CHAPTER 4

FROM CAUSE TO PHENOMENON
D'Alembert's Foundation of Mechanics

The first causes are absolutely unknown to us, but they are subject to simple laws and constants, to be discovered by observation, and whose study is the aim of Natural Philosophy.
Jean-Baptiste Joseph Fourier[1]

4.1 INTRODUCTION

In the previous chapters I have shown that in Newton's view, force was a substance, and in Leibniz's view analogous to it. This means that the underlying idea about the action of force was that force is the cause of changes in motion and is simultaneously the change itself, which is, as it were, transferred. It would, however, be wrong to consider this double meaning as vagueness or ambiguity, since these are two aspects of one and the same thing. It is more appropriate to call the concept of force a unity-in-duality. To clarify this, I have used the metaphor of water pouring from one barrel to another. This metaphor justifies my calling this a remnant of the substantial concept of force in scholastics. After all, when scholasticism turned to the concept of impetus for a solution to the problem of motion, the cause of the *perseverance* of motion was equated to the cause of the *change* in motion.

This unity-in-duality in the concept of force provides us with a common denominator for some of the idiosyncrasies of Newton's mathematization of continuously acting external forces, and of Leibniz's attempt to mathematize living forces. For example, Leibniz's distinction between dead force and living force referred only to a quantitative difference—he had no intention of denying their qualitative similarity. Nonetheless, in retrospect one could say that this distinction had the potential to deconstruct the unity of force. After all, later thinking on the role of space and time in the integration of dead force to living force showed that a qualitative distinction

[1] "Les causes primordiales ne nous sont point connues, mais elles sont assujetties à des lois simples et constantes, que l'on peut découvrir par l'observation, et dont l'étude est l'objet de la Philosophie naturelle" (Fourier, *Théorie analytique de la chaleur* (1822), v (opening sentence of the "Discours préliminaire")).

must logically follow the quantitative. In modern physics, this qualitative distinction is expressed through a dimensional distinction.

Looking back on the history of the concept of force, it is interesting to ask how this unity-in-duality was broken down, and how this could occur so thoroughly that the original unity can now be recognized only with great difficulty. An analysis of d'Alembert's significance for the development of the controversy of the true measure of living force, and especially his conception of force, offers a good starting point for answering this question.

4.2 BETWEEN LEIBNIZ AND D'ALEMBERT: A CHANGE IN THE DEFINITION OF THE PROBLEM

4.2.1 Atomism and Conservation Theories

The collision process served as a touchstone in the search for the true measure of force and its conservation. The change in the magnitude and direction of the velocity of the colliding particles necessarily implied that the force of the particles also changed. If one knew the laws governing impacts, it should be possible to discover a conserved quantity, which might serve as the true measure of force. Conversely, if one knew what the true measure of force was, one would also know what conditions the laws of impact had to meet. Therefore the search for the correct laws of impact was linked to the question of the true measure of force.

This link became more complicated when an attempt was made to bring the laws of impact into accordance with the process of collision. This resulted in serious difficulties regarding the concept of hard particles. In 1970, Wilson Scott described this development in the first chapters of *The Conflict between Atomism and Conservation Theory (1644–1860)*. Atomism played an important role in his interpretation, especially in the shape it was given by Newton.

In seventeenth century mechanical philosophy, there were four fundamental polar characteristics of material particles: soft-hard, elastic-inelastic, penetrable-impenetrable, and divisible-indivisible. However, the way these polar characteristics were linked was not immutable. According to some, a hard particle was elastic, according to others, it was inelastic; in the same way impenetrability could be paired with softness or hardness, or with elasticity or inelasticity.

Although Descartes described his particles as hard and impenetrable, he also said that they were brittle, i.e. that they were infinitely divisible. Hard inelastic particles could therefore rebound if they collided with one another. Conservation of mv, regarded as a scalar quantity, was thus not in conflict with his assumptions concerning the fundamental properties of material particles. For Leibniz, however, it was. Leibniz likewise viewed matter as infinitely divisible, but in his view this was paired with elasticity as a basic property. Consequently, mv^2 is conserved in impacts and he could even say that during *apparently* inelastic impacts, living force was transformed into the motion of the smaller internal particles of the colliding bodies. The 'directive' force was transformed into a 'relative' force.[2]

The discussion was greatly influenced by Newton's 1706 definition of hardness. In Query 23 of the Latin edition of his *Opticks*, he postulated his atom theory in which he characterized both hardness and softness as the incapacity to rebound:

> For Bodies which are either absolutely hard, or so soft as to be void of Elasticity, will not rebound from one another.[3]

Regardless of what Newton's exact intentions might have been with his Queries, it is certain that this characterization had a great influence on the discussion concerning the measure and conservation of force. This was because hardness, defined in this way, became a great obstacle for the theory of conservation of mv^2. However, it was not until after the publication of the sensational Leibniz-Clarke correspondence in 1717 that Newtonians became interested in the problem, although it is unclear whether this *post hoc* also entails a *propter hoc*.[4] Although Newton's characterization of hard bodies appeared to imply clear-cut support for Descartes's position, Newtonians were actually divided between the two positions, i.e. the Cartesian standpoint (conservation of mv), and the Leibnizian standpoint (conservation of mv^2).[5] The controversy now began in earnest.

