

PREFACE

1. AN INTRIGUING CHALLENGE

Whoever undertakes the task of compiling a textbook on a relatively new, but already vastly ramified and quickly growing area of logic - and substructural logics are such, at least to some extent - is faced with a baffling dilemma: he can either presuppose a high degree of logical and mathematical expertise on the reader's part, or else require no background at all except for a "working knowledge" of elementary logic. In our specific case, each one of these policies had its own allure. The former strategy promised to speed up the presentation of some advanced topics and to allow a more refined expository style; the latter one, on the other side, would have permitted to reach a wider audience, some members of which might have had the opportunity to study for the first time some elementary, but fundamental results - such as Gentzen's *Hauptsatz* - directly in the perspective of substructural logics. Teaching logic from this point of view to unexperienced, and presumably still unbiased, students seemed to us an irresistibly intriguing challenge - therefore, we opted for the second alternative.

Thus, we assume that the reader of this book has attended an undergraduate course in logic and has a good mastery of the rudiments of propositional logic (Hilbert-style and natural deduction calculi, truth table semantics) and naive set theory. As for the rest, the volume is self-contained and gradually accompanies the reader up to some of the most recent and specialistic research developments in this area. Some prior acquaintance with either predicate logic or algebra is useful, but not indispensable; in particular, the algebraic notions used throughout the book are surveyed in a special glossary (Appendix A).

Of course, this book is not meant only for students. The researcher in the field of substructural logics will find plenty of material she can directly exploit and draw from in her research practice.

It is not easy, it must be confessed, to write a textbook on this subject short after such a wonderful volume as Restall's *An Introduction to Substructural Logics* (Restall 2000) has been sent to the press. Our intellectual debt towards this work is enormous, as the reader will notice. However, offering a different perspective on a same topic can be valuable, sometimes. Restall's book primarily focuses on natural deduction and display calculi, and on frame semantics. Our viewpoint is somewhat more traditional: we privilege ordinary sequent calculi on the proof-theoretical side, and algebraic models on the semantical side. We believe that readers who are scarcely at ease with the "punctuation mark" proof theory in the style of Dunn, Mints, Belnap, or with frame semantics - especially researchers belonging to substructural schools other than the relevant - could perhaps feel more comfortable in a setting like ours. Thus, we are confident that our book and the one by Restall can profitably integrate and supplement each other.

We tried to arrange this book in such a way as to provide a (hopefully) useful tool for readers coming from *any* substructural tradition (linear logic, Lambek calculus, relevance logics, BCK-logic and contraction-free logics, comparative logic) and from a number of different backgrounds (philosophy, mathematics, computer science, linguistics). It is extremely important, in our opinion, that people from diverse provenances and academic environments, who often tackle the same problems using different jargons and being unacquainted with one another's results, can find a common ground for discussion and mutual interaction. Occasionally, some personal biases of the author - who is a philosophically oriented logician and a specialist of comparative logic - may show up. We hope that this won't happen too often, though.

2. OVERVIEW OF THE CHAPTERS

Chapter 1 introduces the topic from both a historical and a philosophical perspective. After discussing the relationships between substructural logics and proof-theoretical semantics, we provide some reasons for dropping some or all of the structural rules in sequent calculi and, finally, we try to find plausible informal interpretations for substructural sequents.

Chapter 2 contains a presentation of the main sequent and Hilbert-style calculi for substructural logics, and of their elementary syntactic properties. The cut elimination theorem for substructural sequent calculi is the heart of *Chapter 3*, where we also illustrate some decision procedures for these systems. *Chapter 4* deals with more advanced formalisms, some of which have been introduced rather recently: we cover a few generalizations of sequent

systems (n -sided sequent calculi, hypersequent calculi, Dunn-Mints and display calculi) and of natural deduction (proofnets), as well as resolution calculi.

Algebraic semantics will be in the foreground in *Chapters 5* and *6*, where we study the models of substructural logics at first in a purely algebraic perspective, and then linking them to the calculi of the preceding chapters by means of appropriate completeness results. *Chapter 7* is concerned with a different kind of semantics, which generalizes Kripke-style semantics for modal and intuitionistic logics. We discuss models for both distributive logics (Routley-Meyer semantics) and logics without distribution (phase semantics).

Appendix A provides a crib of elementary algebra, model theory and graph theory for those readers who are unfamiliar with even the most basic notions of these disciplines (we primarily thought of students in philosophy or linguistics, but also in computer science). Its main aim is letting the book be as self-contained as possible. *Appendix B* surveys some logics which, regrettably enough, had not received adequate attention throughout the main body of the text.

