CHAPTER 5

QUANTITY OF MOTION: THE ORIGIN AND FUNCTION OF THE CARTESIAN CONSERVATION PRINCIPLE:

This chapter picks up the story, began in the previous two chapters, concerning the complex and varied role of Descartes' conservation law for the "quantity of motion" (size times speed), the numerical measure of "force" that, he believes, is conserved in all bodily interactions. After presenting the historical development of this concept in Descartes' work, this chapter will explore in more detail various problematical aspects of Descartes' conservation principle noted in chapter 3 and 4; namely, its largely neglected "dual role" as a measure of both durational motion and instantaneous "tendencies towards motion". Overall, an underlying non-local, or "holistic", element of quantity of motion (largely derived from his statics) will be revealed as central to a full understanding of the conservation principle's conceptual development and intended operation. This insight into the function of the Cartesian conservation principle will not only be of use in responding to some of the criticisms of Descartes' physics aimed at its overall consistency, such as Alan Nelson's recent allegations, but will also provide a potentially powerful means of constructing a successful Cartesian spacetime, as will be later demonstrated in Part III (chapter 6).

5.1. Quantity of Motion: The Contemporary Opinion.

While it is commonplace to praise Descartes for having brought conservation principles to the forefront of natural philosophy, his particular choice of a conservation principle of moving bodies has generally received a less enthusiastic welcome. That is, even though the fruitfulness of the conservation "idea" was quickly perceived (and can be said to largely justify, say, C. Truesdell's judgment that "[Descartes' physics] is the beginning of theory in the modern sense"[1]), it is nevertheless also true that Descartes' handling of his conserved "quantity of motion", or size × speed, was soon challenged by a host of natural philosophers, both Cartesian and non-Cartesian.[2] Much of the early criticism centered upon the apparent incompatibility of the Cartesian conservation principle, and its attendant natural laws, with the basic operation or structure of Descartes' matter-filled universe (or, plenum). This line of critical analysis is still pursued by many commentators; such as, A. Nelson, who has charged that the very concepts of Cartesian matter and motion preclude the successful establishment of Descartes' third natural law, and by implication, the conservation principle.[3]

One of these puzzles, to be more precise, is the apparent dichotomy in Descartes' handling of bodily motion. On the one hand, he stipulates that motion is merely the reciprocal translation of a body from the vicinity of its contiguous neighbors (Pr II 25), while, on the other hand, also insisting that God maintains an unchanging measure of motion in the universe (i.e., the conservation principle). But, how do these two descriptions, or hypotheses, interrelate? On the first hypothesis, the complexity of the entire range of plenum activity seems to be reduced to, and explained by, the *local* level of a relational change of "place". In contrast, the second hypothesis explains the local motions and collisions of bodies from the *global* perspective of a quasi-"property" conferred upon the plenum via God's "immutability" (Pr II 36): i.e., QM is a consequence of God's "sustaining act". In short, one approach is local, reductivist, and kinematic; while the other is global and metaphysical (or, non-reductive and non-kinematic). This underlying tension in Descartes' system can be discerned in the work of various commentators on Cartesian motion, although it is often not openly or deliberately acknowledged. In an influential essay by R. J. Blackwell, for instance, it is observed that "everything in the [Cartesian] universe other than extension is due to local motion."[4] In this same work, nevertheless, he concludes that "for Descartes the word 'conservation' [in conservation principle] refers more to something in God (i.e., immutability) than to anything properly inherent in the structure of the material universe (223)." Once again, how can motion be an entirely local affair, but also such that "if there were no God, the total quantity of motion may well not be conserved in the Cartesian universe (234)?"

Blackwell's analysis is typical of the work on Cartesian motion, and nicely demonstrates the subtle ontological and epistemological issues invariably raised by such investigations, both with respect to corporeal substance and God's sustaining act. This chapter will attempt to come to grips with at least one aspect of this complex metaphysical problem: the local versus the global, or reductivist versus non-reductivist, nature of Cartesian motion. Despite being long favored by commentators, as well as seemingly endorsed by numerous passages in Descartes' corpus, it will be argued that the local, reductivist reading just cannot make sense of the entire Cartesian conception of motion. A global, non-reductive, and somewhat "holistic" element pervades Descartes' thinking and approach—and the conservation of quantity of motion is the key to unlocking this unacknowledged aspect of his theory. In particular, Descartes' early work on statics will be of central importance for our investigation, since the quasi-holistic character of a system of bodies held in an equilibrium of forces, such as weights on a pulley or balance, retained a pervasive influence on his later approach to the problem of motion and force in the plenum. The blending of this statics outlook with his kinematical set of natural laws inevitably generated numerous difficulties and inconsistencies for Descartes' system, as will be in evidence throughout our investigation. Nevertheless, acknowledging the dichotomous nature of Descartes' conservation principle for the quantity of motion will be shown to be oddly useful in the debates surrounding some of the alleged difficulties with Cartesian natural philosophy—and, more importantly, it will have a corresponding beneficial effect on our primary goal of developing a consistent Cartesian spacetime.