---

[2] See also Westfall, *Force in Newton's Physics: The Science of Dynamics in the Seventeenth Century* (1971), 295.

[3] Newton, *Opticks: or, A Treatise of the Reflections, Refractions, Inflections & Colours of Light* (1704), 398. In the first English edition of 1704 there were only 16 Queries; Queries 17-23 were added to the Latin translation of 1706, were subsequently not included in the second English edition of 1717, but were published again in the third edition in 1721 and the following editions. In this and later editions, Query 23 is given the number 31. Here I have cited from the fourth English edition published in 1730. Papineau says that Newton's definition of hardness was already present in Wallis's prize essay about the laws of impact from 1669 (Papineau, "The *Vis Viva* Controversy" (1977), 121). However, although Wallis did make a remark of this type, it did not have much influence on the generally accepted view of hardness. For example, in his controversy with Leibniz between 1692–1694, Huygens maintained his view that hard bodies may rebound (Scott, *The Conflict between Atomism and Conservation Theory (1644–1860)* (1970), 13-15).

[4] A causal connection is made by Pulte, who refers mistakenly to Scott, *The Conflict*, who certainly did not interpret the relationship as causal (Pulte, *Das Prinzip der kleinsten Wirkung und die Kraftkonzeptionen der rationalen Mechanik* (1989), 66). Compare Papineau, "*Vis Viva* Controversy," 132-136.

[5] The Newtonian MacLaurin defended the conservation of scalar mv, the Marquise du Châtelet and Maupertuis, after initially being unclear about their positions, became advocates of the conservation of mv^2 around 1738. Beginning in 1722, 's Gravesande also supported the view that mv^2 remains conserved, to the great disappointment of many Newtonians who had found much support in his *Physices elementa mathematica experimentes confirmata. Sive introductio ad philosophiam newtonianam* (1720–1721). Hankins's idea that the "Newtonians joined the Cartesians in combating *vis viva*" following 1717 is therefore, generally speaking, incorrect (Hankins, "Eighteenth-Century Attempts to Resolve the *Vis Viva* Controversy" (1965), 282). However, there is something to be said for a shift from the Cartesian-Leibnizian opposition to the Newtonian-Leibnizian opposition, as well as for a geographical shift, as appears from a sketch made by Thomas Reid for his "An Essay on Quantity" from 1748; in this sketch he proposed that the controversy about the true measure could be solved by henceforth defining force as mv^2 on the continent and as mv in the English speaking regions (cited by Laudan, "The *Vis Viva* Controversy; A Post-Mortem" (1968), 139).

The Paris Academy of Sciences held two essay competitions for the years 1724 and 1726, concerning the laws of impact for hard and elastic bodies, respectively.[6] In their submissions to the competition, the Newtonian Colin MacLaurin and the Leibnizian Johann I Bernoulli based the laws of conservation on an analysis of the concept of hardness. MacLaurin used Newton's definition: hard bodies are "those whose parts do not yield at all during a collision"; he thus viewed hardness as a limit of non-elastic softness, and concluded from this that the *scalar* mv remains conserved only during collisions in the *same* direction.[7] However, like many other Newtonian mv supporters, MacLaurin no longer defended the *conservation* of force, but the proposition that mv is the *true* measure of force. Johann I Bernoulli based his analysis on Leibniz's continuity principle, which states that instantaneous changes in velocity are impossible. Bernoulli therefore did not interpret hardness according to Newton's characterization but rather as a limiting case of elasticity, concluding from this that the conservation of mv^2 is universally valid.[8]

Scott interprets this difference in answers as a conflict between atomism, at least in its Newtonian form, and conservation theories.[9] However, this resulted in a shift in the original problem. The question of whether there is a quantity that is *conserved*—and if so, which quantity—acquired an intrinsic interest, apart from the question of which quantity expresses the *true measure* of force. While some authors were interested primarily in the problem of how the qualitative concept of force should be mathematized, others were occupied mainly with the problem of which quantities—not necessarily linked with the concept of force—remained constant over time.

