CHAPTER 5

FROM EFFICIENT TO FINAL CAUSES: THE ORIGIN OF THE PRINCIPLE OF LEAST ACTION

5.1 INTRODUCTION

5.1.1 The Greatest Possible Scandal at Court

Voltaire, the glorifier of reason and the world, must certainly have looked back on his time in Berlin with ambivalent feelings. After the death of his sweetheart the Marquise du Châtelet he spent some time at Sanssouci, Frederick the Great's country residence in Potsdam. He soon proved to be more imaginative and more entertaining than the then President of the Royal Academy, Maupertuis, so that they quickly became enemies.[1] The fact that Voltaire had been bypassed for the presidency six years earlier, in favor of Maupertuis, may also have played a role. Voltaire was bent on revenge, which can indeed be sweet.

The opportunity soon arose, in the form of a dispute about who could claim to be the originator of the mechanical principle of least action. Maupertuis, supported by Euler, defended his claim to discovery against the mathematician Samuel König, who claimed that the honor properly belonged to Leibniz. Voltaire then wrote a satire, the "Diatribe du docteur Akakia" (1752), which criticized Maupertuis for his pride: Maupertuis "had to be unbounced."[2] Although there were *two* attempts to burn the complete first printing of the booklet—at the command of Frederick the Great and with the 'co-operation' of Voltaire himself!—it was nevertheless possible to smuggle one copy out of Berlin.

Voltaire wrote a sequel in Dresden, in which König and Maupertuis signed a peace treaty. This contains the following passages, which also target Maupertuis's ally, the mathematician Euler:

> We no longer deride the Germans, and we confess that figures such as a Copernicus, a Kepler, a Leibniz, a Wolff, a Haller, a Mascau and a Gottsched

[1] For the story of the relationship between Voltaire and Maupertuis, which developed from being fellow spirits to the greatest possible enmity, see Tuffet's 1967 introduction to Voltaire's *Histoire du docteur Akakia et du natif de St. Malo*, vii-cxxxi.

[2] Compare the unbouncing of Tigger in Chapter 7 of A.A. Milne's *The House at Pooh Corner*.

> knew a thing or two, and that we have studied under the Bernoullis, and are still studying. (...)
> Moreover (...) our Lieutenant-General, Leonard Euler, empowers us to declare on his behalf (...) that he confesses in all innocence that he has never known anything about philosophy, and that he sincerely repents that he ever allowed us to persuade him of the possibility of knowing without having studied.[3]

This satire was written during the sequel to the discovery of the principle of least action. The scientific and philosophically discussion had already given way to the less exalted question of who should be regarded as the rightful discoverer of this principle which, it was claimed, linked metaphysics, theology and physics with one another and provided a common center for the whole of physics.[4] The booklet was the climax of Voltaire's involvement in this struggle for the honor of discovery.

His involvement began much more quietly in the autumn of 1752 with Voltaire's anonymously published response to the battle between Maupertuis and König and an answer, also anonymous, from Frederick the Great on Maupertuis's behalf—Maupertuis himself having fallen ill in the autumn.[5] Voltaire's satire is intended as an answer to Frederick the Great's open letter. Its secret publication not only led to a definitive break with Frederick the Great, it is also quite likely that, as Brunet writes, the controversy could have been simply ended had the participants calmed down, were it not for Voltaire's petulant intervention.[6] Voltaire's venom left its mark on the evaluation of Maupertuis and Euler at the time, and has indirectly influenced later evaluations. So there is much to say for Frederick's telling remark, in answer to the publication of the *Diatribe*: "If we consider your works, statues should be built in your honor, but if we considered your conduct, you would be thrown in chains."[7]

---

[3] "Nous ne rabaisserons plus tant les allemands, et nous avouerons que les Kopernick, les Kepler, les Leibnitz, les Wolf, les Haller, les Mascau, les Gotsched sont quelque chose, et que nous avons étudié sous les Bernoulli, et nous étudierons encore," and "De plus (...) notre Lieutenant-Général, Léonhard Euler déclare par notre bouche ce qui suit. I. Qu'il confesse ingénuement de n'ávoir jamais apris la Philosophie, et qu'il se repent sincèrement de s'être laissé persuader par nous qu'on pouvoit la savoir sans l'avoir étudiée. II. (...)" (Voltaire, *Traité de paix conclu entre Mr le président de Maupertuis et Mr le professeur Koenig* (1753), 26-27 and 39.

[4] See Du Bois-Reymond, "Maupertuis" (1892); Harnack, *Geschichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* (1900) I.1, 331-345; Fleckenstein, [Foreword to Euler's *Commentationes mechanicae; principia mechanica*] (1957); Tuffet, [Introduction to Voltaire's *Histoire du docteur Akakia*] (1967); Costabel, "L'affaire Maupertuis-König et les 'questions de fait'" (1979).

