CHAPTER 2

THE STRUCTURE OF SPACETIME THEORIES

Having examined the general features of Cartesian space and motion, and Newton's famous criticism of this theory, we can now proceed to the analysis of the underlying theoretical, or structural, components of the theory of space and time presupposed in Newton's argument. This investigation will not only determine the extent of the deficiencies, if any, in Descartes' system, but it will also outline the necessary structural or theoretical remedies necessary to cure the Cartesian theory of its presumed deficiencies

2.1. Newtonian Space and Time

Rather than entertain suspect relational theories, Newton insists that a complete and comprehensive analysis of the phenomenon of motion must invoke the existence of absolute space and time. In *De gravitatione,* he provides a brief synopsis of this theory: "[space] is eternal, infinite, uncreated, uniform throughout, not in the least mobile, nor capable of inducing change of motion in bodies or change of thought in mind. . . (Newton 1962a, 145)." In the modern parlance, Newton's appeal to the uniformity of space would be construed in terms of symmetry requirements. That is, Newtonian spacetime is symmetric under spatial displacements (homogeneity) and reorientations (isotropy); or, quite simply, that all places and directions in absolute space are inherently similar in nature (more on this latter).[1] In addition to the conception of space as infinite, immovable, and incapable of effecting change in the motions of material bodies (all notions which would be questioned in the 20th Century), Newton also declares that the "moment of duration" is the same for all individual parts of space, or, in other words, that the totality of spaces experience an identical moment of temporal passage: "we do not ascribe various durations to the different parts of space, but say they all endure together. The moment of duration is the same at Rome and at London, on the Earth and on the stars, and throughout all the heavens (Newton 1962a, 137)." With respect to the material aspects of time, he states: "absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external (Newton 1962b, 6)." The uniformity of space is matched, consequently, by the regular and homogeneous passage of temporal instants. On this view, moreover, material objects cannot causally influence temporal succession; space and time are the arena, and not active participants, in the "drama" of material interaction. In essence, Newton envisions absolute space and absolute time as separate from the material contents of the world. As for the obvious question, "What are space and time according to Newton?" (substance, property, or something else?), we will return to this point in a later chapter.

Newton's picture of the nature of space and time is often presented in the modern scientific/geometric formalism as a Newtonian, or Full-Newtonian, spacetime (see, for example; Stein 1967, 174-176). That is, the apparatus of differential geometry is employed in an attempt to articulate the physical (and metaphysical) import of Newton's theory. One must not embrace these techniques unconditionally, though; since they can as easily detract from, as assist in, the analysis of Newton's concept of space and time. More specifically, the technical details of the modern spacetime models might obscure and render unintelligible many intended conceptual facets of Newton's natural philosophy. Nevertheless, if employed judiciously, such techniques can offer valuable insights into the underlying structures of spacetime theories.

Overall, if we envision spacetime as the four-dimensional totality of physical events, then Newtonian spacetime splits this structure into a three-dimensional "space" and a one-dimensional "time" each possessing a Euclidean metric (or distance function). This view of the physical world can be summarized as a "space plus time," $E^3 \times E^1$, or "enduring space". Thus, the specification of a body's position in the universe requires a pair of coordinate values: A fix of its spatial location and its moment in time. Since Newtonian spacetime uniquely separates all events into simultaneity classes (see Figure 1),[2] Newton's concept of space is represented by a series of three-dimensional planes or "slices," with each plane comprising an entire collection of simultaneous events. Hence, enduring space can be pictured as an infinite series of spatial planes (one plane for each simultaneous collection). Newtonian time is just the unique order and distance between these "planes of simultaneity" (via the operation of the temporal metric). Given this structure, all the events located on a slice A bear a fixed temporal distance between the events located on a slice B.

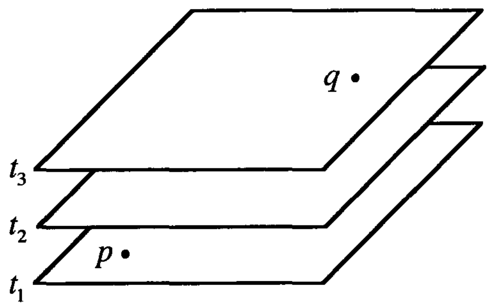

Figure 1. The planes of simultaneity represent the Euclidean three-dimensional space at each instant (in this diagram, p occurs at time t_1, and q at time t_3).

With respect to the geometry on each spatial slice, Newton insists that, "the positions, distances and local motions of bodies are to be referred to the parts of space (Newton 1962a, 137)," a claim that can be interpreted as an appeal to an intrinsic Euclidean spatial "metric" or distance function. In order to determine the actual or absolute (as opposed to relative) spatial lengths between objects and events, Newtonian spacetime thus requires that each point on the planes of simultaneity possess a "built-in" distance function. Essentially, a metric is a function which calculates the distance along a curve by summing over the infinitesimal lengths of the segments defined at each of the curve's points.[3] In Cartesian coordinates, these numbers are often expressed in the familiar Euclidean form $ds = \sqrt{dx^i dx^i}$, where ds signifies the (infinitesimal) length of the three-dimensional vector that connects the point x^i with the point $x^i + dx^i$. Furthermore, since Newton maintains that, "absolute space, in its own nature, without relation to anything external, remains always similar and immovable (Newton 1962b, 6)," the distance between events on each flat simultaneity plane is not conditioned by, or subject to, the altering and distorting influences of material bodies. Unlike the spacetime of General Relativity, where the distribution of matter actually determines the character of the metric geometry, the Newtonian metric is independent of the material objects and observers located in the spacetime.

