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INFORMAL REPRESENTATIONS AND THEIR IMPROVEMENTS

In this book the editors have tried to bring together different studies that might deepen our understanding of symbolization and modeling in the area of mathematical thinking. With this aim in mind it is necessary to reflect also on the question on how this human ability of modeling comes into being? Both symbolization and modeling are conceived of here as ways of representing parts of the physical or cultural reality, and—probably even more importantly—to stabilize a given representation and make it mobile among different places and persons (Latour, 1990). Consequently, inscriptions amplify the human ability of communicating about representations, and it is also a plausible assumption that this faculty has greatly enhanced the development of human thinking both at a cultural and a personal level.

Although it is an intriguing epistemological question whether representations can ever be assumed to mirror reality in a strict sense, such an assumption is not necessary in studies of the human capacity to create tools for organizing realities and making them communicable, which as a result of their successes at least seem to give reliable representations of those realities. In the following chapters the authors will not touch on such epistemological problems concerning modeling. Instead, they all start from the assumption of models as structured forms of symbolization that can be taken as **tools** for organizing a piece of reality and communicating about it. Or to put it more precisely: all studies conceive of symbolic inscriptions as tools that organize human actions in (or on) that reality. And indeed, mathematics is predominantly seen as the area of human thinking that provides the human being with a set of powerful symbolic tools for the reflective organization of reality (cf. Freudenthal, 1991; Verschaffel, Greer & de Corte, 2000).

Given the central position that is assigned to modeling in our modern way of thinking, it also seems logical to try to understand how this ability of creating these tools of the mind comes into being. This is not merely a matter of satisfying scientific curiosity. A deeper understanding of the genesis of modeling might also enhance educators' abilities to optimize the conditions for the improvement of the development of modeling and—a fortiori—of mathematical thinking.

Vygotsky already emphasized the relevance of understanding the *genesis* of psychological functions, but he also argued that development could be seen at different levels. Wertsch (1985) called these levels 'genetic domains' and he

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distinguished at least four levels. With regard to modeling, these can be conceived of in the following way:

(1) *the phylogenetic level* that can be seen as the level of development of the representational function per se of human thinking during phylogenesis. Kaput and Shaffer's study on the genesis of human representational competence during the evolution of the human species (this book) can be taken as a good example of such analysis;

(2) *the sociogenetic level* that can be seen as the process of development of modeling as a cultural device in the history of science. Dijksterhuis' (1961) study of 'The mechanization of the world picture' can be referred to here as an example that demonstrates how mathematics gradually became the main producer of the symbolic machinery for representing reality for a better understanding. This process is expressed quite to the point by Latour (1990, p. 40) as a 'cascade of ever simplified inscriptions'. In the present book, this type of cultural analysis is not directly included, but at the ontogenetic level the process of creating inscriptions that can be improved by series of follow-up innovations will be demonstrated in some of the following articles. It is not too hard to imagine that these microgenetic productions may produce the input for the cascade of inscriptions that occurs in the cultural history of science.

(3) *the ontogenetic level* that tries to describe and explain the developmental process during individual development with regard to the function of symbolization and modeling; it is assumed here that modeling is not an advanced function of thinking that is composed of previously constructed elements, but rather that representational processes are present from an early age; this representational faculty is seen as **emerging** from the cultural activities that the human being is engaged in from an early age. This very faculty is a consequence of the human need for tools for organizing activities and is improved in the interaction between human beings. The chapters by Van Oers, and Lehrer and Pritchard (in this section) can be seen as most directly referring to the ontogenetic level of development of modeling. Both chapters demonstrate the early manifestations of representational activities and give examples of the functions of models or symbolizations proper as tools for organizing either speech about the quantitative aspects of reality (Van Oers), or for organizing the spatial aspects of reality (Lehrer and Pritchard).

(4) *the microgenetic level* that refers to the process of dealing with models in particular situations; diSessa in his chapter shows that students, from the upper grades of primary school on, do have all kinds of assumptions about adequate models that probably regulate their own thinking in situations where modeling is needed; interestingly, diSessa's study shows again the tool character of models: in his study, modeling appears to be particularly stimulated in situations where designing schematic representations is necessary. It turns out that students are able to create the tools for their design activities, and that this is an iterative process. Tools are never invented in a final form, but they gradually develop as a result of reflection guided by epistemological or even aesthetic values. Similarly, but in much more detail, Meira demonstrates how mathematical notations are created by primary school pupils in a step-by-step way. The dynamics of this microgenetic process are,

according to Meira, related to the material and social circumstances the pupils are involved in, the goals that emerge in these situations, and the efficacy of the tools that they construct in those circumstances. Similarly, in the Lehrer and Pritchard study it is demonstrated that young children can make inscriptions that help them to stabilize their representations of spatial relations in reality and also to enhance the mobility of the inscriptions, but it is also obvious that the initial inscriptions are never final and they will be innovated in more and more condensed forms in an ongoing process. As such we see here in micro-form a beginning of the cascade of inscriptions. In these studies it can be seen that the construction of inscriptions is a complex process that is influenced by many situational, social and psychological factors. We must assume, however, that this process is not a completely rational process. There are reasons to assume that the construction of inscriptions and the innovation of follow-up reconstructions are confined by the limits of an individual's reflective imagination. As Van Oers tried to demonstrate, this is strongly a language-based process. It is appealing to hypothesize that pupils at school often can't do much better than provisionally adopt condensed scientific inscriptions as language-like terms and master how they can be applied in an acceptable way. But appropriation of the meanings of an inscription may require a form of 'ascending the cascade' and reconstructing in detail all the meanings that previous generations of scientists have put together in this condensed form.

Taken together, these studies can be seen as demonstrations of the complexities of the gradual evolution of modeling, both in the ontogenetic and the microgenetic sense. This evolution is never a straightforward process that can be programmed for all pupils in a unified standard way. Not only do the different social and material conditions in the situations create differences that introduces variety in the process, the variety of background knowledge of individual pupils also creates different valuations of any individual with the tools created. Like Wertsch (1998) already demonstrated there is essentially an irreducible tension between an agent and the tools he is dealing with, which evokes a permanent need for development in the agent and/or the tools. This very tension between agent and tool (in the context of the cultural practices that children are involved in from a very early age) may be assumed to be one of the basic conditions for the emergence of modeling during ontogeny.

Given these findings it may be assumed that it makes sense to introduce reflections on forms of representation in young children's activities, by all means when representing can be made a useful tool for the children. Collaborative reflection on meanings and inscriptions may provoke children's representational abilities and gradually contribute to the improvement of the representational competence (including the symbolic machinery from the mathematical speech register and the epistemological and aesthetic values that regulate the constructive process of tool production). The studies presented in this section add to the plausibility of this point of view. A detailed theory of the conditions that promote the emergence of modeling may—in an educator's hands—eventually contribute to the improvement of mathematics education. We are, however, just at the beginning of understanding the complexities of this process.

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