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## INTRODUCTION TO SECTION III

### *Models, situated practices, and generalization*

The contributions of the first section of the book looked in detail at symbolizing and modeling by making fine-grained analyses of the spontaneous and informal processes of construction and use of mathematical symbols and models by children and students in more or less experimental instructional settings. In the second section of the book, the pro-active role of teachers and of instructional design was highlighted. Section III addresses some remaining theoretical issues related to learning and teaching of mathematical symbols and models, thereby relying on multiple perspectives including cultural, sociological, philosophical and historical ones.

In section III mathematical modeling is considered in connection to situated practices on the one hand, and generalization and transfer on the other hand. Key issues addressed in this section are the nature of mathematical models and generalizations, and the circumstances in which they are used. Recent theorizing argues that this is a complex matter that cannot be fully understood on the basis of symbolic tools or problem solving abilities alone. A deeper grasp of human symbolizing capacity also requires reflection on the evolution of brain functions and the language faculty. Hence, the issue of ‘models, situated practices and generalization’ is here not only addressed from a microgenetic perspective, but also from a sociogenetic and phylogenetic point of view. Furthermore, sociocultural perspectives about the important role of the classroom context and culture on the nature of the process of mathematical modeling are considered. Finally, some important consequences of these analyses for (the goals of) education in general, and mathematics education in particular, are addressed.

Nemirovsky’s chapter starts with a critical review of the many studies and debates on the notions of transfer and generalization of learning developed during the twentieth century. Then he outlines his own view on ‘situated generalization’. He describes the following three aspects that characterize such a generalization: (1) where the generalization takes place, (2) the values associated with generalizing, and (3) how generalizations relate to particular instances. In the second part of his chapter, Nemirovsky illustrates and grounds his notion of ‘situated generalization’ through an in-depth analysis of an interview with Clio, an 11-year old girl, working with problems involving the graphical representation of the motion of a train through a tunnel.

Verschaffel, Greer, and De Corte review a set of empirical studies providing evidence that, after several years of traditional mathematics instruction, students have developed a tendency to reduce mathematical modeling to selecting *the* correct formal-arithmetic operation with the numbers given in the problem, without seriously taking into account their common-sense knowledge and realistic considerations about the problem context. This evidence was obtained by means of a series of especially designed word problems with problematic modeling assumptions from a realistic point of view, administered in the context of a mathematical lesson. After having discussed these empirical studies, the results are interpreted against the background of schooling in general, and the mathematics classroom in particular. Afterwards studies aimed at changing students' perceptions of word problem solving by taking a radical (realistic) modeling perspective are reported. Taking into account all the research findings reported in their chapter, the authors end with a series of instructional recommendations, the most important one being their proposal to reconceptualise word problems as exercises in mathematical modeling.

Kaput and Shaffer review the evolutionary psychologist Merlin Donald's (1991) four-stage account of the evolution of human representational competence, and suggest that we are now entering a new, fifth stage of evolution where students will become more skilled at modeling as they gain access to more flexible and powerful means of automatically representing problems. It is suggested that the evolutionary perspective needs to complement mathematics educators' other ways of understanding the learning and use of mathematics, especially the semiotic side of the subject. In the first part of the chapter the Merlin Donald analysis is recounted. Afterwards the co-evolution of writing and the earliest mathematics is examined, and this is followed by a description of the new stage into which we are emerging.

Section III ends with a discussion chapter by Carraher and Schliemann. Taking into account that the three chapters approach mathematical symbolizing, modeling and/or tool use in quite diverse ways, the discussion consists of three sections – one for each chapter.

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RICARDO NEMIROVSKY

## ON GUESSING THE ESSENTIAL THING

**Abstract.** This paper relates to the ongoing discussion in the domain of psychology of (mathematics) education about the consequences of the ‘situated cognition’ paradigm for the issues of generalization and transfer of learning. This chapter focuses on the nature on generalizing. A first issue that is examined is the relationship between principles, laws, and definitions, on the one hand, and the circumstances in which they are validly applied. In this respect, a distinction is made between formal and situated generalizations across the following three areas: (a) where the generalization takes place, (b) values associated with generalizing, and (c) how generalizations relate to particular instances. A second part to be published elsewhere elaborates on transfer of learning. These papers articulate the claims through an in-depth analysis of an interview with Clio, an 11-year old girl working with problems involving the graphical representation of motion.

### 1. INTRODUCTION

The issue examined in this chapter is the relationship between principles, laws, or definitions and the circumstances in which they are valid, applied, or used in some way. Generalization is often seen as the process of inferring principles *from* a set of instances or applying a principle *to* a set of instances. In spite of the apparent simplicity of this formulation, it is often very complex to ascertain in actual human talk and interaction what is being generalized and on what basis. In other words, it is easy to recognize an isolated sentence, particularly if it is of the form ‘all  $x$  are  $y$ ’, as the statement of a generalization, but if we consider utterances—in the Bakhtinian sense of words said by someone, to someone else, in actual contexts and intentions—then questions about the nature and origins of generalizing cease to be straightforward. Sometimes a generalization reflects lack of familiarity. It is common to hear people saying things like ‘all babies look the same,’ or ‘Russian sounds like repeating the same sounds over and over.’ It is often uttered by someone who is unfamiliar with babies or Russian and who, from his ‘outsider’ point of view, fails to notice salient differences among the population or language in question. This is a kind of flattening or leveling-off generalization; it is a generalization that simplifies by overlooking.

On the other hand we find in everyday life examples of generalizing that do not ‘flatten’ the phenomena in question. A worker in an infant day care is likely to talk about babies ‘in general’ without losing sight of their individuality, or manifest some common patterns in how she treats babies but maintaining an ability to adapt those patterns to the idiosyncrasies of each baby and being sensitive to their particular needs. This is a generalizing that brings up and preserves complexity. It is a generalizing that is always close to examples, cases, and stories as well as to

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