[5] [Voltaire], *Response d'un académicien de Berlin à un académicien de Paris* (18 September 1752); [Frederick the Great], *Lettre d'un académicien de Berlin à un académicien de Paris* (11 November 1752).

[6] Brunet, *Maupertuis I. Étude biographique* (1929), 157.

[7] "si vos ouvrages méritent qu'on vous érige des statues, votre conduite vous mériterait des chaînes" (cited in: Voltaire, *Mélanges*, 1439).

INTERMEZZO
THE MATHEMATICAL FORMULATION OF THE PRINCIPLE OF LEAST ACTION

To give the reader some sense of the substance of the principle of least action, without having to assume any great knowledge of its original or contemporary context, I will introduce the principle in the mathematical form given to it by Euler and Lagrange. They understand it, in formal terms, as the principle that of all the possible mechanical changes, the change that actually occurs is that for which a certain quantity, called the 'action', is a minimum.

What does this mean for a simple example such as the trajectory of a ball that is thrown into the air? Or in more general terms, what does it mean for a moving particle that is subject to external central forces such as gravity (see Figure 5.1)

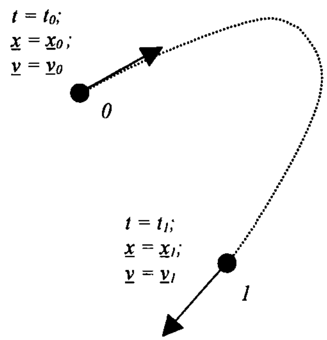

Figure 5.1 A particle in motion under the influence of external central forces

Suppose that the particle begins to move with an initial speed $\underline{v}_0$ at point $\underline{x}_0$ and time t_0. At time t_1 it will arrive at point $\underline{x}_1$, moving at speed $\underline{v}_1$. Now, it is possible to calculate the path it has traveled, point by point, on the basis of the forces at work and the initial conditions. We then determine the continuous changes that result from these two factors, by regarding the force as a succession of infinitesimal actions, and calculating their net effect by integration. But we can also calculate the path based on a principle of the *whole*. That is, for all possible paths, we determine the integral of a function of the characteristics of that path, known as the action. This integral is then minimal for the actual path taken.

In the case given, this integral has the form $_0\int^1 m\underline{v}d\underline{s}$, where *0* and *1* refer to the initial and final situations. However it will be clear that this principle entails some conditions regarding the possibility of variations. These are in part mathematical conditions, such as that the variations should be zero in the beginning and end situation. There are also physical conditions, such as that the conservation of mv^2 applies in all variations. Both conditions can already be found implicitly in Euler, and they were explicitly formulated by Lagrange (*Méchanique analitique*, 274-281).

For the sake of completeness it should be noted here that the conditions as found in Euler and Lagrange are not necessary. Nowadays the principle is presented in the more general formulation of Hamilton: $S=\int £(q, dq/dt, t)dt$ is a minimum. Here £ is the Lagrangian function, S the action, q a generalized coordinate and t is the time. For a mathematically-based introduction to the principle, see Whittaker, *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies* (1904), 245-262; and for a more popular approach see d'Abro, *The Rise of the New Physics* I (1939), 256-268, or Feynman, *The Feynman Lectures on Physics* (1961–1963) I.26, 1-8 and II.19, 1-14.

5.1.2 *The Significance of the Principle of Least Action for Mechanics*

Voltaire's ridicule of Maupertuis's arrogant attitude may have been the basis of historians' often cynical attitude to Maupertuis's claim to a scientific reputation. But when we remove the remainders of the web that Voltaire wrapped around his prey, we can see that Maupertuis's principle of least action is not without interest, and that it has considerable significance for the science of mechanics. The historical significance of the principle lies in the first place in its being the first successful attempt to gain new insights in mechanics from an approach based on final causes. While such an approach was fiercely disputed in mechanical philosophy, it was nevertheless a common element in the natural sciences of the eighteenth century. However, teleological considerations were generally limited to reflections in natural theology. Except in optics, they were little used in the scientific theories proper. Maupertuis succeeded, by means of his principle, in making room for teleology in mechanics.

The principle was to suffer a remarkable fate after it was brought into the world by Maupertuis and Euler. Although the principle in all its variations and developments has borne and still bears the name that they chose to express its teleological character, this teleological character has been just as consistently denied, almost from the beginning. It has been recognized only as a mathematical principle. This is another reason why the principle of least action occupies a special place among eighteenth century teleological views.

