

## Index

- A priori, a posteriori probability 123
- Absorbing state, 271
- Absorption probability, 301
- Absorption time, 256
- Allocation models, 54
- Almost surely, 244
- American put, 324
- Aperiodic class, 299
- Arbitrage
  - meaning of, 362
  - opportunity, portfolio, 362
- Arbitrage-free market, 363
- Area, 20, 41
- Arithmetical density, 38
- Artin, 175
- Asset
  - see “financial instrument”, 323
- Asset return
  - see “return”, 325
- Asset return distribution, 340
  - continuous compounding, 341
  - logarithmic scale, 341
  - with fat tails, 341
- Asset risk, *see* risk
- Asymptotically equal, 218
- Axioms for probability, 24
  
- Banach’s match problem, 72
- Bayes’ theorem, 123
- Bernoulli’s formula, 38
- Bernoulli, J., 235
- Bernoullian random variable, 93, 175, 187
- Bertrand’s paradox, 100
- Binomial coefficient, 51
  - generalized, 135
  - properties, 60, 197
  - properties(, 57
- Binomial distribution, 93
  
- Birth-and-death process, 295
- Birthday problem, 65
- Black–Scholes formula, 361
- Bond, 324
  - maturity date, 324
  - par value, 324
  - zero-coupon, 324
- Boole’s inequality, 31
- Borel, 99
- Borel field, 103
- Borel’s theorem, 240
- Branching process, 305
- Brownian motion, 259
- Buffon’s needle problem, 160
  
- Call option, 353
- Capital asset pricing model, 339
- Card shuffling, 314
- Cardano’s paradox, 172
- Cauchy distribution, 341, 346
- Cauchy functional equation, 160
- Cauchy–Schwarz inequality, 174
- Central limit theorem, 229
- Certificate of deposit, 324
- Chapman–Kolmogorov equations, 266
- Characteristic function, 190
  - see stable distribution, 344
- Chebyshev’s inequality, 236, 243, 358
- Chi-square distribution, 244
- Chinese dice game, 72
- Class of states, 271
- Class property, 278
- Coin-tossing scheme, 36
- Communicating states, 271
- Complement, 3
- Conditional expectation, 128
  - filtration, 371
  - tower property, 370

Index **393**

- Conditional probability, 115
  - basic formulas, 115–122
- Contingent claim, 353
- Contingent claim (see also “option”, “financial derivative”), 353
- Convergence of distributions, 229
- Convolution, 186, 197
- Coordinate variable, 75
- Correlation, 175
- Countable additivity, 32
- Countable set, 23
- Coupon collecting problem, 164
- Covariance, 175
- Cramér, 231
- Credibility of testimony, 157
  
- D’Alembert’s argument, 27, 53
- De Méré’s dice problem, 143
- De Moivre–Laplace theorem, 223
- De Morgan’s laws, 7
- Density function, 94
- Derivative security, 353
- Dice patterns, 72
- Difference, 8
- Difference equations, 253
- Discount bond, *see* “bond, zero-coupon”
- Discount rate, 357
- Discrete, 95
- Disjoint, 10
- Distribution function, 84, 95, 105
  - stable, 342–344
- Diversification, *see* portfolio diversification
  - misfortunes with lack of, 340
- Dividend, *see* “stock dividend”
- Doob, 320
- Doubling the bet, 195
- Doubly stochastic matrix, 295
- Duration of play, 288
  
- Efficient frontier, 335, 336
- Ehrenfest model, 269, 297, 312
- Einstein, 127
- Elementary probabilities, 85
- Empty set, 2
- Enron, 340
- Equally likely, 21, 26, 34
- Equity-type securities, 323
  
- Ergodic theorem, 241
- Errors in measurements, 172
- European option price, 361
- Event, 26, 34
- Exchangeable events, 138
- Expectation, 86, 114, 161
  - addition theorem, 162, 166
  - approximation of, 97
  - expression by tail probabilities, 193, 194
  - multiplication theorem, 170
  - of function of random variable, 88, 96
- Expected return time, 288
- Exponential distribution, 101
  - memoryless property, 119, 160
  
- Factorial, 50
- Favorable to, 146
- Feller, 72, 112, 241
- Fermat–Pascal correspondence, 29, 143
- Financial derivative, 324, 355
  - equity-type, 324
  - see also* derivative security, 355
- Financial instrument, 323
  - equity-, debt-type, 323
- Finite additivity, 31
- First entrance time, 273
  - decomposition formula, 276
- Fourier transform, 190
- Frequency, 21, 241
- Fundamental rule (of counting), 45
  