(Henceforth, 'QM' will designate quantity of motion, and not the general notion of a conservation principle, which can take any number of forms different from "size times speed".)

This chapter will bring together many seemingly disparate issues. In section 2, we will examine the origins and structure of Descartes' universally conserved motive quantity. This analysis will help to clarify the ontological status and unifying importance of QM in his universe, as well as its dual instantaneous/durational and dual static/dynamic character. In section 3, the conservation law will be put to work in order to resolve a series of alleged difficulties with Cartesian matter in its plenum setting. The aforementioned interrelated themes of reduction and holism will be a principal component of this investigation, and will be shown to play an important part in the relationship between Descartes' problematic thesis of local, relative motion and his conserved force.

5.2. Quantity of Motion in Cartesian Natural Philosophy

As D. Garber has carefully documented, Descartes' conservation principle only reached its canonical formulation, in his *Principles of Philosophy* (1644), after a long process of development and gestation. In his earlier, suppressed *The World* (1633), Descartes claims that God "always conserves [in all of matter] the same amount of [motion] (AT XI 43)." But it is not clear, at least in this work, that QM (size × speed) is the "motion" conserved.[5] More than likely, the source of Descartes' interest in conserved properties of motion was his friend and mentor, I. Beeckman, who formulated an early version of the momentum law, or size × velocity.[6] Regardless of the source of his inspiration, however, Descartes' references to a conserved QM become more explicit by the late 1630's (e.g., see AT II 543), culminating in an unambiguous presentation in the *Principles:*

> [Motion] has a certain and determinate quantity, which we can understand easily to be able to remain always the same in the whole universe, even though it may change in its individual parts. That is why we might think that when one part of matter moves twice as fast as another which is twice as large, there is the same amount of motion in the smaller as in the larger (Pr II 36).

Of course, in this treatise, the conservation principle forms the foundation of the three laws of nature, which describe the basic motions of material bodies in the Cartesian plenum. As noted in chapter 3, QM is particularly conspicuous in the third natural law, and the accompanying seven collision rules which constitute its specific instances, since this law dictates that impacting bodies either retain all their motion or transfer as much motion as they lose (and thus conserve QM; Pr II 40). Descartes justifies the conservation principle through an appeal to God's "immutability," as he dubs it (Pr II 37). He states: "It seems obvious to me that it is none other than God himself, who created in the beginning both motion and rest in the whole of matter, and now preserves through his normal concourse alone the same amount of motion and rest as he placed in it at that time (Pr II 36)."

Yet, to a host of Cartesian scholars reared on the "a priori" epistemology of the *Meditations*, the form of Descartes' conservation law (for QM) must appear a rather unmotivated and problematic choice. Whereas other Cartesian hypotheses could conceivably have a rational foundation based on God's immutability (albeit improbably; e.g., extension as the sole attribute of matter, Pr II 11, and even the existence of a conserved property of motion), it is not at all obvious, in the a priori sense, why the totality of plenum motion must equal the size × speed of all plenum bodies. Why this particular quantity? Why not "size + speed", or "size × speed^2"? In short, Descartes' conservation principle incorporates two key assumptions which demand justification: first, the choice of simples or primitives ("size", "speed"), and second, the mathematical relationship that ties the primitives together (product function, " × "). As for the domain of primitives, Descartes reckons size and speed to be "modes" of extended substance (Pr II 27); which, in the *Principles* at least, are defined as the particular ways in which a corporeal body manifests its extension. Size and speed thus appear to fit into the Cartesian ontological scheme, since they are a natural correlate of his a priori identification of extension as matter's essential attribute. There are problems with this selection, however. Not only is Descartes' concept of "size" notoriously difficult to interpret, but other presumably basic properties of Cartesian matter and motion are excluded without explanation, such as the direction of motion (or, approximately, "determination").[7] A similar puzzle surrounds the choice of the product relationship, ×, to interrelate his simples. On the whole, it does not appear that any a priori justification can be given, based on God's immutability, for privileging the product function over the range of all possible quantitative relationships (e.g., addition, subtraction, division, etc.). This dilemma is nicely summarized by S. Nadler: "It is plausible to claim that [God's] immutability implies *some* conservation law. But that it implies the conservation of the total quantity of motion in the universe is questionable."[8]

