Preface

In mathematical modeling of processes occurring in industrial systems, logistics, management science, operations research, networks, and control theory one often encounters optimization problems involving more than one objective function, so that multiobjective optimization (or vector optimization, initiated by W. Pareto) has received new impetus. The growing interest in multiobjective problems, both from the theoretical point of view and as it concerns applications to real problems, asks for a general scheme that embraces several existing developments and stimulates new ones. With this book we intend to give direct access to new results and new applications of this quickly growing field.

Mathematical Background

In particular, we discuss basic tools of partially ordered spaces and apply them to variational methods in nonlinear analysis and for optimization problems; i.e., we present the relevant functional analysis for our presentations, especially separation theorems for not necessarily convex sets, which are important for the characterization of solutions, for the proof of existence results and optimality conditions in multicriteria optimization. We use the optimality conditions in order to derive numerical algorithms for special classes of vector optimization problems.

Purpose of This Book

We believe that our book will be of interest to graduate students in mathematics, economics, and engineering as well as researchers in pure and applied mathematics, economics, engineering, geography, and town planning. A sound knowledge of linear algebra and introductory real analysis should provide readers with sufficient background for this book.

On the one hand, the book has the character of a monograph, because the authors use many of their own results and applications; on the other hand, it is a textbook, because we would like to present in a sense a state of the art of the field in an understandable, useful, and teachable way.

Organization

Firstly, we shall give some simple examples to show which kinds of problems can be handled with the methods of the book. Then the three main chapters follow.

In the first of them we deal with connections between order structures and topological structures of sets, give a new nonconvex separation theorem, which is very useful for scalarization, and study different properties of multifunctions.

The second of them contains our results concerning the theory of multicriteria optimization and equilibrium problems directly. Approximate efficiency, scalarization, new existence results with respect to different order relations, well-posedness, sensitivity, duality, and optimality conditions with respect to general vector optimization problems and a big section on minimal point theorems belong to this chapter as well as new results of vector equilibrium problems and their applications to vector-valued variational inequalities.

Those new theoretical results are applied in the last chapter of the book in order to construct numerical algorithms, especially proximal-point algorithms and geometrical algorithms based on duality assertions. It is possible to use the special structure of several classes of multicriteria optimization problems (location problems, approximation problems, fractional programming problems, multicriteria control problems) for deriving optimality conditions and corresponding algorithms. We discuss concrete applications (approximation problems, location problems in town planning, multicriteria equilibrium problems, fractional programming) with solution procedures and in some cases with corresponding software. Here and in the whole book there are examples to illustrate the results or to check stated conditions. The chapters are followed by a list of references, a list of symbols and a big enough index.

The book was written by four authors, we wrote it together, and it was at every time stimulating and profitable to consider the problems from different sides. A. Göpfert, Chr. Tammer, and C. Zălinescu contributed to Sections 1.1, 2.1–2.3, 3.1, and 3.10; C. Zălinescu wrote Sections 2.4–2.7 and 3.2–3.6; H. Riahi wrote Sections 3.8, 3.9, 4.2.4, and 4.2.5; and A. Göpfert and Chr. Tammer wrote Sections 1.2–1.6, 3.7, 3.11, 4.1, 4.2.1–4.2.3, and 4.3–4.6.

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