Preface

Our goal in this book is to explore some of the connections between control theory and geometric mechanics; that is, we link control theory with a geometric view of classical mechanics in both its Lagrangian and Hamiltonian formulations and in particular with the theory of mechanical systems subject to motion constraints. This synthesis of topics is appropriate, since there is a particularly rich connection between mechanics and nonlinear control theory. While an introduction to many important aspects of the mechanics of nonholonomically constrained systems may be found in such sources as the monograph of Neimark and Fufaev [1972], the geometric view as well as the control theory of such systems remains largely scattered through various research journals. Our aim is to provide a unified treatment of nonlinear control theory and constrained mechanical systems that will incorporate material that has not yet made its way into texts and monographs.

Mechanics has traditionally described the behavior of free and interacting particles and bodies, the interaction being described by potential forces. It encompasses the Lagrangian and Hamiltonian pictures and in its modern form relies heavily on the tools of differential geometry (see, for example, Abraham and Marsden [1978] and Arnold [1989]). From our own point of view, our papers Bloch, Krishnaprasad, Marsden, and Murray [1996], Bloch and Crouch [1995], and Baillieul [1998] have been particularly influential in the formulations presented in this book.

Control Theory and Nonholonomic Systems. Control theory is the theory of prescribing motion for dynamical systems rather than describing their observed behavior. These systems may or may not be mechanical in nature, and in fact traditionally, the underlying system is not assumed to be mechanical. Modern control theory began largely as linear theory, having its roots in electrical engineering and using linear algebra, complex variable theory, and functional analysis as its principal tools. The nonlinear theory of control, on the other hand, relies to a large extent again on differential geometry.

Nonholonomic mechanics describes the motion of systems constrained by nonintegrable constraints, i.e., constraints on the system velocities that do not arise from constraints on the configurations alone. Classic examples are rolling and skating motion. Nonholonomic mechanics fits uneasily into the classical mechanics, since it is not variational in nature; i.e., it is neither Lagrangian nor Hamiltonian in the strict sense of the word. It has a close cousin (variational axiomatic mechanics—a term coined by Arnold, Kozlov, and Neishtadt [1988]); which is variational for systems subject to nonintegrable constraints but does not describe the motion of mechanical systems. It is important, however, for the theory of optimal control, as will be developed in the main text.

There is a close link between nonholonomic constraints and controllability of nonlinear systems. Nonholonomic constraints are given by nonintegrable distributions; i.e., taking the bracket of two vector fields in such a distribution may give rise to a vector field not contained in this distribution. It is precisely this property that one wants in a nonlinear control system so that we can drive the system to as large a part of the state space as possible.

A key concept for studying the control and geometry of nonholonomic systems, as well as many other mechanical systems, is the notion of a fiber bundle and an associated connection. The bundle point of view not only gives us a way of organizing variables in a physically meaningful way, but gives us basic ideas on how the system behaves physically, and on how to prescribe controls. A bundle connection relates base and fiber variables in the system, and in this sense one can take a gauge theoretic point of view of nonholonomic control systems.

Optimal Control. There is a beautiful link between optimal control of nonholonomic systems and so-called sub-Riemannian geometry. For a large class of physically interesting systems, the optimal control problem reduces to finding geodesics with respect to a singular (sub-Riemannian) metric. The geometry of such geodesic flows is exceptionally rich and provides guidance for designing control laws. See Montgomery [2002] for additional information.

Physical Examples. One of the aims of this book is to illustrate the elegant mathematics behind many simple, interesting, and useful mechanical examples. Among these are the rigid body and rolling rigid body, the rolling ball on a rotating turntable, the rattleback top, the rolling penny, and the satellite with momentum wheels. There are clearly a number of points in common between these systems, among them the fact that rotational motion and the existence of constraints, either externally imposed or dynamically generated (conserved momenta), play a key role. In one sense these notions—rotation and constraints—form the heart of the book and are vital to studying both the dynamics and control of these systems. Further, one of the delights of this subject is that although these systems may have many features in common, their behavior is quite different and often quite unexpected. Why does a rattleback top rotate in only one direction? What is the behavior of a ball on a rotating turntable? Why does a tennis racket not want to spin about its middle axis? How do I roll a penny to a particular point on a table, parallel to an edge, and with Lincoln's head in the upright position?