Apart from the fact that it gave the physicists of the time an opportunity to vent their bile over metaphysics, the principle initially had no great significance for mechanics, not any more than for physics in general. Limiting conditions were formulated and the formulation itself was refined, but the principle was generally regarded as superfluous. As early as 1800 the principle seemed to have been forgotten. Hegel wrote nothing about this principle in his philosophy of nature, although he was well informed about the state of physics at that time, and the principle would have fitted consistently into his speculative dialectic philosophy, because of its unifying character.

Nevertheless the principle had a belated flowering, beginning around 1860, with Thomson and Tait in England and Helmholtz in Germany.[8] The principle seemed to develop into a universal overarching principle, from which all fundamental physical laws such as the laws of the conservation of energy and momentum and the equations of motion could be derived. But the revolutionary storms around the turn of the century prevented further growth. While it survived the theory of relativity with

[8] Thomson and Tait, *Treatise on Natural Philosophy* (1867). They predicted a great future for the principle, not only in dynamics, but also in "several branches of physical science now beginning to receive dynamical explanations" (*ibidem*, 337). In Germany, Hermann von Helmholtz extended the principle for domains beyond dynamics, such as electrodynamics, and pointed to the importance of the principle for all reversible natural phenomena (see Chapter 1, note 67).

flying colors,[9] in quantum mechanics its significance is reduced to providing the transition to classical mechanics.[10]

Although this later history does not concern us here, it is important to observe that, from this period on, the quantity of action became a purely mathematical magnitude, which could not really be visualized in the way that it was possible, up to a certain point, to visualize concepts such as mass, force and energy. In so far as action had any physical meaning, this could only be known through its mathematical expression.

As has been said, the situation at the beginning of the principle's development was different. In the years 1740–1750, when the principle was explicitly formulated by Maupertuis, Euler, König and some others, a physical and even metaphysical significance was attributed to it: the concept was closely linked to the action of forces. For example, the physical meaning of the concept of action was manifest in attempts to provide an ontological foundation for the concept and the associated minimal principle in particular situations. At that time the principle was part of a metaphysical debate—that to us counts as outmoded, but which must be seen as the historical basis of also the modern effort to find minimal principles in physics.

What concerns me here is mainly to clarify this physical and metaphysical issue, and to examine the significance of the principle for the development of mechanics. Events at the Berlin Academy between 1740 and 1751 were central to the physical and metaphysical issue. During that period a great deal of attention was given to the metaphysical and physical aspect of the principle, with Maupertuis and Euler playing key roles.

A new phase began in 1751, when König wrote an article in the famous Leipzig journal *Acta eruditorum*, in which he not only demonstrated that Maupertuis's

[9] In 1915 Max Planck even claimed that the principle "deserves the highest place among all physical laws" ("unter allen physikalischen Gesetzen die höchste Stelle einzunehmen geeignet ist"), because the principle, at least in its Hamiltonian form, has the characteristic of being Lorentz-invariant (Planck, "Das Prinzip der kleinsten Wirkung," 701).

[10] Modern handbooks generally do not deal with the principle explicitly. However, this need not mean that the principle cannot have any overarching significance. The principle is given a remarkably positive treatment in Landau and Lifshitz's series *Course in Theoretical Physics*. In the volume on *Mechanics* they regard the principle, in the form that Hamilton later gave it, as "[t]he most general formulation of the law governing the motion of mechanical systems" (—, *Mechanics* (^{2}1969), 2). The principle has the same status in their treatment of relativistic mechanics (—, *The Classical Theory of Fields* (1989=^{4}1975), 24). However, in other fields they do not mention the principle. It is only when they come to non-relativistic quantum mechanics that they are able to assign it any significance. This is in the approximation of the point of transition from quantum to quasi-classical mechanics, in which the quantity of action S is said to be proportional to the phase φ in the wave function $\psi = ae^{i\varphi}$, where Planck's constant, $\hbar = 1.054 \cdot 10^{-27}$ erg sec, is the constant of proportionality, so that $\psi = ae^{iS/\hbar}$ (—, *Quantum Mechanics; A Shorter Course* II, 24-26). However, Kemble considers that this 'action function' is not identical to that in the principle of least action, because the latter depends on the path whereas the former does not (Kemble, *The Fundamental Principles of Quantum Mechanics* (1937), 44). In recent years there has been renewed interest in the principle. It is accorded an important position in a modern textbook for relativistic quantum field theory: "it leads to the path integral formulation of quantum mechanics" (Wit and Smith, *Field Theory in Particle Physics* I (1986), 1).

http://www.springer.com/978-1-4020-0233-5

What was Mechanical about Mechanics
The Concept of Force between Metaphysics and
Mechanics from Newton to Lagrange
Boudri, J.C.
2002, XVI, 280 p., Hardcover
ISBN: 978-1-4020-0233-5