KOENO GRAVEMEIJER

INTRODUCTION TO SECTION II

The role of Models, Symbols and Tools in instructional design

In the chapters in this section the focus is on the role of models, symbols and tools in instructional design. The chapters reflect the shift in the way models, symbols and tools are viewed in instructional design. Instead of the conventional focus on models that embody the formal mathematics to be taught, the emphasis is on alternative perspectives. Within these perspectives the ways in which symbols are used and the meanings they come to have, are seen to be mutually constitutive. Another common thread is formed by the instructional design theory for realistic mathematics education (RME). The RME theory features as an example of a design theory that takes the aforementioned dialectic relation into account. Key design heuristics of the RME theory are 'guided reinvention' (Freudenthal, 1973), 'didactical phenomenology' (Freudenthal, 1983), and 'emergent models' (Gravemeijer, 1999). It is especially the latter heuristic that is relevant for this book, since the emergent-models heuristic aims at the design of instructional activities that support the evolution of ways of symbolizing as part of a process of fostering the development of mathematical meaning.

In this section two examples of this heuristic are discussed, one from primary and one from lower secondary education.

It may be worth noting however, that the RME-modeling heuristic is also used in upper-secondary education—for instance, by Doorman (Gravemeijer & Doorman, 1999) for *calculus*, and by Rasmussen (1999) for *differential equations*. Doorman takes his point of departure in modeling problems are tackled with discrete approximations, inscribed by discrete graphs initially. Later, similar graphs—first discrete and later continuous—form the basis for more formal calculus. Rasmussen starts out by presenting students the problem of how to use a rate of change equation about the number of a certain species to approximate the future number of that species. In subsequent activities, reflections and discussions, the slope field emerges as an initial record of students' mathematical activity, which later on evolves into a tool for students to reason about solution functions to differential equations.

In the first chapter of this section, Gravemeijer and Stephan elaborate this heuristic on the basis of a design experiment in which an instructional sequence on linear measurement is combined with a sequence on flexible arithmetic. They show that in *the model* in the RME heuristic is to be understood in a holistic,

Gravemeijer, K., Lehrer, R., Oers, B. van and Verschaffel, L. (eds.), Symbolizing, Modeling and Tool Use in Mathematics Education, 141–143.

metaphorical, sense. The model functions as an overarching concept that guides the instructional design. This overarching model is concretized in a series of symbolizations in the actual sequence. In the measurement/arithmetic example, this overarching model consists of the idea of a ruler. In the instructional activities, tacit ruler-type tools emerge as records of iterating measurement units. Such records are first used as measurement tools, but gradually develop into tools for reasoning about measures or (later) for reasoning about quantities in general. In terms of the emergent-models heuristic, the model evolves from a *model of* the students' informal mathematical activity (measuring by iterating measurement units) to a *model for* more mathematical reasoning (in this case reasoning with numerical relationships). The authors argue that this shift coincides with the development of a framework of number relations, within which numbers acquire an object-like quality. They further show that this shift comes about in a dynamic, reflexive process in which symbolizations and meaning co-evolve.

In conjunction with Gravemeijer and Stephan, Cobb explicitly takes a social perspective in his chapter. His point of departure is that it is essential to account for the mathematical learning of the classroom community taken as a unit of analysis in its own right. The central theoretical construct in this type of analysis is that of a classroom mathematical practice. The latter can be characterized by three interrelated mathematical norms: 1) a taken-as-shared purpose, 2) taken-as-shared ways of reasoning with and talking about tools and symbols, and 3) taken-as-shared forms of mathematical argumentation. This central construct is connected with instructional design in that an envisioned learning trajectory can be viewed as encompassing an anticipated sequence of classroom mathematical practices. Symbol use can be seen as a constituent part of the mathematical practices in which students come to participate.

Cobb presents an analysis of the mathematical practices established during a seventh-grade classroom teaching experiment—based on RME theory—that focused on statistical data analysis. The issue of how symbolizing, modeling and tool use relate to the emergence of what is traditionally called mathematical content is elaborated against this background. The analysis of the mathematical practices is supplemented with a description of the taken-as-shared ways in which two computer-based analysis tools were used in the classroom. Key to these analyses is that they treat people's activity with symbols as an integral aspect of their mathematical reasoning rather than as external aids to it. Cobb further points to the distinction between the conception of models in RME-design theory and the way the term mathematical model is used within the discipline. A mathematical model is typically seen as separate from the situation modeled. Modeling, then, can be described in terms of 'translation' and 'fit'. In contrast, in the RME approach there is no firm distinction between the model and what is being modeled. Here, modeling is primarily seen as organizing or mathematizing—with the objective of structuring situations in terms of mathematical relationships. As a consequence, the model and the situation that is being modeled are mutually constituted.

In the third chapter of this section, Patrick Thompson extends the discussion of emergent models by arguing that it is useful to complement RME with theories of quantitative reasoning. In his view, RME and quantitative reasoning provide complementary foci in design of instruction, as well as in evaluating it. According to him, with RME the focus seems to be on ways of influencing students' activity, while a theory of quantitative reasoning attends to the mathematical understandings one hopes students will come to have. In elaborating theories for quantitative reasoning, Thompson adheres to radical constructivism as a background theory—which lays down a set of commitments and constraints on acceptable types of explanations and interpretations. Against this background, he discusses the implications of a constructivist conception of intersubjectivity for instructional design. From a constructivist perspective intersubjectivity does not imply shared meaning, but meaning that is taken-as-shared. Therefore, one will have to design instruction that takes into account a variety of understandings and supports individual growth in the intended direction. In this type of design, objects of discussion are claimed to be key design elements. For, as Thompson argues, conceptual analysis of what one hopes students will eventually understand, entails imagining students thinking about something in the context of a *discussion* of it. He coins the term 'didactical object' to refer to such a 'thing to talk about' and elaborates this in a few examples. He further introduces the term 'didactical model' to refer to a scheme of meanings, actions, and interpretations that constitutes the designer's image of all that needs to be understood for someone to make sense of the didactical object in the intended manner. The didactical model should ideally also capture aspects of how the intended understanding might develop. Described in this manner, didactical models are highly compatible with the idea of a learning trajectory (Simon, 1995). The advantage, however, is to be found in a clear separation between descriptions of instruction and descriptions of learning—which allows for thinking of the two in relation to each other.

REFERENCES

Freudenthal, H. (1973). *Mathematics as an Educational Task.* Dordrecht: Reidel.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures.* Dordrecht: Reidel.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning.* 1 (2), 155-177.

Gravemeijer, K., and Doorman, M. (1999). Context problems in realistics mathematics education: A calculus course as an example. *Educational Studies in Mathematics,* 39, 111-129.

Rasmussen, Ch. L. (1999). *Symbolizing and Unitizing in Differential Equations*, paper presented at the annual meeting of the National Council of Teachers of Mathematics Research Presession, San Francisco, April 1999.

Simon, M.A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education*, 26, 114-45.

Koeno Gravemeijer
Freudenthal Institute/Department of Educational Sciences
Utrecht University,
PO Box 9432, 3506 GK Utrecht
The Netherlands

Springer

http://www.springer.com/978-1-4020-1032-3

Symbolizing, Modeling and Tool Use in Mathematics Education

Gravemeijer, K.P.; Lehrer, R.; van Oers, H.J.; Verschaffel, L. (Eds.)

2002, IV, 308 p., Hardcover

ISBN: 978-1-4020-1032-3