While we have attempted to cover a substantial amount of material, this book is very much written from the authors' perspective, and there is much fascinating work in this area that we have had to omit.

A Path Through the Book. This book can be read on many different levels. On the one hand, there are numerous physical examples that are analyzed in elementary terms, as in Chapter 1. On the other hand, there are theoretical sections that use some sophisticated analysis and geometry. There are also sections on the background mathematics used, and our hope is that this book mixes these ingredients in an instructive and useful way. Depending on one's background and preferences, this book can be read in a linear or nonlinear fashion (in the sense of progression through the pages).

Many of the examples are returned to later in the book as illustrations of the general theory. These later returns to the examples vary in difficulty from again quite elementary to more sophisticated demonstrations of the theory. We urge the reader to use them to understand the theory.

The theory itself varies greatly in difficulty—some of it is again quite elementary and easy to read—usually at the beginning of each section or chapter, but some of it quite technical and based on various pieces of the research work of the authors and sometimes their collaborators. Many technical sections may be omitted on first reading or without loss of continuity; we also refer the reader to the Internet supplement for additional material. This is available on the book's web site, where errata, reprint data, and other information may be found:

`http://www.cds.caltech.edu/mechanics_and_control`

We have gone to some trouble to fill in the necessary background for the general theory and to put in elementary illustrations of it. For example, we discuss the theory of connections and the geodesic flow on the line.

Scope of the Book. We should also emphasize that while this book cuts quite a large swath through an area of mechanics and nonlinear control, it is very much mechanics and control as seen by the authors. There is a huge and exciting literature on mechanics, nonholonomic mechanics, nonlinear control, and optimal control that we have not discussed at all and indeed have not even been able to reference in many cases. We urge the reader to follow up on related areas both through the references that are here and through references in those papers.

Prerequisites. This book is intended for graduate students who wish to learn this subject as well as for researchers in the area who wish to enhance their techniques. Chapter 1 is written in a way that assumes rather few prerequisites and is intended to motivate people to read further and to acquire the needed background for that task. Chapters 2, 3, and 4 contain some of the needed background in geometry, mechanics, and control. They are necessarily brief in nature and are meant, in part, to summarize topics that are treated in other courses. There is, however, new expository material on various topics that we describe in more detail below. In addition, we have collected material that is hard to find in any one source. From Chapter 5 on, we assume that the reader is knowledgeable about these topics.

A knowledge of basic mechanics, as in the well-known book of Goldstein [1980], is helpful, although we do in fact develop both Lagrangian and Hamiltonian mechanics as well as the theory of control from first principles. So for a reader who knows nothing of these fields but has the usual dose of "mathematical sophistication" it is quite possible to read and benefit from this book. Similarly, it it not necessary to know anything about nonholonomic mechanics.

Some parts of Chapters 2 and 3 are based on *Mechanics and Symmetry* (Marsden and Ratiu [1999]) and can be skipped by the knowledgeable reader or consulted as the need arises. Another piece of useful background is the recently published Beijing lecture notes of Roger Brockett (see Brockett [2000]), whose spirit certainly pervades much of the nonlinear control theory in this book. Similarly, the collection *Mathematical Control Theory*, written in honor of Roger Brockett's 60th birthday, is a useful adjunct (see Baillieul and Willems [1999]).

This book can be viewed as somewhere between a research monograph and a textbook. It has been successfully used as a textbook for courses at Caltech and Michigan as well as for lecture series at the Technical University of Vienna and the IIIe Cycle Romand de Mathématique, Les Diablerets, Switzerland. In this regard there are a number of exercises, particularly in the earlier chapters, meant to help readers gauge their understanding, but this is not a main focus of the book.

A Brief Rundown of the Chapters in This Book. We will now give a brief synopsis of the various chapters in the book. Specific citations to the works of authors that are mentioned here are given in the main text.