4.2.2 Experimental Modes of Response

Up to this point, we have only discussed the metaphysical and mathematical sides of the controversy: the question concerning the perfection of God, the definition of the concept of force, the nature of matter, and problems surrounding integral calculus. Around 1720, the discussion was unexpectedly given a new impulse: an attempt was made to find an answer through experimentation. Although this did not alter the basic principles of the discussion, it was greatly clarified by being made more tangible.[10]

---

[6] I derived the information about these competitions from Scott, *The Conflict*, Chapter 2.

[7] "ceux dont les parties ne cèdent point du tout dans le choc" (MacLaurin, *Démonstration des loix du choc des corps* (1724) (from Scott, *The Conflict*, 43n.16)).

[8] Johann I Bernoulli, *Discours sur les lois de la communication du mouvement* (1724–1726) (paraphrased in Scott, *The Conflict*, 43n3.).

[9] Scott, *The Conflict*, Chapter 2.

[10] According to Hankins, experimentation meant the "most decisive progress" in the controversy. It was very significant for d'Alembert and Desaguliers, for instance (Hankins, *Science and the Enlightenment* (1985), 31).

I will mention only the experiments conducted by Willem Jacob 's Gravesande during the 1720s.[11] Soon after the publication of his explanation of Newton's *Principia mathematica*, which was an important contribution to the acceptance of Newton's gravitational theory on the European continent, 's Gravesande turned to an investigation of the issue of living force. With an ingenious apparatus he determined the size of the hollows made by copper balls falling onto clay. The volume of the hollows proved to be proportional to the quantity mv^2 of the balls. In this way 's Gravesande became convinced that 'living force' was not just a word, but something real. He later wrote that, although the term 'force' often has many meanings, "what had been a mere argument about words at first, became an argument about the thing itself" in relation to collision processes.[12]

His conclusions caused a tumult among the Newtonians. The already waning dividing line between the competing camps of Newton and Leibniz supporters was under an even greater threat! In the following years 's Gravesande's success led other Newtonians to free themselves from Newton's metaphysical views.[13] The theological principles on which Newton based his rejection of the conservation of living force lost their absolute value. When 's Gravesande responded to Clarke's attacks, he emphasized that he followed the *method* and not the *person.*[14]

However, the 'method' also demonstrated the opposite. If two inelastic balls collide with one another, they come to rest if the quantity of motion in both balls is exactly equal. In 1722, 's Gravesande also carried out this experiment. Perhaps to force nature to pronounce judgment, he then developed an experiment in which *both* effects, the change in velocity and indentation, were achieved simultaneously. He allowed two equally large hollow copper balls with different masses, but with the same quantity of motion, to collide with a motionless hanging clay ball. His conclusion was that a moving body is associated with two effects: indentation and change in velocity, and accordingly with two causes, force and inertia, respectively.[15]

---

[11] 's Gravesande published his findings in "Essai d'une nouvelle théorie sur le choc des corps" (1722), "Sup[p]lement à l'essai sur le choc des corps" (1722), in the editions of his *Introductio* after 1723, and in "Remarques sur la force des corps en mouvement & sur le choc" (1729). He later developed yet another experiment which he published in "Nouvelles experiences sur la force des corps en mouvement" (1733). Concerning the apparatus used by 's Gravesande for his experiments see: De Clercq, "The 's Gravesande Collection in the Museum Boerhaave, Leiden" (1988). Even earlier than 's Gravesande, the Italian Giovanni Poleni performed similar experiments (Poleni, *De castellis per quae derivantur Fluviorum aquae (...)* Padua: Comini 1718). It is interesting that for him the context of the problem was not theoretical but technological: the management of river flows. In 1733 's Gravesande published a summary of Poleni's findings in his *Journal historique* (Poleni, "Epistolae mathematicae, second extrait," *Journal historique de la république des letters* 1733 (2, 2), 220-229). For the relationship between the experiments of Poleni and 's Gravesande see: Maffioli, "Italian Hydraulics and Experimental Physics in Eighteenth-Century Holland. From Poleni to Volta" (1989), 263-266 and De Clercq, *At the Sign of the Oriental Lamp* (1997), 169-171.

[12] "ce qui n'étoit d'abord qu'une Dispute de Mots, devient une Dispute sur la Chose même" ('s Gravesande, "Remarques sur la force," 197).

[13] See also above, note 5.

[14] 's Gravesande, "Remarques sur la force," 195.

[15] 's Gravesande, "Nouvelle experiences."

What was Mechanical about Mechanics
The Concept of Force between Metaphysics and
Mechanics from Newton to Lagrange
Boudri, J.C.
2002, XVI, 280 p., Hardcover
ISBN: 978-1-4020-0233-5