

2. The construction of life tables

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This chapter first recalls the probabilistic background of the estimation of mortality intensity by age and sex during a year of observation. It leads to the fundamental pair of relationships –hereafter numbered [24] and [32]– yielding the number of deaths inside each triangle of the Lexis diagram.

Three methods (the last one with two additional sub-variants) of practical computation of the probability of dying between two consecutive ages are considered: the first one (*A*) is based on the number of deaths inside each *triangle*, taking into account the distribution of *birthdays* within each of the two birth-cohorts concerned, the second one (*B*) is based on the number of deaths inside each *square* and the distribution of birthdays, the third one (*C*) simply consists in the computation of *rates*: ratio of the number of deaths in the square to the *mid-year* population. Method *C* takes the *observed rate* as an estimate of the *risk of mortality* q , while method C_2 takes it as an estimate of the *probability of dying* Q . Method C_1 is intermediate between *C* and C_2 ¹.

¹ Methods *C*, C_1 and C_2 consist in deriving the estimate $\hat{Q}$ of the probability of dying, Q , from the observed *rate* r :

$$r = \frac{2D}{P_1 + P_2}$$

by :

$$\text{Method } C : \hat{Q} = 1 - e^{-r}$$

$$\text{Method } C_1 : \hat{Q} = \frac{r}{1 + r/2}$$

$$\text{Method } C_2 : \hat{Q} = r$$

It is shown that, except for ages 0 and 1 and for higher ages (above 80), and except for cohorts born in a period of *abrupt* changes in the birth-rate, method *C*, and even methods C_1 and C_2 , yield satisfactory results. But for cohorts born during world wars, it is really worthwhile to use methods *A* or *B* and –at higher ages– to use method *A*. Moreover if methods *C* and C_1 yield quite comparable results, method C_2 becomes inaccurate above age 60.

1 | The estimation of a constant risk

Let us consider a set of N individuals exposed to the occurrence of a *non repeatable* event, denoted E . Individual j , $j = 1, 2, \dots, N$, is exposed to E between times b_j and e_j , beginning and end of the *exposure* period, respectively.

Let us assume that the *intensity* of occurrence of E , denoted q , also called the *risk*² or the *instantaneous quotient* of occurrence of E , is *constant* over time and *identical* for all individuals.

Under these assumptions, the probability that event E occurs to individual j between times t and $t + dt$, given that it did not occur to that individual before time t , is, for any j and t :

$$q \, dt. \quad [1]$$

Let $P(b, t)$ denote the probability that event E does not occur to a given individual exposed to E between times b and t . We may write:

$$P(b, t + dt) = P(b, t)(1 - q \, dt), \quad [2]$$

expressing that the probability $P(b, t + dt)$ of no occurrence between times b and $t + dt$ is:

- the probability $P(b, t)$ of no occurrence between times b and t multiplied by
- the probability $1 - q \, dt$ of no occurrence between times t and $t + dt$, given its non-occurrence between times b and t .

² If event E is *death*, q is called the *force of mortality*.

From [2], it follows that P satisfies:

$$\frac{\partial \{ \text{Log}[P(b, t)] \}}{\partial t} = -q, \quad [3]$$

which implies, taking into account that $P(b, b) = 1$, that:

$$P(b, t) = e^{-q(t-b)}. \quad [4]$$

Therefore, the probability that E occurs to individual j between times t and $t + dt$ is:

$$e^{-q(t-b_j)} q dt. \quad [5]$$

The probability of occurrence of E during a *unitary* period -i.e. of length equal to 1- is denoted Q . Using [4], the probability of no occurrence during a unitary period, $1 - Q$, is:

$$1 - Q = e^{-q},$$

so that Q is related to q by:

$$Q = 1 - e^{-q} \approx \frac{q}{1 + \frac{q}{2}} \quad \text{if } q \text{ is small.} \quad [6]$$

Let us now consider a set of N individuals, *independently* exposed to the occurrence of event E . The probability that E occurs to individuals $j_1, j_2, \dots, j_n$ at times $t_{j_1}, t_{j_2}, \dots, t_{j_n}$ and *not* to the other $N - n$ individuals, is:

$$L = \prod_{j=j_1}^{j_n} \left[e^{-q(t_j - b_j)} q \right] \prod_{j \in (j_1, \dots, j_n)} \left[e^{-q(e_j - b_j)} \right]. \quad [7]$$

Let T_j be, for individual j , the time elapsed from the beginning b_j of the exposure period until:

- the time of occurrence of E , if E did occur to individual j ;

The Life Table

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