- Gambler’s ruin problem, 253, 257
- Gamma distribution, 196
- Gauss–Laplace distribution, 224
- Generating function, 183
  - as expectation, 189
  - multiplication theorem, 187, 190
  - of binomial, 187
  - of geometric, 188
  - of negative binomial, 188
- Genetical models, 150, 304, 313
- Genotype, 150
- Geometrical distribution, 91
- Geometrical probability problems, 98, 99
- Gross return, 326

**394** Index

- Hardy–Weinberg theorem, 152
- Hereditary problem, 153
- Holding time, 207
- Homogeneous Markov chain, *see* “Markov chain”
- Homogeneity, 210
  - in space, 268
  - in time, 210, 263
- Homogeneous chaos, 217
- Identically distributed, 230
- Independent events, 36, 140
- Independent random variables, 139, 141
- Indicator, 13, 168
- Infinitely often, 258, 280
- Initial distribution, 264
- Insider trading, 369
- Integer-valued random variable, 88
- Intensity of flow, 207
- Interarrival time, 167, *see also* “waiting time”
- Intersection, 4
- Joint density function, 105
- Joint distribution function, 107
- Joint probability distribution, 105
- Joint probability formula, 121
- Keynes, 118, 126, 133, 358
  - and short-term investors, 339
- Khinchine, 236
- Kolmogorov, 143
- Lévy, 231, 259
- Laplace, 123, *see also under* “De Moivre” and “Gauss”
  - law of succession, 127
- Laplace transform, 190
- Last exit time, 275
  - decomposition formula, 276
- Law of large numbers, 235
  - J. Bernoulli’s, 235
  - strong, 240
- Law of small numbers, 204
- Leading to, 271
- Limited liability, 325
- Loan
  - interest, 324
  - principal, 324
- Lognormal distribution, 346, 347
- Long position, 332
- Lottery problem, 163
- Marginal density, 107
- Marginal distribution, 105
- Markov, 236, 262
- Markov chain, 263
  - examples, 267–271
  - nonhomogeneous, 263, 270
  - of higher order, 318
  - positive-, null-recurrent, 295
  - recurrent-, nonrecurrent, 284
  - reverse, 318
  - two-state, 293
- Markov property, 263
  - strong, 282
- Markowitz, 331
- Martingale, 319
  - discounted stock price process as, 360, 365
- Matching problems, 66, 168, 176
- Mathematical expectation, *see* “expectation”
- Maximum and minimum, 145
- Mean-variance optimization
  - definition, 331–332
  - effect of riskless security, 337–339
  - equilibrium, 339
  - risky assets example, 332–335
  - risky assets generalization, 335–337
- Measurable, 25, 113
- Median, 112
- Moments, 172
- Money market instrument, 324, 328
- Montmort, 198
- Multinomial coefficient, 52
- Multinomial distribution, 178, 179
- Multinomial theorem, 177
- Multiperiod model, 326
  - dynamic replication, 367
  - European option price, 369
  - horizon, 326
  - self-financing strategy, 367
  - successive returns, 326
- Multiperiod portfolio strategy, 369
- Mutual fund, 339

Index **395**

- Negative binomial distribution, 188
- Neyman-Pearson theory, 157
- Non-Markovian process, 271
- Nonhomogeneous Markov chain, 263, 270
- Nonmeasurable, 40
- Nonrecurrent, 278, *see also* under “recurrent”
- Normal distribution, 224
  - convergence theorem, 230
  - moment-generating function, moments, 226
  - positive, 244
- Normal family, 227
- Null-recurrent, 295
- Numéraire invariance principle, 366–367
- Occupancy problems, 192, *see also* “allocation models”
- Occupation time, 288
- One-period model, 326
  - European option price, 363
- Option, 353
  - 1-period model, 361–367
  - American, 353
  - as insurance, 354, 359
  - Black-Scholes formula, 361
  - buyer/holder of, 356
  - call, 353
  - European, 353
  - exercise, strike price, 353
  - exotic, 354
  - expiration/maturity date, 353
  - Fundamental pricing theorems, 371
  - multi-period model, 367–372
  - payoff, 355
  - premium, 361
  - price, 356
  - pricing probability, 365
  - put, 353
  - standard, 354
  - underlying security, 353
  - writer/seller of, 356
- Optional time, 281
- Ordered  $k$ -tuples, 46
- Pólya, 133, 231, 270
- Pairwise independence, 147
- Pareto, 349
- Pareto distribution, 346, 349
- Partition problems, 55
- Pascal’s letters to Fermat, 29, 143
- Pascal’s triangle, 58
- Permutation formulas, 50–52
- Persistent, *see* recurrent
- Poincaré’s formula, 168
- Poisson, 133
- Poisson distribution, 199, 211
  - models for, 204–206
  - properties, 214–216
- Poisson limit law, 202
- Poisson process, 212
  - distribution of jumps, 217
  - finer properties, 244
- Poisson’s theorem on sequential sampling, 133
- Poker hands, 71
- Portfolio
  - allocation, 329
  - diversification, 330
  - multi-period, 369
  - return, 329
  - risk, 329
  - weight, 329
- Portfolio frontier, 335
- Position
  - long, 332
  - short, 332
- Positive-recurrent, 295
- Pricing probability, 365
  - equivalent, 365
- Probability (classical definition), 24
- Probability distribution, 85
- Probability measure, 24
  - construction of, 34
- Probability of absorption, 301
- Probability of extinction, 307
- Problem (for other listings *see* under key words)
  - of liars, 157
  - of points, 28, 197
  - of rencontre, 168
  - of sex, 119
- Put option, 353
- Put-call parity, 373

