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# 1. The life table:

## A demographic overview

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### 1 | Introduction

The purpose of this preliminary chapter is to present an elementary overview of the life table, its objectives and its construction, and to point out some of the problems related to the background assumptions of life table methodology. The life table is one of the oldest tools of demographic analysis. Indeed, it seems that life expectancies were already estimated during the Roman Empire, though the lack of data led to more or less informed guesses rather than to hard facts! The method was however perfected during the 17th and especially the 18th century, by such well-known figures as Graunt, Halley, and de Moivre. It was during the 19th century however that the method gained wide acceptance thanks to the recent availability of data derived from the census and the vital registration system. Presently, any first-year student in demography can easily calculate a life table, and several software packages are on the market: life table construction has thus become ordinary routine. The method is not without problems, however, as we shall see in the following sections.

### 2 | The life table functions

In this paper, only the *period life table* will be considered, having resort to the fictitious or synthetic cohort approach. Consider the general case of calculating probabilities of dying  $q_x$  (or  ${}_1q_x$ ) from exact age  $x$  to exact age  $x+1$ , in the various cohorts observed during two calendar years  $j$  and  $j+1$ . In actual practice, risks or rates of dying are often computed over a period of several years, in

order to attenuate random fluctuations, either by averaging the number of deaths over several years or by averaging the risks or rates themselves.

We wish to estimate the 'true' *risk of dying* one would have observed if there had been no disturbances (e.g. emigration). A detailed analysis of the estimation of this risk is presented in the chapter by G. Calot and A. Franco in this volume. In an elementary approach, assuming *independence* between the risks of dying and those of emigrating as well as linear survivorship functions, one can use the so-called "*exact Berkson formula*" for the 'net' probability:

$$q_x = 1 + 0.5 \left( \frac{d_x}{S_x} - \frac{\delta_x}{S_x} \right) - \sqrt{1 + 0.5 \left( \frac{d_x}{S_x} - \frac{\delta_x}{S_x} \right)^2} - 2 \frac{d_x}{S_x}$$

where  $d_x$  and  $\delta_x$  are respectively the number of deaths and the number of emigrations at completed age  $x$ , and  $S_x$  is the number of survivors at exact age  $x$ .

An excellent approximation to the exact Berkson formula has been derived by H. Le Bras and M. Artzrouni (1980):

$$q_x \approx \frac{d_x}{S_x} \left( 1 - \frac{\delta_x}{d_x + \delta_x - 2 S_x} \right)$$

This approximation gives practically the same result as the exact Berkson formula. At the price of a greater approximation, it is possible to derive a simple expression for the corrected value of  $q_x$  excluding disturbances (or competing risks), usually called the "*approximate Berkson formula*":

$$q_x \approx \frac{d_x}{S_x - 0.5 \delta_x}$$

Other methods for obtaining net probabilities have been developed. For example, if one supposes that the *forces* of mortality and emigration are constant or proportional over the interval, one obtains the so-called *Elveback formula* (Chiang, 1968). In this case one has:

$$q_x = 1 - \left( \frac{S_x - \delta_x - d_x}{S_x} \right)^{d_x / (d_x + \delta_x)} = 1 - \left( \frac{S_{x+1}}{S_x} \right)^{d_x / (d_x + \delta_x)}$$

This formula has been very slightly improved by Keyfitz and Frauenthal (1975). In fact, the 'exact' Berkson formula and the Elveback formula yield very similar results; the difference between the 'exact' and 'approximate' Berkson formulas is slightly larger.

In practice, let  $d'$ ,  $d''$ ,  $e'$ ,  $e''$ , and  $i'$ ,  $i''$ , respectively represent the number of deaths, out-migrants, and in-migrants occurring in cohort  $j-x$  during the two calendar years  $j$  and  $j+1$ . Moreover, let  $S_x$  be the population surviving at *exact* age  $x$ , and let  $Z_x$  (or  $S_{x+1/2}$  on average) be the population enumerated at age  $x$  in *completed* years on the 31st of December of year  $j$ . In order to obtain the net probability of dying corrected for disturbances or competing risks (in- and out-migration), one can have resort to one of the above formulas, for example the approximate Berkson formula, which easily takes immigration into account too:

$$q_x \approx \frac{d' + d''}{S_x - 0.5(e' + e'') + 0.5(i' + i'')} = \frac{d' + d''}{S_x - 0.5E_x + 0.5I_x} \quad [1.1]$$

where  $E_x = e' + e''$  and  $I_x = i' + i''$ . As  $Z_x$  represents the mid-year population (on December 31st, year  $j$ ), one has

$$S_x = Z_x + d' + e' - i'$$

Writing  $S_{x+1/2}$  for  $Z_x$  in order to avoid confusion, the corrected probability of dying can also be written:

$$q_x \approx \frac{d' + d''}{S_{x+1/2} + d' - \frac{e'' - e'}{2} + \frac{i'' - i'}{2}} \quad [1.2]$$

If there is no net migration (i.e. if  $E_x = I_x$ ), relation [1.1] becomes simply equal to

$$q_x \approx \frac{d' + d''}{S_x} \approx \frac{d' + d''}{S_{x+1/2} + d' + e' - i'}$$

Furthermore, if  $e' = i'$ , the above relation is reduced to

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