

Contents

Preface	vii
1 Introduction	1
1.1 The classical partial differential equations	1
1.2 Well-posed problems	3
1.3 The one-dimensional wave equation	5
1.4 Fourier's method	9
2 Preparations	15
2.1 Complex exponentials	15
2.2 Complex-valued functions of a real variable	17
2.3 Cesàro summation of series	20
2.4 Positive summation kernels	22
2.5 The Riemann–Lebesgue lemma	25
2.6 *Some simple distributions	27
2.7 *Computing with δ	32
3 Laplace and Z transforms	39
3.1 The Laplace transform	39
3.2 Operations	42
3.3 Applications to differential equations	47
3.4 Convolution	53
3.5 *Laplace transforms of distributions	57
3.6 The Z transform	60

3.7	Applications in control theory	67
	Summary of Chapter 3	70
4	Fourier series	73
4.1	Definitions	73
4.2	Dirichlet's and Fejér's kernels; uniqueness	80
4.3	Differentiable functions	84
4.4	Pointwise convergence	86
4.5	Formulae for other periods	90
4.6	Some worked examples	91
4.7	The Gibbs phenomenon	93
4.8	*Fourier series for distributions	96
	Summary of Chapter 4	100
5	L^2 Theory	105
5.1	Linear spaces over the complex numbers	105
5.2	Orthogonal projections	110
5.3	Some examples	114
5.4	The Fourier system is complete	119
5.5	Legendre polynomials	123
5.6	Other classical orthogonal polynomials	127
	Summary of Chapter 5	130
6	Separation of variables	137
6.1	The solution of Fourier's problem	137
6.2	Variations on Fourier's theme	139
6.3	The Dirichlet problem in the unit disk	148
6.4	Sturm–Liouville problems	153
6.5	Some singular Sturm–Liouville problems	159
	Summary of Chapter 6	160
7	Fourier transforms	165
7.1	Introduction	165
7.2	Definition of the Fourier transform	166
7.3	Properties	168
7.4	The inversion theorem	171
7.5	The convolution theorem	176
7.6	Plancherel's formula	180
7.7	Application 1	182
7.8	Application 2	185
7.9	Application 3: The sampling theorem	187
7.10	*Connection with the Laplace transform	188
7.11	*Distributions and Fourier transforms	190
	Summary of Chapter 7	192

8 Distributions	197
8.1 History	197
8.2 Fuzzy points – test functions	200
8.3 Distributions	203
8.4 Properties	206
8.5 Fourier transformation	213
8.6 Convolution	218
8.7 Periodic distributions and Fourier series	220
8.8 Fundamental solutions	221
8.9 Back to the starting point	223
Summary of Chapter 8	224
9 Multi-dimensional Fourier analysis	227
9.1 Rearranging series	227
9.2 Double series	230
9.3 Multi-dimensional Fourier series	233
9.4 Multi-dimensional Fourier transforms	236
Appendices	
A The ubiquitous convolution	239
B The discrete Fourier transform	243
C Formulae	247
C.1 Laplace transforms	247
C.2 Z transforms	250
C.3 Fourier series	251
C.4 Fourier transforms	252
C.5 Orthogonal polynomials	254
D Answers to selected exercises	257
E Literature	265
Index	267



<http://www.springer.com/978-0-387-00836-3>

Fourier Analysis and Its Applications

Vretblad, A.

2003, XII, 272 p., Hardcover

ISBN: 978-0-387-00836-3