As previously mentioned, however, one must proceed carefully when interpreting Newton's theory with the aid of modern geometric techniques; since such methods can present a somewhat misleading and distorted picture of Newton's actual intentions if not interpreted carefully. For instance, one should not construe Figure 1 as representing spacetime by a series of instantaneous spatial slices, since this is not only an incorrect interpretation of the figure, but of Newton's concept of substantival (absolute) space, as well. Figure 1 depicts $E^3 \times \{t\}$, for each specific moment of time t (i.e., t_1, t_2, t_3, etc.), and not $E^3 \times E^1$. The structure $E^3 \times \{t\}$ is useful for a graphic presentation of Newton's intuitions about the "moment of duration", but it can easily mislead one into thinking that $E^3 \times E^1$ is a union of individual spatial slices (which is, moreover, an misunderstanding of $E^3 \times \{t\}$). Consequently, Figure 1 would not seem to capture adequately Newton's overall concept of space as a persisting or enduring entity—it is a *single* thing which exists in or through time, not a collection of nearly identical things or spatial slices. His numerous claims on the status of absolute space, such as, "space is eternal in duration (Newton 1962a, 137)," would thus seem to corroborate the view that $E^3 \times E^1$ is the most natural modern-day geometric rendering of Newton's theory.

Notwithstanding these complications, however, the structure of Newtonian spacetime is generally quite useful in exhibiting other aspects of Newton's theory (as presented in the *De Gravitatione* argument). In particular, our four-dimensional approach can disclose the spacetime symmetries intrinsic to different methods of connecting or "stitching together" the instantaneous spatial slices. As explained, Newton obtains the velocity or directed speed of a body against the motionless backdrop of absolute space. Hence, his demand that spatial positions endure through time can be accommodated by isolating, or identifying within the larger structure $E^3 \times E^1$, the "fixed" points p of E^3 that form the set, $\{p\} \times E^1$. Each of the "lines" formed by these sets, $\{p\} \times E^1$, thus represents a point p of space that endures through time in Newton's sense, comprising a sort of spacetime "rigging" that cut the planes of simultaneity equidistantly in every slice.[4] Informally, the rigging can be pictured as a path or line which uniquely connects each spatial point with the same point on all the preceding and succeeding planes of simultaneity (see Figure 2). By way of this structural feature, one can meaningfully discuss in Newtonian spacetime whether or not a specific object occupies the same spatial location through time (since the rigging identifies the "same spatial location" on each spatial slice). Moreover, all objects (or events) in spacetime can be partitioned into one of two classes; those at rest with respect to absolute space (i.e., those that do not leave their spatial location), and those moving with respect to absolute space (i.e., those that do). As a result, all objects will possess "absolute velocities" and "absolute accelerations" relative to "motionless" absolute space. (The details of this process will be explained in the next section.) Moreover, it is important to note that our Newtonian spacetime is not equipped with an intrinsic geometrical structure which correlates or connects *in a unique manner* the one-dimensional time lines $\{p\} \times E^1$ with the flat Euclidean three-dimensional planes $E^3 \times \{t\}$. Put another way, many different sets of points $\{p\} \times E^1$ can be picked out in our Newtonian $E^3 \times E^1$ spacetime that stitch together, often rather differently, the $E^3 \times \{t\}$ spatial planes. A unique rigging thus cannot be derived from the structure and properties of the spatial slices and the temporal ordering: rather, a unique rigging must be incorporated into the our theory as an additional postulate, a fact that will assume importance below.

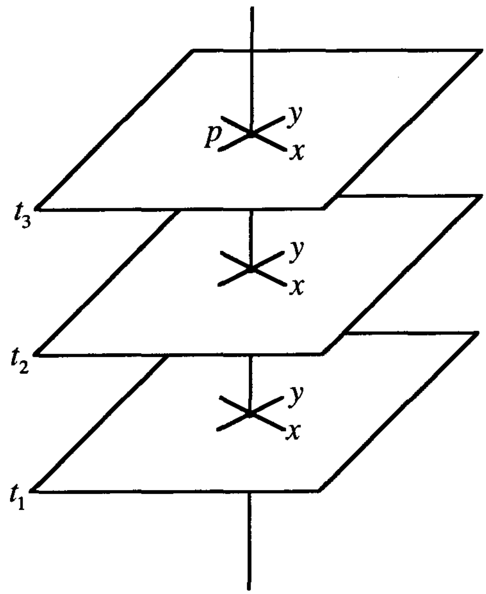

Figure 2. The spacetime rigging identifies the same spatial location p, which is the origin of a coordinate system (x, y), on each plane of simultaneity. The third spatial dimension of the coordinate frame has been repressed in this illustration.

Despite its great amount of structure, there remains a significant group of coordinate transformations that represent the inherent symmetries of our Newtonian spacetime. Briefly, there exists a class of coordinate structures or frames that can be characterized in terms of the given underlying structure of our Newtonian spacetime, and within this class of coordinates one can identify the coordinate transformations that "display" the inherent symmetries of that underlying spacetime structure. This interpretation of "symmetry" relies heavily on the concept of a "passive transformation" of coordinates, which is commonly understood as the functions that correlate

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Cartesian Spacetime
Descartes' Physics and the Relational Theory of Space
and Motion
Slowik, E.
2002, XII, 252 p., Hardcover
ISBN: 978-1-4020-0265-6