

Preface

The rapid increase of the industrial use of structures made of advanced composite materials, *e.g.*, laminated materials, has necessitated the development of new analytical tools that are suitable for the analysis and study of the mechanical behaviour of such structures.

The study of three-dimensional continua has been a traditional part of graduate and postgraduate education in solid mechanics. With rational simplifications to the three-dimensional theory of elasticity, two-dimensional engineering theories of plates and shells can be derived. These simplified theories have been successfully applied to many engineering structures composed of homogeneously isotropic materials, *e.g.*, metals. However, it has been recognised that the conventional two-dimensional approaches are sometime too conservative and incapable of giving satisfactory predictions over the full ranges of performance of composite materials, due to the strong influence of anisotropy and high level of transverse stresses, *etc.* To overcome these difficulties, various “refined” and “higher order” theories have been proposed. Unfortunately, almost all these theories are based on some form of thin plate or shell approximations, and have serious limitations in predicting interfacial stresses at material interfaces of a lamina. Thus, none of the theories seems to have gained wide acceptance.

An alternative approach, which forms the main part of this book, is to abandon all the approximations based on the idea of a thin plate or shell and seek solutions directly from the three-dimensional equations of elasticity. In this respect, significant attention has, over the last few decades, been focused on the search for efficient three-dimensional solutions for laminated composite plates and shells. These include various analytical and numerical methods, *e.g.*, stress function, series expansion and finite element methods.

It is not the intention of this book to give a complete coverage of three-dimensional elasticity. The book will focus on the introduction and review of the applications of the state space method to the three-dimensional solution of laminated plates and shells. The first few applications of this type of solution (also called the method of transfer matrix, propagator matrix or initial functions) are probably due to Vlasov (1957) and Bufler (1971) where isotropic three-dimensional bodies were considered. Applying the method to laminated plate problems, along with a recursive formulation, Fan and Ye (1990a,b) were the first to apply the method systematically to solve various composite plate and shell problems, either analytically or numerically. One of the unique features of this method is that it can provide accurate three-dimensional solutions for laminated

plates and shells that guarantee a continuous transverse stress field across the material interfaces. Numerical versions of the method, for example, the state space finite element method, inherit this unique feature such that the continuity of interface tractions at perfect interfaces is automatically satisfied. It is very difficult, computationally expensive and, sometimes, even unpractical to achieve this when a traditional three-dimensional finite element method is used. Other features of the method include that the number of unknown functions or unknown node variables is independent of the number of material layers in a lamina. This is a particularly useful feature for the analysis of multi-layered laminated composites.

With the requirement for a strong grounding in the engineering theories of plates and shells, this book is intended for those readers who possess knowledge of the theory of elasticity, solid mechanics, solution of differential equations and numerical methods. In writing this book consideration was given to the possibility of reading the individual chapters as separate units. This was partly achieved at the cost of some repetition.

The fundamental theories covered in the first three chapters have been well documented in the literature and can be found easily elsewhere. In Chapter 1 a brief introduction to the classification and properties of composites is given. Chapter 2 presents the fundamental equations of three-dimensional elasticity without proofs. The special forms of these equations in a number of commonly used co-ordinate systems are also presented. Chapter 3 is an introduction to the state space representations of linear systems. The state space concept, the solution to the state equation and some related topics are briefly discussed. Chapter 4 and 5 deal with vibration, stress and stability of laminated rectangular and circular plates, respectively. Chapters 6 and 7 are solutions to laminated shells, including cylindrical, spherical, doubly-curved shells. Chapter 8 describes the application of the method to the analysis of edge effects in laminated shells. Chapter 9 is devoted to the numerical implementation of the state space method.

This book attempts to provide a comprehensive, but by no means exhaustive, coverage of the available solutions using the state space approach. The book is meant to be of use not only for research, but also as an advanced reference. Many of the numerical results presented in the book are important for both their own interest and also as test problems for validating new numerical methods.

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