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NAÏVE PROBABILITY

The theory of mental models and extensional probability

Suppose that someone tells you: "If the director is in the office, then her secretary is in the office too". You start to think about the different possibilities compatible with the conditional. You think of the possibility of the director in the office, and so her secretary is in the office too. You think about what happens if the director is not in the office: in one possibility, the secretary is in the office; in another possibility, the secretary is not in the office, either. You have envisaged the three possibilities that are compatible with the truth of the conditional assertion, which we summarize as follows, using "¬" to denote negation:

Director in office	Secretary in office
¬ Director in office	Secretary in office
¬ Director in office	¬ Secretary in office

Following philosophers and logicians, we refer to such possibilities as the "extensions" of the conditional assertion, i.e., possibilities to which it refers. And when individuals infer probabilities by considering the extensions of assertions, we shall say that they are reasoning *extensionally.*

You can tackle the same problem in a different way. You know that directors are unlikely to spend as much time in the office as their secretaries. This stereotype may have occurred to you as you were thinking about the problem, and you might have based your inference on it. When you think in this way, you do not consider the extensions of assertions, but rather you use some index – some evidence or knowledge – to infer a probability. We use "non-extensional" as an umbrella term to cover the many ways in which people can arrive at probabilities without thinking about extensions. Of course, you might think about a problem both extensionally and non-extensionally.

M.C. Galavotti (ed.), Observation and Experiment in the Natural and Social Sciences, 43-55.

Given a problem about a set of events, you can consider its partition, that is, the exhaustive set of possible conjunctions of individual events. In the problem about the director and the secretary, there are four such possibilities, which comprise this "partition" for the problem:

Director in office	Secretary in office
Director in office	¬ Secretary in office
¬ Director in office	Secretary in office
¬ Director in office	¬ Secretary in office

Once you know the probabilities for each possibility in a partition, you know everything that is to be known from a probabilistic standpoint. So let us introduce some probabilities, which for convenience we state as chances out of a hundred:

		Chances
Director in office	Secretary in office	50
Director in office	¬ Secretary in office	0
¬ Director in office	Secretary in office	30
¬ Director in office	¬ Secretary in office	20

You can now deduce the probability of any assertion about the domain, including conditional probabilities, such as:

> The probability that the director is not in the office given that the secretary is in the office: 30/80

The mental model theory postulates that each mental model represents a possibility, and that its structure and content capture what is common to the different ways in which the possibility might occur. For example, when individuals understand that either the director or else the secretary is in the office, but not both, they construct two mental models to represent the two possibilities:

director

secretary

where each line represents an alternative model, "director" denotes a model of the director in the office, and "secretary" denotes a model of the secretary in the office. Likewise, a conjunction, such as:

The director is in the office and the secretary is in the office

has only a single mental model:

director secretary

Granted that individuals construct mental models to represent the possibilities described in assertions, they can reason by formulating a conclusion that holds in their mental models, and they can test its *validity* by checking whether it holds in all possible models of the discourse. They can establish the invalidity of a conclusion by finding a counterexample, i.e., a model of the discourse in which the conclusion is false.

The theory makes a fundamental assumption, which is known as the principle of *truth:*

> Individuals represent assertions by constructing sets of mental models in which, first, each model represents a true possibility, and, second, the clauses in the assertions, affirmative or negative, are represented in a mental model only if they are true in the possibility.

Consider an exclusive disjunction in which only one of the two clauses is true:

The director is not in the office or else the secretary is in the office.

The mental models of the disjunction represent only the two true possibilities, and within them, they represent only the two clauses in the disjunction when they are true within a possibility:

¬ director

secretary

The first model represents the possibility that the director is not in the office, but it does not represent explicitly that it is false that the secretary is in the office. The second model represents the possibility that the secretary is in the office, but it does not represent explicitly that it is false that the director is not in the office (i.e. the director *is* in the office).

The mental models of conditionals are simple. For a conditional, such as:

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