, y_1), (x_2, y_2), \dots, (x_{113}, y_{113})$, which is a process which involves theories, models, and practices relating to telescopes, the human eye and visual system, light, movement of the earth, clocks, and so on, and so on.

Suppes (ibid., pp.27-29) argues that the fundamental theory and descriptions of apparatus are two extremes of hierarchy — in between are the models of the theory, the models of the experiment, and the models of the data. He (in Morgenbesser, 1967, p.62) emphasises over and over again that to be able to "connect" experimental data to a relevant theory, the data have to be put through a "conceptual grinder", which refers to this conceptual hierarchy he sets out from the "raw" observations to the final "fundamental" scientific theory. The theory of the experiment is the definition of all the possible realisations of the theory that is the first "step down from the abstract level" (Suppes, 1989, p.28) of the fundamental theory. A possible realisation of the theory of the experiment is a model of the theory if the experimental conditions are satisfied. Models of the experiment represent experimental data in canonical form, but when is a possible realisation of the data a model of the data? Suppes (ibid., p.29) remarks

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