**396** Index

- Quality control, 62
- Queuing process, 315–316
- Random mating, 150
- Random variable, 77, 113
  - continuous, 95
  - countable vs. density case, 96
  - discrete, 95
  - function of, 78
  - range of, 84
  - with density, 95
- Random vector, 75, 105
- Random walk, 250
  - free, 267
  - generalized, 268–269
  - in higher dimensions, 270, 285
  - on a circle, 294
  - recurrence of, 257, 284–285
  - with barriers, 268
- Randomized sampling, 216
- Rate of return
  - see “return”, 326
- Recurrent, 278, 280
  - Markov chain, 284
  - random walk, 258
- Renewal process, 313
- Repeated trials, 35
- Replicating strategy, 363
- Return, 325, 326
  - annualization, 326, 327
  - compounding effect, 327
  - continuous compounding, 341
  - distribution, 340
  - distribution with fat tails, 341
  - gross, 326
- Riemann sums, 97
- Risk, 328
  - definition, 328
  - lack of, 328
- Risk–return tradeoff, 331
- Risk-neutral probability, *see* pricing probability
- Riskless security, 328
- Sample function, 212
- Sample point, space, 2
- Sampling (with or without replacement)
  - vs. allocating, 55
  - with ordering, 48
  - without ordering, 50–52
- Sequential sampling, 129
- Sharpe, 339
- Sharpe ratio, 387
- Short position, 332
- Significance level, 234
- Simpson’s paradox, 148
- Size of set, 2
- St. Petersburg paradox, 111, 321
- Stable distribution, 342–344
  - characteristic function, 344
  - Lévy’s characterization, 345
- Stable distribution type, 343
- Stable law, *see* stable distribution
- Standard deviation, 172
- State of the economic world, 325
- State space, 262
- Stationary distribution, 292
- Stationary process, 139, 153, 292
- Stationary transition probabilities, 263
- Steady state, 287
  - equation for, 290
- Stirling’s formula, 219, 247
- Stochastic independence, *see* independent events, random variables
- Stochastic matrix, 266
- Stochastic process, 129, 213
  - stock price evolution as, 360
- Stochastically closed, 299
- Stock dividend, 323
- Stopping time, 281
- Strong law of large numbers, 240
- Strong Markov property, 282
- Submartingale, 357
  - discounted stock price process as, 360
  - expectation under, 358
- Summable, 161
- Supermartingale, 357
  - discounted stock price process as, 360
  - expectation under, 358
  - in example of greed, 358
- Symmetric difference, 9
- Symmetric distribution, 345

Index **397**

Taboo probabilities, 275, 317  
Tauberian theorem, 288  
Time parameter, 129  
Tips for counting problems, 61  
Total probability formula, 122  
Transient, *see* “nonrecurrent”  
Transition matrix, 266  
Transition probability, 262, 266  
    limit theorems for, 288, 299  
Tulipmania, 356  
  
Uniform distribution, 89, 99  
Union, 4  
  
Variance, 172  
    addition theorem, 173  
  
Waiting time, 91, 101, 188  
Wald’s equation, 91  
Wiener process, 259  
  
Zero-or-one law, 309



<http://www.springer.com/978-0-387-95578-0>

Elementary Probability Theory  
With Stochastic Processes and an Introduction to  
Mathematical Finance

Chung, K.L.; AitSahlia, F.

2003, XIV, 404 p., Hardcover

ISBN: 978-0-387-95578-0