3. WHAT HAS BEEN LEFT OUT

Although we tried to cover as many topics as possible, due to obvious limitations of size we could not help making choices. In order to delimit the bounds of our enterprise, we imposed ourselves four constraints:

- *The propositional constraint.* Throughout this book, we shall remain within the boundaries of propositional logic. There exist interesting inquiries concerning quantified substructural logics, or even substructural arithmetic or set theory (see e.g. Meyer 1998), but in our opinion such a work will remain somehow foreign to the spirit of substructural logics so far as the difference between lattice-theoretical and group-theoretical quantifiers is not properly understood. We think that taking a firm grip on such a distinction is, at present, the most important task with which substructural logicians are confronted (a promising start is in O'Hearn and Pym 1999).

- *The commutative constraint.* We shall not consider logics without exchange rules, i.e. logics whose group-theoretical disjunction and conjunction connectives are not commutative. These logics pose tricky technical problems which by now, however, are beginning to find acceptable solutions. Some of the current work into noncommutative logics is reported in Appendix B; see also Abrusci and Ruet (2000), Bayu Surarso and Ono (1996), Ono (1999).

- *The classical constraint.* We shall focus on logics with an involutive negation, disregarding systems with minimal or intuitionistic negations.

Subintuitionistic logics are briefly surveyed in Appendix B, where the interested reader will find appropriate references to the literature.

■ *The ℓ -constraint.* Although we shall generally consider logics with more than one pair of disjunction and conjunction connectives, in each case at least one such pair will exhibit lattice properties. Logics whose underlying algebraic structures are not lattice-ordered have recently emerged in the context of the "unsharp approach" to quantum logics (see e.g. Giuntini 1996), but the connection between these systems and substructural logics is still unclear.

Besides abiding by these constraints, we had to leave out of this book other topics which would have surely deserved attention. For example, we neglected some items which have been exhaustively illustrated in the handbook by Restall - e.g. natural deduction, the Curry-Howard isomorphism for substructural logics, the semantics of proofs. Other important references for this constructive approach to our subject are Girard et al. (1989) and Wansing (1993). We shall spend nothing but a few words on Gabbay's approach to substructural logics in the framework of *labelled deductive systems* (Gabbay 1996), which represents one of the most innovative perspectives in contemporary logical research. Dunn's *gaggle theory* and Urquhart's inquiry into the feasibility of the decision problem for substructural calculi (Urquhart 1990) have been passed over as well, except for some occasional mentions.

4. ACKNOWLEDGEMENTS

Our first heartfelt thanks obviously go to Ettore Casari, who first introduced us into logic in the mid-eighties, and into substructural logics, some years later. Studying and working under his guidance has been one of the luckiest opportunities we had throughout our scientific *iter*. His work on pregroups and comparative logic was, needless to say, a main source of inspiration for the general framework underlying the present book. We also thank Ettore Casari for consistently supporting in many ways the project of this volume.

We are greatly indebted to Daniele Mundici for his encouragement and his invaluable suggestions, as well as for putting us in contact with his dynamic and stimulating research group. We gratefully acknowledge the friendly support and help provided by Roberto Giuntini and Maria Luisa Dalla Chiara.

We feel extremely grateful to Heinrich Wansing, who supported this enterprise - from its very beginning - more than one could have asked for; to André Fuhrmann, who first led us into the territories of relevance logics; and to Pierluigi Minari, whose papers and oral remarks helped us to understand many things concerning these topics.

Several people read portions of the manuscript and suggested precious improvements: among them, let us mention with immense gratitude Ettore Casari, Agata Ciabattoni, Enrico Moriconi, Hiroakira Ono and Heinrich Wansing.

We also thank Matthias Baaz, Antonio Di Nola, Steve Giambrone, Sandor Jenei, Edwin Mares, Bob Meyer, Mario Piazza, Greg Restall, Giovanni Sambin, Harold Schellinx, John Slaney, Richard Zach, who answered questions, provided insights or discussed with us (orally or *via* e-mail) about relevant issues.

Finally, we want to express our gratitude to an anonymous referee, for his/her precious remarks, and to Tamara Welschot and the editorial staff of the series *Trends in Logic* for their kind and competent assistance.



<http://www.springer.com/978-1-4020-0605-0>

Substructural Logics: A Primer

Paoli, F.

2002, XIII, 305 p., Hardcover

ISBN: 978-1-4020-0605-0