If Descartes did not procure QM based on any sort of a priori reflection, on what grounds did he select this particular quantity (and from where did it originate)? The most plausible explanation is to assume that Descartes simply borrowed this quantity, which was well-known to his predecessors in both statics and impetus theory, and generalized it to cover all material interactions in his plenum. Descartes probably knew the many Scholastic theories grouped under the general title "impetus" from his early Jesuit training in natural philosophy. Recalling our discussion from chapter 3, impetus theories were developed as a sort of a forerunner to the modern concept of inertial motion, and Descartes' own laws of motion. A body's impetus was conceived as a force or quasi-property causally responsible for its "violent" motion away from its naturally resting terrestrial state (i.e., naturally at rest when confined to the surface of the earth). This force was often measured by the speed or velocity of a body and its quantity of matter, as in the case of John Buridan.[9] In statics, further quantities bearing a close resemblance to QM were the various dynamical renditions of the "law of the lever," later dubbed in their mature form, the principle of "virtual work." Statics is the branch of mechanics that examines the forces of bodies in equilibrium, such as weights suspended in a pulley or balance. Descartes was well-acquainted with the theoretical aspects of these types of simple machines, as is evident in several letters from the late 1630s. For example:

> To raise a 100-pound weight to a height of one foot twice is the same as raising a 200-pound weight to a height of one foot, and also the same as raising a 100-pound weight to the height of two feet. From this it clearly follows that . . . the force required to support it at a certain position and prevent it from falling, is to be measured at the beginning of the motion which the power supporting it must provide if it is going to raise it or if it is going to accompany its falls (AT II 229—see Figure 17).

The measured quantity in Descartes' rendition of the law of the lever incorporates only very small displacements (along with weight), thus rendering it roughly analogous to the later virtual work principle: "Note that I say 'begin to fall', and not simply 'fall', since it is only the beginning of the fall [of the weights] that we need to consider (AT II 233)."[10] Following A. Gabbey, we will label Descartes' treatment of the lever law, the "General Statical Principle," or GSP.[11] The similarities between QM and the GSP are quite evident, as are their differences. Despite the obvious parallel use of the product function, the GSP's elements are weight and infinitesimal displacement (where "infinitesimal" simply denotes "minute" or "small", and not its later mathematical usage). QM, on the other hand, employs size and non-instantaneous speed (Pr II 39; see below).

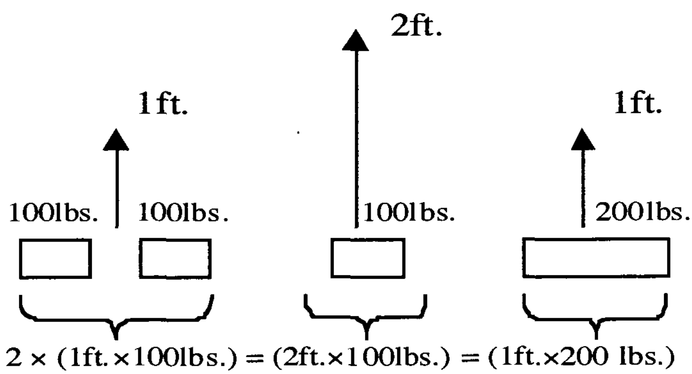

Figure 17. Three different instances of raised bodies with identical values for the quantity of motion.

Nevertheless, the differences between QM and the GSP are not as great as they might at first seem. In one sense, of course, QM is non-instantaneous; i.e., it includes the Cartesian concept of speed, which is only manifest over a non-instantaneous temporal period (as best exemplified in the collision rules, Pr II 46-52). Yet, QM is also closely tied to the instantaneous property[12] which we can loosely entitle, "tendency" (*tendere,* Pr II 39), although the *Principles* employs several designations interchangeably in depicting this notion (e.g., "striving," *conatus,* Pr III 56; "first preparation for motion," *prima praeparatio ad motum,* Pr III 63). To summarize the conclusions from chapters 3 and 4 above: tendencies are

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