Chapter 1 consists of a little preliminary mechanics but mainly of examples that are used later in the book. Key examples include the vertical and falling rolling disks and various versions of a skate on ice as well as the rolling ball. More complicated examples include the roller racer and rattleback top. There are also mechanical examples that are holonomic but that are used later to illustrate basic Lagrangian and Hamiltonian mechanics and control. These include the Toda lattice, the free and controlled rigid body, and the pendulum on a cart.

Chapter 2 is devoted to various mathematical preliminaries and can be used as desired according to the mathematical expertise of the reader. It goes all the way from the basic theory of manifolds and ordinary differential equations to the theory of Ehresmann connections. The most original part and that which is least likely to be familiar to readers is that on connections. We have gone to a great deal of trouble here to analyze how Ehresmann connections specialize to principal connections and to affine connections and Riemannian connections. These ideas are also illustrated with very simple examples such as the geodesic flow on the line! Connections play a vital role in mechanics and in particular nonholonomic mechanics, where they arise from constraints.

Chapter 3 gives general background in geometric mechanics, and parts of it can again be skipped by the knowledgeable reader. There are, however, new things here: a new exposition of the theory of forces; a description of the Murray-Ostrowski view of mechanical systems and the mechanical connection, together with the example of the spacecraft with rotors; a detailed description of coupled planar rigid body motion as developed by Oh, Sreenath, Krishnaprasad, and Marsden; and a description of phases and holonomy as developed by Marsden and Ostrowski.

Chapter 4 gives general background in nonlinear control theory including basic definitions of controllability and accessibility, some theory on averaging and motion planning (including work of Leonard and Krishnaprasad), a proof of Brockett's necessary condition for stabilization, and some of the theory of Hamiltonian and Lagrangian control systems following work of Brockett, van der Schaft, Willems, and others.

Chapter 5 is the basic chapter on nonholonomic mechanics and owes much in exposition to the paper of Bloch, Krishnaprasad, Marsden, and Murray as well as to work of Bloch and Crouch. We discuss the basic geometric approach in these papers. The basic interaction of symmetries and constraints in nonholonomic systems and how they lead to the nonholonomic momentum equation is discussed. Explicit examples of the momentum map are given. In addition the role of "almost" Hamiltonian structure is discussed, building on work of Bates and Sniatycki, van der Schaft and Maschke, as well as that of Marsden and Koon.

Chapter 6 discusses various aspects of control and stabilization of nonholonomic systems both for kinematic and dynamic systems. Open loop controls are discussed for the Brockett canonical form following Murray and Sastry, and its discontinuous stabilization is discussed based on the work of Bloch, Drakunov, and Kinyon. The Coron approach to smooth time-varying stabilization is also briefly discussed. Following the work of Bloch, Reyhanoglu, and McClamroch, control and stabilization of dynamic nonholonomic control systems is described. Control of nonholonomic systems on Riemannian manifolds is discussed following work of Bloch and Crouch.

Chapter 7 is devoted to optimal control. It begins by discussing the relationship of variational nonholonomic control systems and the classical Lagrange problem to optimal control. A brief introduction to the maximum principle is given. We then discuss sub-Riemannian (kinematic) optimal control problems based on the work of Bloch, Crouch, and Ratiu and building on the work of Brockett and Baillieul. We give a brief discussion of abnormal extremals following work of Montgomery and Sussmann. Dynamic optimal control is discussed following work of Bloch and Crouch and Silva, Leite, and Crouch. Related work on integrable systems is discussed in the internet supplement.

Chapter 8 discusses an energy–momentum-based approach to the stability of nonholonomic systems. This is based on the thesis work of Zenkov and related work with Bloch and Marsden. Also described are notions of asymptotic stability in Euler–Poincaré–Suslov systems following work of Kozlov and its connection to the Toda lattice following work of Bloch.

Chapter 9 discusses some recent and still developing research on energy-based techniques for mechanical and nonholonomic systems. A brief description of the controlled Lagrangian or matching technique of Bloch, Leonard, and Marsden is given with some recent applications to certain nonholonomic systems based on work with D. Zenkov. Finally, work of Baillieul is described on second-order averaging methods and their connections with classical geometry.

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